

## Miscellaneous Exercise

### Question 1:

Evaluate:  $\left[ i^{18} + \left( \frac{1}{i} \right)^{25} \right]^3$

### Solution 1:

$$\begin{aligned} & \left[ i^{18} + \left( \frac{1}{i} \right)^{25} \right]^3 \\ &= \left[ i^{4 \times 4 + 2} + \frac{1}{i^{4 \times 6 + 1}} \right]^3 \\ &= \left[ (i^4)^4 \cdot i^2 + \frac{1}{(i^4)^6 \cdot i} \right]^3 \\ &= \left[ i^2 + \frac{1}{i} \right]^3 \quad [i^4 = 1] \\ &= \left[ -1 + \frac{1}{i} \times \frac{i}{i} \right]^3 \quad [i^2 = -1] \\ &= \left[ -1 + \frac{i}{i^2} \right]^3 \\ &= [-1 - i]^3 \\ &= (-1)^3 [1 + i]^3 \\ &= -[1^3 + i^3 + 3 \cdot 1 \cdot i(1 + i)] \\ &= -[1 + i^3 + 3i + 3i^2] \\ &= -[1 - i + 3i - 3] \\ &= -[-2 + 2i] \\ &= 2 - 2i \end{aligned}$$

### Question 2:

For any two complex numbers  $z_1$  and  $z_2$ , prove that

$$\operatorname{Re}(z_1 z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

### Solution 2:

Let  $z_1 = x_1 + iy_1$  and  $z_2 = x_2 + iy_2$

$$\begin{aligned} \therefore z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= x_1 x_2 + ix_1 y_2 + iy_1 x_2 + i^2 y_1 y_2 \end{aligned}$$

$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \quad [i^2 = -1]$$

$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$

$$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$

$$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

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### Question 3:

Reduce  $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$  to the standard form

### Solution 3:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i) - 2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$

$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$

$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \quad [\text{On multiplying numerator and denominator by } (14+5i)]$$

$$= \frac{462+165i+434i+155i^2}{2[(14)^2 - (5i)^2]} = \frac{307+599i}{2(196-25i^2)}$$

$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

This is the required standard form.

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### Question 4:

If  $x-iy = \sqrt{\frac{a-ib}{c-id}}$  prove that  $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$

### Solution 4:

$$x-iy = \sqrt{\frac{a-ib}{c-id}}$$

$$= \sqrt{\frac{a-ib}{c-id} \times \frac{c+id}{c+id}} \quad [\text{On multiplying numerator and denominator by } (c+id)]$$

$$= \sqrt{\frac{(ac+bd)+i(ad-bc)}{c^2+d^2}}$$

$$\begin{aligned}\therefore (x-iy)^2 &= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2} \\ \Rightarrow x^2 - y^2 - 2ixy &= \frac{(ac+bd)+i(ad-bc)}{c^2+d^2}\end{aligned}$$

On comparing real and imaginary parts, we obtain

$$x^2 - y^2 = \frac{ac+bd}{c^2+d^2}, -2xy = \frac{ad-bc}{c^2+d^2} \dots\dots(1)$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{ad-bc}{c^2+d^2}\right)^2$$

[Using (1)]

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2+d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2+d^2)^2}$$

$$= \frac{a^2(c^2+d^2) + b^2(c^2+d^2)}{(c^2+d^2)^2}$$

$$= \frac{(c^2+d^2)(a^2+b^2)}{(c^2+d^2)^2}$$

$$= \frac{a^2+b^2}{c^2+d^2}$$

Hence, proved.

### Question 5:

Convert the following in the polar form:

(i)  $\frac{1+7i}{(2-i)^2}$ ,

(ii)  $\frac{1+3i}{1-2i}$

### Solution 5:

(i) Here,  $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$$

$$= -1+i$$

Let  $r \cos \theta = -1$  and  $r \sin \theta = 1$

On squaring and adding, we obtain  $r^2(\cos^2 \theta + \sin^2 \theta) = 1$

$$\Rightarrow r^2(\cos^2 \theta + \sin^2 \theta) = 2$$

$$\Rightarrow r^2 = 2 \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in II quadrant}]$$

$$\therefore z = r \cos \theta + i r \sin \theta$$

$$= \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

This is the required polar form.

(ii) Here,  $z = \frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+2i+3i-6}{1+4}$$

$$= \frac{-5+5i}{5} = -1+i$$

Let  $r \cos \theta = -1$  and  $r \sin \theta$

$= 1$  on squaring and adding, we obtain  $r^2(\cos^2 \theta + \sin^2 \theta)$

$$= 1+1$$

$$\Rightarrow r^2(\cos^2 \theta + \sin^2 \theta) = 2$$

$$\Rightarrow r^2 = 2 \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow r = \sqrt{2} \quad [\text{Conventionally, } r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in II quadrant}]$$

$$\therefore z = r \cos \theta + i r \sin \theta$$

$$= \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

This is the required polar form.

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### Question 6:

Solve the equation  $3x^2 - 4x + \frac{20}{3} = 0$

**Solution 6:**

The given quadratic equation is  $3x^2 - 4x + \frac{20}{3} = 0$

This equation can also be written as  $9x^2 - 12x + 20 = 0$

On comparing this equation with  $ax^2 + bx + c = 0$ , we obtain  $a = 9, b = -12$  and  $c = 20$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}i}{18} && [\sqrt{-1} = i] \\ &= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3}i \end{aligned}$$

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**Question 7:**

Solve the equation  $x^2 - 2x + \frac{3}{2} = 0$

**Solution 7:**

The given quadratic equation is  $x^2 - 2x + \frac{3}{2} = 0$

This equation can also be written as  $2x^2 - 4x + 3 = 0$

On comparing this equation with  $ax^2 + bx + c = 0$ , we obtain  $a = 2, b = -4$  and  $c = 3$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}i}{4} && [\sqrt{-1} = i] \\ &= \frac{2 \pm \sqrt{2}i}{2} = 1 \pm \frac{\sqrt{2}}{2}i \end{aligned}$$

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**Question 8:**

Solve the equation  $27x^2 - 10x + 1 = 0$

**Solution 8:**

The given quadratic equation is  $27x^2 - 10x + 1 = 0$

On comparing this equation with  $ax^2 + bx + c = 0$ , we obtain  $a = 27, b = -10$  and  $c = 1$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}i}{54} && [\sqrt{-1} = i] \\ &= \frac{5 \pm \sqrt{2}i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27}i \end{aligned}$$


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### Question 9:

Solve the equation  $21x^2 - 28x + 10 = 0$

### Solution 9:

The given quadratic equation is  $21x^2 - 28x + 10 = 0$

On comparing this equation with  $ax^2 + bx + c = 0$ , we obtain  $a = 21, b = -28$  and  $c = 10$

Therefore, the discriminant of the given equation is

$$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$$

Therefore, the required solutions are

$$\begin{aligned} \frac{-b \pm \sqrt{D}}{2a} &= \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}i}{42} && [\sqrt{-1} = i] \\ &= \frac{28 \pm 2\sqrt{14}i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i = \frac{2}{3} \pm \frac{\sqrt{14}}{21}i \end{aligned}$$


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### Question 10:

If  $z_1 = 2 - i, z_2 = 1 + i$ , find  $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$

### Solution 10:

$$z_1 = 2 - i, z_2 = 1 + i$$

$$\therefore \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right| = \left| \frac{(2 - i) + (1 + i) + 1}{(2 - i) - (1 + i) + 1} \right|$$

$$= \left| \frac{4}{2 - 2i} \right| = \left| \frac{4}{2(1 - i)} \right|$$

$$= \left| \frac{2}{1 - i} \times \frac{1 + i}{1 + i} \right| = \left| \frac{2(1 + i)}{(1^2 - i^2)} \right|$$

$$= \left| \frac{2(1 + i)}{1 + 1} \right| \quad [i^2 = -1]$$

$$= \left| \frac{2(1 + i)}{2} \right|$$

$$=|1+i|=\sqrt{1^2+1^2}=\sqrt{2}$$

Thus, the value of  $\left| \frac{z_1+z_2+1}{z_1-z_2+1} \right|$  is  $\sqrt{2}$ .

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**Question 11:**

If  $a+ib = \frac{(x+i)^2}{2x^2+1}$ , prove that  $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$

**Solution 11:**

$$\begin{aligned} a+ib &= \frac{(x+i)^2}{2x^2+1} \\ &= \frac{x^2+i^2+2xi}{2x^2+1} \\ &= \frac{x^2-1+i2x}{2x^2+1} \\ &= \frac{x^2-1}{2x^2+1} + i\left(\frac{2x}{2x^2+1}\right) \end{aligned}$$

On comparing real and imaginary parts, we obtain

$$a = \frac{x^2-1}{2x^2+1} \quad \text{and} \quad b = \frac{2x}{2x^2+1}$$

$$\begin{aligned} \therefore a^2+b^2 &= \left(\frac{x^2-1}{2x^2+1}\right)^2 + \left(\frac{2x}{2x^2+1}\right)^2 \\ &= \frac{x^4+1-2x^2+4x^2}{(2x^2+1)^2} \\ &= \frac{x^4+1+2x^2}{(2x^2+1)^2} \\ &= \frac{(x^2+1)^2}{(2x^2+1)^2} \\ \therefore a^2+b^2 &= \frac{(x^2+1)^2}{(2x^2+1)^2} \end{aligned}$$

Hence, proved.

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**Question 12:**

Let  $z_1 = 2-i$ ,  $z_2 = -2+i$ . Find (i)  $\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$ , (ii)  $\operatorname{Im}\left(\frac{1}{z_1 \bar{z}_1}\right)$

**Solution 12:**

$$z_1 = 2 - i, z_2 = -2 + i$$

$$(i) \quad z_1 z_2 = (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$$

$$\overline{z_1} = 2 + i$$

$$\therefore \frac{z_1 z_2}{\overline{z_1}} = \frac{-3 + 4i}{2 + i}$$

On multiplying numerator and denominator by  $(2 - i)$ , we obtain

$$\begin{aligned} \frac{z_1 z_2}{\overline{z_1}} &= \frac{(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{2^2 + 1^2} \\ &= \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11}{5}i \end{aligned}$$

On comparing real parts, we obtain

$$\operatorname{Re}\left(\frac{z_1 z_2}{\overline{z_1}}\right) = \frac{-2}{5}$$

$$(ii) \quad \frac{1}{z_1 \overline{z_1}} = \frac{1}{(2 - i)(2 + i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5}$$

On comparing imaginary parts, we obtain

$$\operatorname{Im}\left(\frac{1}{z_1 \overline{z_1}}\right) = 0$$

**Question 13:**

Find the modulus and argument of the complex number  $\frac{1 + 2i}{1 - 3i}$

**Solution 13:**

Let  $z = \frac{1 + 3i}{1 - 3i}$ , then

$$\begin{aligned} z &= \frac{1 + 2i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i} = \frac{1 + 3i + 2i + 6i^2}{1^2 + 3^2} = \frac{1 + 5i + 6(-1)}{1 + 9} \\ &= \frac{-5 + 5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i \end{aligned}$$

Let  $z = r \cos \theta + ir \sin \theta$

$$\text{i.e., } r \cos \theta = \frac{-1}{2} \text{ and } r \sin \theta = \frac{1}{2}$$

On squaring and adding, we obtain

$$r^2 (\cos^2 \theta + \sin^2 \theta) = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\Rightarrow r = \frac{1}{\sqrt{2}} \quad [\text{Conventionally, } r > 0]$$



$$\therefore \frac{1}{\sqrt{2}} \cos \theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2}$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad [\text{As } \theta \text{ lies in the II quadrant}]$$

Therefore, the modulus and argument of the given complex number are  $\frac{1}{\sqrt{2}}$  and  $\frac{3\pi}{4}$  respectively.

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#### Question 14:

Find the real numbers  $x$  and  $y$  if  $(x - iy)(3 + 5i)$  is the conjugate of  $-6 - 24i$ .

#### Solution 14:

$$\text{Let } z = (x - iy)(3 + 5i)$$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$

$$\therefore \bar{z} = (3x + 5y) - i(5x - 3y)$$

It is given that,  $\bar{z} = -6 - 24i$

$$\therefore (3x + 5y) - i(5x - 3y) = -6 - 24i$$

Equating real and imaginary parts, we obtain

$$3x + 5y = -6 \quad \dots\dots(i)$$

$$5x - 3y = 24 \quad \dots\dots(ii)$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$9x + 15y = -18$$

$$25x - 15y = 120$$

$$\hline 34x = 102$$

$$\therefore x = \frac{102}{34} = 3$$

Putting the value of  $x$  in equation (i), we obtain

$$3(3) + 5y = -6$$

$$\Rightarrow 5y = -6 - 9 = -15$$

$$\Rightarrow y = -3$$

Thus, the values of  $x$  and  $y$  are 3 and  $-3$  respectively.

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#### Question 15:

Find the modulus of  $\frac{1+i}{1-i} - \frac{1-i}{1+i}$ .

#### Solution 15:

$$\begin{aligned} \frac{1+i}{1-i} - \frac{1-i}{1+i} &= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \\ &= \frac{1+i^2+2i-1-i^2+2i}{1^2+1^2} \\ &= \frac{4i}{2} = 2i \\ \therefore \left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| &= |2i| = \sqrt{2^2} = 2 \end{aligned}$$


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**Question 16:**

If  $(x+iy)^3 = u+iv$ , then show that:  $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

**Solution 16:**

$$\begin{aligned} (x+iy)^3 &= u+iv \\ \Rightarrow x^3 + (iy)^3 + 3 \cdot x \cdot iy(x+iy) &= u+iv \\ \Rightarrow x^3 + i^3 y^3 + 3x^2 yi + 3xy^2 i^2 &= u+iv \\ \Rightarrow x^3 - iy^3 + 3x^2 yi - 3xy^2 &= u+iv \\ \Rightarrow (x^3 - 3xy^2) + i(3x^2 y - y^3) &= u+iv \end{aligned}$$

On equating real and imaginary parts, we obtain

$$u = x^3 - 3xy^2, \quad v = 3x^2 y - y^3$$

$$\begin{aligned} \therefore \frac{u}{x} + \frac{v}{y} &= \frac{x^3 - 3xy^2}{x} + \frac{3x^2 y - y^3}{y} \\ &= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y} \end{aligned}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4(x^2 - y^2)$$

$$\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

Hence, proved.

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**Question 17:**

If  $\alpha$  and  $\beta$  are different complex numbers with  $|\beta|=1$ , then find  $\left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right|$

**Solution 17:**

Let  $\alpha = a+ib$  and  $\beta = x+iy$

It is given that,  $|\beta|=1$

$$\therefore \sqrt{x^2 + y^2} = 1$$

$$\Rightarrow x^2 + y^2 = 1 \dots\dots(i)$$

$$\left| \frac{\beta - \alpha}{1 - \bar{\alpha}} \right| = \left| \frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)} \right|$$

$$= \left| \frac{(x - a) + i(y - b)}{1 - (ax + aiy - ibx + by)} \right|$$

$$= \left| \frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)} \right|$$

$$= \left| \frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)} \right| \quad \left[ \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right]$$

$$= \frac{\sqrt{(x - a)^2 + (y - b)^2}}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}}$$

$$= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2x^2 + b^2y^2 - 2ax + 2abxy - 2by + b^2x^2 + a^2y^2 - 2abxy}}$$

$$= \frac{\sqrt{(x^2 + y^2) + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2(x^2 + y^2) + b^2(y^2 + x^2) - 2ax - 2by}}$$

$$= \frac{\sqrt{1 + a^2 + b^2 - 2ax - 2by}}{\sqrt{1 + a^2 + b^2 - 2ax - 2by}} \quad [\text{Using (1)}]$$

$$\therefore \left| \frac{\beta - \alpha}{1 - \bar{\alpha}\beta} \right| = 1$$

**Question 18:**

Find the number of non-zero integral solutions of the equation  $|1 - i|^x = 2^x$

**Solution 18:**

$$|1 - i|^x = 2^x$$

$$\Rightarrow \left( \sqrt{1^2 + (-1)^2} \right)^x = 2^x$$

$$\Rightarrow (\sqrt{2})^x = 2^x$$

$$\Rightarrow 2^{x/2} = 2^x$$

$$\Rightarrow \frac{x}{2} = x$$

$$\Rightarrow x = 2x$$

$$\Rightarrow 2x - x = 0$$

$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of nonzero integral solutions of the given equation is 0.

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**Question 19:**

If  $(a+ib)(c+id)(e+if)(g+ih) = A+iB$ , then show that:

$$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2.$$

**Solution 19:**

$$(a+ib)(c+id)(e+if)(g+ih) = A+iB$$

$$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$$

$$|(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \quad \because [|z_1 z_2| = |z_1| |z_2|]$$

$$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$$

On squaring both sides, we obtain

$$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2. \text{ Hence proved.}$$

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**Question 20:**

If  $\left(\frac{1+i}{1-i}\right)^m = 1$  then find the least positive integral value of m.

**Solution 20:**

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$$\Rightarrow i^m = i^{4k}$$

$\therefore m = 4k$ , where  $k$  is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of  $m$  is  $4 (= 4 \times 1)$ .

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