



# NCERT Class 11 Maths

## Solutions

### Miscellaneous Exercise

#### Question 1:

Prove that:  $2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$

#### Solution 1:

$$\text{L.H.S.} = 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13}$$

$$\begin{aligned}
&= 2 \cos \frac{\pi}{3} \cos \frac{9\pi}{13} + 2 \cos \left( \frac{\frac{3\pi}{13} + \frac{5\pi}{13}}{2} \right) \cos \left( \frac{\frac{3\pi}{13} - \frac{5\pi}{13}}{2} \right) \\
&\left[ \cos x + \cos y = 2 \cos \left( \frac{x+y}{2} \right) \cos \left( \frac{x-y}{2} \right) \right] \\
&= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \left( \frac{-\pi}{13} \right) \\
&= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \frac{\pi}{13} \\
&= 2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + 2 \cos \frac{4\pi}{13} \cos \frac{\pi}{13} \\
&= 2 \cos \frac{\pi}{13} \left[ \cos \frac{9\pi}{13} + \cos \frac{4\pi}{13} \right] \\
&= 2 \cos \frac{\pi}{13} \left[ 2 \cos \left( \frac{\frac{9\pi}{13} + \frac{4\pi}{13}}{2} \right) \cos \frac{\frac{9\pi}{13} - \frac{4\pi}{13}}{2} \right] \\
&= 2 \cos \frac{\pi}{13} \left[ 2 \cos \frac{\pi}{2} \cos \frac{5\pi}{26} \right] \\
&= 2 \cos \frac{\pi}{13} \times 2 \times 0 \times \cos \frac{5\pi}{26} \\
&= 0 = \text{R.H.S.}
\end{aligned}$$


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**Question 2:**

Prove that:  $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

**Solution 2:**

$$\begin{aligned}
\text{L.H.S.} &= (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x \\
&= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x \\
&= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x) \\
&= \cos(3x - x) - \cos 2x \quad \left[ \cos(A - B) = \cos A \cos B + \sin A \sin B \right] \\
&= \cos 2x - \cos 2x \\
&= 0 \\
&= \text{R.H.S.}
\end{aligned}$$


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**Question 3:**

Prove that:  $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4 \cos^2 \frac{x+y}{2}$

**Solution 3:**

$$\begin{aligned}
\text{L.H.S.} &= (\cos x + \cos y)^2 + (\sin x - \sin y)^2 \\
&= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y \\
&= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y) \\
&= 1 + 1 + 2\cos(x + y) \quad [\cos(A + B) = (\cos A \cos B - \sin A \sin B)] \\
&= 2 + 2\cos(x + y) \\
&= 2[1 + \cos(x + y)] \\
&= 2\left[1 + 2\cos^2\left(\frac{x + y}{2}\right) - 1\right] \quad [\cos 2A = 2\cos^2 A - 1] \\
&= 4\cos^2\left(\frac{x + y}{2}\right) = \text{R.H.S}
\end{aligned}$$


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**Question 4:**

Prove that:  $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x - y}{2}$

**Solution 4:**

$$\begin{aligned}
\text{L.H.S.} &= (\cos x - \cos y)^2 + (\sin x - \sin y)^2 \\
&= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y \\
&= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y] \\
&= 1 + 1 - 2[\cos(x - y)] \quad [\cos(A - B) = \cos A \cos B + \sin A \sin B] \\
&= 2[1 - \cos(x - y)] \\
&= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x - y}{2}\right)\right\}\right] \quad [\cos 2A = 1 - 2\sin^2 A] \\
&= 4\sin^2\left(\frac{x - y}{2}\right) = \text{R.H.S}
\end{aligned}$$


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**Question 5:**

Prove that:  $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

**Solution 5:**

It is known that  $\sin A + \sin B = 2\sin\left(\frac{A + B}{2}\right) \cdot \cos\left(\frac{A - B}{2}\right)$

$$\begin{aligned}
\text{L.H.S.} &= \sin x + \sin 3x + \sin 5x + \sin 7x \\
&= (\sin x + \sin 5x) + (\sin 3x + \sin 7x) \\
&= 2\sin\left(\frac{x + 5x}{2}\right) \cdot \cos\left(\frac{x - 5x}{2}\right) + 2\sin\left(\frac{3x + 7x}{2}\right) \cos\left(\frac{3x - 7x}{2}\right)
\end{aligned}$$

$$\begin{aligned}
&= 2 \sin 3x \cos(-2x) + 2 \sin 5x \cos(-2x) \\
&= 2 \sin 3x \cos 2x + 2 \sin 5x \cos 2x \\
&= 2 \cos 2x [\sin 3x + \sin 5x] \\
&= 2 \cos 2x \left[ 2 \sin \left( \frac{3x+5x}{2} \right) \cdot \cos \left( \frac{3x-5x}{2} \right) \right] \\
&= 2 \cos 2x [2 \sin 4x \cdot \cos(-x)] \\
&= 4 \cos 2x \sin 4x \cos x = \text{R.H.S.}
\end{aligned}$$


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**Question 6:**

Prove that:  $\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

**Solution 6:**

It is known that

$$\sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cdot \cos \left( \frac{A-B}{2} \right), \cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cdot \cos \left( \frac{A-B}{2} \right)$$

$$\begin{aligned}
\text{L.H.S.} &= \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} \\
&= \frac{\left[ 2 \sin \left( \frac{7x+5x}{2} \right) \cdot \cos \left( \frac{7x-5x}{2} \right) \right] + \left[ 2 \sin \left( \frac{9x+3x}{2} \right) \cdot \cos \left( \frac{9x-3x}{2} \right) \right]}{\left[ 2 \cos \left( \frac{7x+5x}{2} \right) \cdot \cos \left( \frac{7x-5x}{2} \right) \right] + \left[ 2 \cos \left( \frac{9x+3x}{2} \right) \cdot \cos \left( \frac{9x-3x}{2} \right) \right]} \\
&= \frac{[2 \sin 6x \cdot \cos x] + [2 \sin 6x \cdot \cos 3x]}{[2 \cos 6x \cdot \cos x] + [2 \cos 6x \cdot \cos 3x]} \\
&= \frac{2 \sin 6x [\cos x + \cos 3x]}{2 \cos 6x [\cos x + \cos 3x]} \\
&= \tan 6x \\
&= \text{R.H.S.}
\end{aligned}$$


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**Question 7:**

Prove that:  $\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$

**Solution 7:**

$$\begin{aligned}
\text{L.H.S.} &= \sin 3x + \sin 2x - \sin x \\
&= \sin 3x + \left[ 2 \cos \left( \frac{2x+x}{2} \right) \sin \left( \frac{2x-x}{2} \right) \right] \quad \left[ \sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) \right] \\
&= \sin 3x + \left[ 2 \cos \left( \frac{3x}{2} \right) \sin \left( \frac{x}{2} \right) \right]
\end{aligned}$$

$$\begin{aligned}
&= \sin 3x + 2 \cos \frac{3x}{2} \sin \frac{x}{2} \\
&= 2 \sin \frac{3x}{2} \cdot \cos \frac{3x}{2} + 2 \cos \frac{3x}{2} \sin \frac{x}{2} \quad [\sin 2A = 2 \sin A \cdot \cos B] \\
&= 2 \cos \left( \frac{3x}{2} \right) \left[ \sin \left( \frac{3x}{2} \right) + \sin \left( \frac{x}{2} \right) \right] \\
&= 2 \cos \left( \frac{3x}{2} \right) \left[ 2 \sin \left\{ \frac{\left( \frac{3x}{2} \right) + \left( \frac{x}{2} \right)}{2} \right\} \cos \left\{ \frac{\left( \frac{3x}{2} \right) - \left( \frac{x}{2} \right)}{2} \right\} \right] \\
&= 2 \cos \left( \frac{3x}{2} \right) \cdot 2 \sin x \cos \left( \frac{x}{2} \right) \\
&= 4 \sin x \cos \left( \frac{x}{2} \right) \cos \left( \frac{3x}{2} \right) = \text{R.H.S.}
\end{aligned}$$


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**Question 8:**

Find  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$ , if  $\tan x = -\frac{4}{3}$ ,  $x$  in quadrant II

**Solution 8:**

Here,  $x$  is in quadrant II.

$$\text{i.e., } \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

There,  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$

are lies in first quadrant.

$$\text{It is given that } \tan x = -\frac{4}{3}$$

$$\sec^2 x = 1 + \tan^2 x = 1 + \left( -\frac{4}{3} \right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

$$\therefore \cos^2 x = \frac{9}{25}$$

$$\Rightarrow \cos x = \pm \frac{3}{5}$$

As  $x$  is in quadrant II,  $\cos x$  is negative.

$$\cos x = \frac{-3}{5}$$

$$\text{Now, } \cos x = 2\cos^2 \frac{x}{2} - 1$$

$$\Rightarrow \frac{-3}{5} = 2\cos^2 \frac{x}{2} - 1$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = \frac{2}{5}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$$

$$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}} \quad \left[ \because \cos \frac{x}{2} \text{ is positive} \right]$$

$$\therefore \cos \frac{x}{2} = \frac{\sqrt{5}}{5}$$

$$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$$

$$\Rightarrow \sin^2 \frac{x}{2} + \left( \frac{1}{\sqrt{5}} \right)^2 = 1$$

$$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{2}{\sqrt{5}} \quad \left[ \because \sin \frac{x}{2} \text{ is positive} \right]$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left( \frac{2}{\sqrt{5}} \right)}{\left( \frac{1}{\sqrt{5}} \right)} = 2$$

Thus, the respective values of  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  are  $\frac{2\sqrt{5}}{5}$ ,  $\frac{\sqrt{5}}{5}$ , and 2.

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### Question 9:

Find,  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  for  $\cos x = -\frac{1}{3}$ ,  $x$  in quadrant III

### Solution 9:

Here,  $x$  is in quadrant III.

$$\text{i.e., } \pi < x < \frac{3\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Therefore,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  are negative, where  $\sin \frac{x}{2}$  as is positive.

It is given that  $\cos x = -\frac{1}{3}$ .

$$\cos x = 1 - 2\sin^2 \frac{x}{2}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{1 - \left(-\frac{1}{3}\right)}{2} = \frac{\left(1 + \frac{1}{3}\right)}{2} = \frac{4/3}{2} = \frac{2}{3}$$

$$\Rightarrow \sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \quad \left[ \because \sin \frac{x}{2} \text{ is positive} \right]$$

$$\therefore \sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now  $\cos x = 2\cos^2 \frac{x}{2} - 1$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} = \frac{1 + \left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \quad \left[ \because \cos \frac{x}{2} \text{ is negative} \right]$$

$$\therefore \cos \frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  are  $\frac{\sqrt{6}}{3}$ ,  $\frac{-\sqrt{3}}{3}$ , and  $-\sqrt{2}$ .

### Question 10:

Find  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$  for  $\sin x = \frac{1}{4}$ ,  $x$  in quadrant II

### Solution 10:

Here,  $x$  is in quadrant II.

$$\text{i.e., } \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore,  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$ ,  $\tan \frac{x}{2}$  are all positive.

It is given that  $\sin x = \frac{1}{4}$

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4} \quad [\cos x \text{ is negative in quadrant II}]$$

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} = \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 + \sqrt{15}}{8}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}} \quad \left[ \because \sin \frac{x}{2} \text{ is positive} \right]$$

$$= \sqrt{\frac{4 + \sqrt{15}}{8}} \times \frac{2}{2}$$

$$= \sqrt{\frac{8 + 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 + 2\sqrt{15}}}{4}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} = \frac{1 + \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 - \sqrt{15}}{8}$$

$$\Rightarrow \cos \frac{x}{2} = \sqrt{\frac{4 - \sqrt{15}}{8}} \quad \left[ \because \cos \frac{x}{2} \text{ is positive} \right]$$

$$= \sqrt{\frac{4 - \sqrt{15}}{8}} \times \frac{2}{2}$$

$$= \sqrt{\frac{8 - 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 - 2\sqrt{15}}}{4}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{8 + 2\sqrt{15}}}{4}\right)}{\frac{\sqrt{8 - 2\sqrt{15}}}{4}} = \frac{\sqrt{8 + 2\sqrt{15}}}{\sqrt{8 - 2\sqrt{15}}}$$

$$= \sqrt{\frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}}} \times \frac{8 + 2\sqrt{15}}{8 + 2\sqrt{15}}$$



$$= \sqrt{\frac{(8+2\sqrt{15})^2}{64-60}} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$$

Thus, the respective values are  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$

are  $\frac{\sqrt{8+2\sqrt{15}}}{4}$ ,  $\frac{\sqrt{8-2\sqrt{15}}}{4}$  and  $4+\sqrt{15}$

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