Exercise 9.2

Question 1:
Find the sum of odd integers from 1 to 2001.

Solution 1:
The odd integers from 1 to 2001 are 1, 3, 5 …… 1999, 2001.
This sequence forms an A.P.
Here, first term, $a=1$
Common difference, $d=2$
Here, $a+(n-1)d=2001$

$\Rightarrow 1+(n-1)(2)=2001$

$\Rightarrow 2n-2=2000$

$\Rightarrow n=1001$

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$\therefore S_n=\frac{1001}{2}\left[2\times1+(1001-1)\times2\right]$

$=\frac{1001}{2}\left[2+1000\times2\right]$

$=1001\times1001$

$=1002001$

Thus, the sum of odd numbers from 1 to 2001 is 1002001.

Question 2:
Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

Solution 2:

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, …. 995.

Here, $a = 105$ and $d = 5$

$$a + (n-1)d = 995$$

$$\Rightarrow 105 + (n-1)5 = 995$$

$$\Rightarrow (n-1)5 = 995 - 105 = 890$$

$$\Rightarrow n - 1 = 178$$

$$\Rightarrow n = 179$$

$$\therefore S_n = \frac{179}{2}\left[2(105) + (179-1)(5)\right]$$

$$= \frac{179}{2}\left[2(105) + (178)(5)\right]$$

$$= 179\left[105 + (89)5\right]$$

$$= 179(105 + 445)$$

$$= (179)(550)$$

$$= 98450$$

Thus, the sum of all natural numbers lying between 100 and 1000, which are multiples of 5, 98450.

Question 3:

In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution 3:

First term = 2

Let d be the common different of the A.P.

Therefore, the A.P. is $2, 2+d, 2+2d, 2+3d\ldots$

Sum of first five terms $= 10 + 10d$

Sum of next five terms $= 10 + 35d$

According to the given condition,

$$10 + 10d = \frac{1}{4}(10 + 35d)$$

$$\Rightarrow 40 + 40d = 10 + 35d$$

$$\Rightarrow 30 = -5d$$

$$\Rightarrow d = -6$$

$$\therefore a_{20} = a + (20-1)d = 2 + (19)(-6) = 2 - 114 = -112$$

Thus, the 20^{th} of the A.P. is -112.

Question 4:

How many terms of the A.P. $-6, -\frac{11}{2}, -5$, ….. are needed to give the sum -25?

Solution 4:

Let the sum of n terms of the given A.P. be -25.

It is known that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

Where n = number of terms,   first term, and d = common difference

Here, $a = -6$

$$d = -\frac{11}{2}+6 = \frac{-11+12}{2} = \frac{1}{2}$$

Therefore, we obtain

$$-25 = \frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$

$$\Rightarrow -50 = n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$

$$\Rightarrow -50 = n\left[-\frac{25}{2}+\frac{n}{2}\right]$$

$$\Rightarrow -100 = n(-25+n)$$

$$\Rightarrow n^2 - 25n + 100 = 0$$

$$\Rightarrow n^2 - 5n - 20n + 100 = 0$$

$$\Rightarrow n(n-5) - 20(n-5) = 0$$

$$\Rightarrow n = 20 \text{ or } 5$$

Question 5:

In an A.P., if p^{th} term is 1/q and q^{th} term is 1/p, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.

Solution 5:

It is known that the general term of an A.P. is $a_n = a+(n-1)d$

$\therefore$ According to the given information,

$$p^{th} \text{ term} = a_p = a+(p-1)d = \frac{1}{q} \qquad ......(1)$$

$$q^{th} \text{ term} = a_q = a+(q-1)d = \frac{1}{p} \qquad ......(2)$$

Subtracting (2) from (1), we obtain

$$(p-1)d-(q-1)d=\frac{1}{q}-\frac{1}{p}$$

$$\Rightarrow (p-1-q+1)d=\frac{p-q}{pq}$$

$$\Rightarrow (p-q)d=\frac{p-q}{pq}$$

$$\Rightarrow d=\frac{1}{pq}$$

Putting the value of d in (1), we obtain

$$a+(p-1)\frac{1}{pq}=\frac{1}{q}$$

$$\Rightarrow a=\frac{1}{q}-\frac{1}{q}+\frac{1}{pq}=\frac{1}{pq}$$

$$\therefore S_{pq}=\frac{pq}{2}\left[2a+(pq-1)d\right]$$

$$=\frac{pq}{2}\left[\frac{2}{pq}+(pq-1)\frac{1}{pq}\right]$$

$$=1+\frac{1}{2}(pq-1)$$

$$=\frac{1}{2}pq+1-\frac{1}{2}=\frac{1}{2}pq+\frac{1}{2}$$

$$=\frac{1}{2}(pq+1)$$

Thus, the sum of first pq terms of the A.P. is $=\frac{1}{2}(pq+1)$.

Question 6:

If the sum of a certain number of terms of the A.P. 25, 22, 19, …… is 116. Find the last term

Solution 6:

Let the sum of n terms of the given A.P. be 116.

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

Here, $a=25$ and $d=22-25=-3$

$$\therefore S_n=\frac{n}{2}\left[2\times 25+(n-1)(-3)\right]$$

$$\Rightarrow 116=\frac{n}{2}[50-3n+3]$$

$$\Rightarrow 232=n(53-3n)=53n-3n^2$$

$\Rightarrow 3n^2 - 53n + 232 = 0$
$\Rightarrow 3n^2 - 24n - 29n + 232 = 0$
$\Rightarrow 3n(n-8) - 29(n-8) = 0$
$\Rightarrow (n-8)(3n-29) = 0$
$\Rightarrow n = 8$ or $n = \frac{29}{3}$
However, n cannot be equal to $\frac{29}{3}$ therefore, $n = 8$
$\therefore a_8 = \text{Last term} = a + (n-1)d = 25 + (8-1)(-3)$
$= 25 + (7)(-3) = 25 - 21$
$= 4$
Thus, the last term of the A.P. is 4.

Question 7:
Find the sum to n terms of the A.P., whose k^{th} term is $5k+1$.

Solution 7:
It is given that the k^{th} term of the A.P. is $5k+1$.
k^{th} term $= a_k + (k-1)d$
$\therefore a + (k-1)d = 5k + 1$
$a + kd - d = 5k + 1$
$\therefore$ Comparing the coefficient of k, we obtain $d = 5$;
$\Rightarrow a - d = 1$
$\Rightarrow a - 5 = 1$
$\Rightarrow a = 6$
$S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[2(6) + (n-1)(5)]$
$= \frac{n}{2}[12 + 5n - 5]$
$= \frac{n}{2}[5n + 7]$

Question 8:
If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

Solution 8:

It is known that: $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

According to the given condition,

$$\frac{n}{2}\left[2a+(n-1)d\right] = pn+qn^2$$

$$\Rightarrow \frac{n}{2}\left[2a+nd-d\right] = pn+qn^2$$

$$\Rightarrow na+n^2\frac{d}{2}-n\cdot\frac{d}{2} = pn+qn^2$$

Comparing the coefficients of n^2 on both sides, we obtain

$$\frac{d}{2} = q$$

$$\therefore d = 2q$$

Thus, the common difference of the A.P. is $2q$.

Question 9:

The sums of n terms of two arithmetic progressions are in the ratio $5n+4:9n+6$. Find the ratio of their 18^{th} terms.

Solution 9:

Let a_1, a_2 and d_1, d_2 be the first terms and the common difference of the first and second arithmetic progression respectively.

According to the given condition,

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]} = \frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} = \frac{5n+4}{9n+5} \quad ....(1)$$

Substituting $n=35$ in (1), we obtain

$$\frac{2a_1+34d_1}{2a_2+34d_2} = \frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2} = \frac{179}{321} \quad ....(2)$$

$$\frac{18^{th} \text{ term of first}}{18^{th} \text{ term of second A.P.}} = \frac{a_1+17d_1}{a_2+17d_2} \quad .... (3)$$

From (2) and (3), we obtain

$$\frac{18^{th} \text{ term of first}}{18^{th} \text{ term of second A.P.}} = \frac{179}{321}$$

Thus, the ratio of 18th term of both the A.P.s is 179 : 321.

Question 10:

If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first $(p+q)$ terms.

Solution 10:

Let a and d be the first term and the common difference of the A.P. respectively.

Here,

$$S_p = \frac{p}{2}[2a+(p-1)d]$$

$$S_q = \frac{p}{2}[2a+(q-1)d]$$

According to the given condition,

$$\frac{p}{2}[2a+(p-1)d] = \frac{q}{2}[2a+(q-1)d]$$

$$\Rightarrow p[2a+(p-1)d] = q[2a+(q-1)d]$$

$$\Rightarrow 2ap + pd(p-1) = 2aq + qd(q-1)$$

$$\Rightarrow 2a(p-q) + d[p(p-1) - q(q-1)] = 0$$

$$\Rightarrow 2a(p-q) + d[p^2 - p - q^2 + q] = 0$$

$$\Rightarrow 2a(p-q) + d[(p-q)(p+q) - (p-q)] = 0$$

$$\Rightarrow 2a(p-q) + d[(p-q)(p+q-1)] = 0$$

$$\Rightarrow 2a + d(p+q-1) = 0$$

$$\Rightarrow d = \frac{-2a}{p+q-1} \qquad ......(1)$$

$$\therefore\ S_{p+q} = \frac{p+q}{2}[2a+(p+q-1)\cdot d]$$

$$\Rightarrow S_{p+q} = \frac{p+q}{2}\left[2a+(p+q-1)\left(\frac{-2a}{p+q-1}\right)\right] \qquad \text{[From (1)]}$$

$$= \frac{p+q}{2}[2a-2a]$$

$$= 0$$

Thus, the sum of the first $(p+q)$ terms of the A.P is 0.

Question 11:

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively. Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Solution 11:

Let a_1 and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$$S_p=\frac{p}{2}\left[2a_1+(p-1)d\right]=a$$

$$\Rightarrow 2a_1+(p-1)d=\frac{2a}{p} \qquad ....(1)$$

$$S_q=\frac{q}{2}\left[2a_1+(q-1)d\right]=b$$

$$\Rightarrow 2a_1+(q-1)d=\frac{2b}{q} \qquad ......(2)$$

$$S_r=\frac{r}{2}\left[2a_1+(r-1)d\right]=c$$

$$\Rightarrow 2a_1+(r-1)d=\frac{2c}{r} \qquad ......(3)$$

Subtracting (2) from (1), we obtain

$$(p-1)d-(q-1)d=\frac{2a}{p}-\frac{2b}{q}$$

$$\Rightarrow d(p-1-q+1)=\frac{2aq-2bp}{pq}$$

$$\Rightarrow d(p-q)=\frac{2aq-2bp}{pq}$$

$$\Rightarrow d=\frac{2(aq-bp)}{pq(p-q)} \qquad ......(4)$$

Subtracting (3) from (2), we obtain

$$(q-1)d-(r-1)d=\frac{2b}{q}-\frac{2c}{r}$$

$$\Rightarrow d(q-1-r+1)=\frac{2b}{q}-\frac{2c}{r}$$

$$\Rightarrow d(q-r)=\frac{2br-2qc}{qr}$$

$$\Rightarrow d=\frac{2(br-qc)}{qr(q-r)} \qquad ......(5)$$

Equating both the values of d obtained in (4) and (5), we obtain

$$\frac{aq-bp}{pq(p-q)}=\frac{br-qc}{qr(q-r)}$$

$$\Rightarrow qr(q-r)(aq-bq)=pq(q-q)(br-qc)$$
$$\Rightarrow r(aq-bp)(q-r)=p(br-qc)(p-q)$$
$$\Rightarrow (aqr-bpr)(q-r)=(bpr-pqc)(p-q)$$
Dividing both sides by pqr, we obtain
$$\left(\frac{a}{p}-\frac{b}{q}\right)(q-r)=\left(\frac{b}{q}-\frac{c}{r}\right)(p-q)$$
$$\Rightarrow \frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$
$$\Rightarrow \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$
Thus, the given result is proved.

Question 12:

The ratio of the sums of m and n terms of an A.P. is $m^2:n^2$. Show that the ratio of m^{th} and term is $(2m-1):(2n-1)$.

Solution 12:

Let a and b be the first term and the common difference of the A.P. respectively. According to the given condition,
$$\frac{\text{Sum of } m \text{ terms}}{\text{Sum of } n \text{ terms}}=\frac{m^2}{n^2}$$
$$\Rightarrow \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{m^2}{n^2}$$
$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n} \quad \text{.......(1)}$$
Putting $m=2m-1$ and $n=2n-1$, we obtain
$$\frac{2a+(2m-2)d}{2a+(2n-2)d}=\frac{2m-1}{2n-1}$$
$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1} \quad \text{........ (2)}$$
$$\frac{m^{th} \text{ term of A.P.}}{n^{th} \text{ term of A.P.}}=\frac{a+(m-1)d}{a+(n-1)d} \quad \text{......(3)}$$
From (2) and (3), we obtain
$$\frac{m^{th} \text{ term of A.P.}}{n^{th} \text{ term of A.P.}}=\frac{2m-1}{2n-1}$$
Thus, the given result is proved.

Question 13:

If the sum of n terms of an A.P. of $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Solution 13:

Let a and b be the first term and the common difference of the A.P. respectively.

$a_m = a+(m-1)d = 164 \qquad ......(1)$

Sum of n terms: $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Here,

$\frac{n}{2}\left[2a+nd-d\right] = 3n^2+5n$

$\Rightarrow na+n^2\cdot\frac{d}{2}-\frac{nd}{2} = 3n^2+5n$

Comparing the coefficient of n^2 on both sides, we obtain

$\frac{d}{2}=3$

$\Rightarrow d=6$

Comparing the coefficient of n on both sides, we obtain

$a-\frac{d}{2}=5$

$\Rightarrow a-3=5$

$\Rightarrow a=8$

Therefore, from (1), we obtain

$8+(m-1)6=164$

$\Rightarrow (m-1)6=164-8=156$

$\Rightarrow m-1=26$

$\Rightarrow m=27$

Thus, the value of m is 27.

Question 14:

Insert five numbers between 8 and 26 such that resulting sequence is an A.P.

Solution 14:

Let A_1, A_2, A_3, A_4 and A_5 be five numbers between 8 and 26 such that $8, A_1, A_2, A_3, A_4, A_5, 26$ is an A.P.

Here, $a=8,\ b=26, n=7$

Therefore, $26=8+(7-1)d$

$\Rightarrow 6d=26-8=18$

$\Rightarrow d=3$

$A_1=a+d=8+3=11$

$A_2=a+2d=8+2\times3=8+6=14$

$A_3 = a+3d = 8+3\times3 = 8+9 = 17$

$A_4 = a+4d = 8+4\times3 = 8+12 = 20$

$A_5 = a+5d = 8+5\times3 = 8+15 = 23$

Thus, the required five numbers between 8 and 26 are 11, 14, 17, 20 and 23.

Question 15:

If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between a and b, then find the value of n.

Solution 15:

A.M. of a and b $= \frac{a+b}{2}$

According to the given condition,

$\frac{a+b}{2} = \frac{a^n+b^n}{a^{n-1}+b^{n-1}}$

$\Rightarrow (a+b)(a^{n-1}+b^{n-1}) = 2(a^n+b^n)$

$\Rightarrow a^n + ab^{n-1} + ba^{n-1} + b^n = 2a^n + 2b^n$

$\Rightarrow ab^{n-1} + a^{n-1}b = a^n + b^n$

$\Rightarrow ab^{n-1} - b^n = a^n - a^{n-1}b$

$\Rightarrow b^{n-1}(a-b) = a^{n-1}(a-b)$

$\Rightarrow b^{n-1} = a^{n-1}$

$\Rightarrow \left(\frac{a}{b}\right)^{n-1} = 1 = \left(\frac{a}{b}\right)^0$

$\Rightarrow n-1 = 0$

$\Rightarrow n = 1$

Question 16:

Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and $(m-1)^{th}$ numbers is 5:9. Find the value of m.

Solution 16:

Let $A_1, A_2, ...A_m$ be m numbers such that $1, A_1, A_2, ...A_m, 31$ is an A.P.

Here, $a=1,\ b=31,\ n=m+2$

$\therefore 31 = 1+(m+2-1)(d)$

$\Rightarrow 30 = (m+1)d$

$\Rightarrow d = \frac{30}{m+1}$ ......(1)

$A_1 = a+d$

$A_2 = a + 2d$

$A_3 = a + 3d$

$\therefore\ A_7 = a + 7d$

$A_{m-1} = a + (m-1)d$

According to the given condition,

$$\frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \frac{1+7\left(\frac{30}{(m+1)}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9} \qquad [\text{From}(1)]$$

$$\Rightarrow \frac{m+1+7(30)}{m+1+30(m-1)} = \frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$$

$\Rightarrow 9m + 1899 = 155m - 145$

$\Rightarrow 155m - 9m = 1899 + 145$

$\Rightarrow 146m = 2044$

$\Rightarrow m = 14$

Thus, the value of m is 14.

Question 17:

A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs. 5 every month, what amount he will pay in the 30th installment?

Solution 17:

The first installment of the load is Rs. 100.

The second installment of the load is Rs. 105 and so on.

The amount that the man repays every month forms an A.P.

The A.P. is 100, 105, 110 …

First term, $a = 100$

Common difference, $d = 5$

$A_{30} = a + (30-1)d$

$= 100 + (29)(5)$

$= 100 + 145$

$= 245$

Thus, the amount to be paid in the 30th installment is Rs. 245.

Question 18:

The difference between any two consecutive interior angles of a polygon is $5°$. If the smallest angle is $120°$, find the number of the sides of the polygon.

Solution 18:

The angles of the polygon will form an A.P. with common difference d as $5°$ and first term a as $120°$.

It is known that the sum of all angles of a polygon with n sides is $180(n-2)$.

$$\therefore S_n = 180°(n-2)$$

$$\Rightarrow \frac{n}{2}\left[2a+(n-1)d\right] = 180°(n-2)$$

$$\Rightarrow \frac{n}{2}\left[240°+(n-1)5°\right] = 180°(n-2)$$

$$\Rightarrow n\left[240+(n-1)5\right] = 360(n-2)$$

$$\Rightarrow 240n + 5n^2 - 5n = 360n - 720$$

$$\Rightarrow 5n^2 - 125n + 720 = 0$$

$$\Rightarrow n^2 - 25n + 144 = 0$$

$$\Rightarrow n^2 - 16n - 9n + 144 = 0$$

$$\Rightarrow n(n-16) - 9(n-16) = 0$$

$$\Rightarrow (n-9)(n-16) = 0$$

$$\Rightarrow n = 9 \text{ or } 16$$