

Miscellaneous Exercise

Question 1:

Solve the inequality $2 \leq 3x - 4 \leq 5$

Solution 1:

$$2 \leq 3x - 4 \leq 5$$

$$\Rightarrow 2 + 4 \leq 3x - 4 + 4 \leq 5 + 4$$

$$\Rightarrow 6 \leq 3x \leq 9$$

$$\Rightarrow 2 \leq x \leq 3$$

Thus, all the real numbers, x , which are greater than or equal to 2 but less than or equal to 3, are the solutions of the given inequality. The solution set for the given inequality is $[2, 3]$.

Question 2:

Solve the inequality $6 \leq -3(2x - 4) < 12$

Solution 2:

$$6 \leq -3(2x - 4) < 12$$

$$\Rightarrow 2 \leq -(2x - 4) < 4$$

$$\Rightarrow -2 \geq 2x - 4 > -4$$

$$\Rightarrow 4 - 2 \geq 2x > 4 - 4$$

$$\Rightarrow 2 \geq 2x > 0$$

$$\Rightarrow 1 \geq x > 0$$

Thus, the solution set for the given inequality is $(0,1]$.

Question 3:

Solve the inequality $-3 \leq 4 - \frac{7x}{2} \leq 18$

Solution 3:

$$-3 \leq 4 - \frac{7x}{2} \leq 18$$

$$\Rightarrow -3 - 4 \leq -\frac{7x}{2} \leq 18 - 4$$

$$\Rightarrow -7 \leq -\frac{7x}{2} \leq 14$$

$$\Rightarrow 7 \geq \frac{7x}{2} \geq -14$$

$$\Rightarrow 1 \geq \frac{x}{2} \geq -2$$

$$\Rightarrow 2 \geq x \geq -4$$

Thus, the solution set for the given inequality is $[-4,2]$.

Question 4:

Solve the inequality $-15 < \frac{3(x-2)}{5} \leq 0$

Solution 4:

$$-15 < \frac{3(x-2)}{5} \leq 0$$

$$\Rightarrow -75 < 3(x-2) \leq 0$$

$$\Rightarrow -25 < x-2 \leq 0$$

$$\Rightarrow -25+2 < x \leq 2$$

$$\Rightarrow -23 < x \leq 2$$

Thus, the solution set for the given inequality is $(-23,2]$.

Question 5:

Solve the inequality $-12 < 4 - \frac{3x}{-5} \leq 2$

Solution 5:

$$-12 < 4 - \frac{3x}{-5} \leq 2$$

$$\Rightarrow -12 - 4 < \frac{-3x}{-5} \leq 2 - 4$$

$$\Rightarrow -16 < \frac{3x}{5} \leq -2$$

$$\Rightarrow -80 < 3x \leq -10$$

$$\Rightarrow \frac{-80}{3} < x \leq \frac{-10}{3}$$

Thus, the solution set for the given inequality is $\left(\frac{-80}{3}, \frac{-10}{3}\right]$.

Question 6:

Solve the inequality $7 \leq \frac{(3x+11)}{2} \leq 11$

Solution 6:

$$7 \leq \frac{(3x+11)}{2} \leq 11$$

$$\Rightarrow 14 \leq 3x+11 \leq 22$$

$$\Rightarrow 14 - 11 \leq 3x \leq 22 - 11$$

$$\Rightarrow 3 \leq 3x \leq 11$$

$$\Rightarrow 1 \leq x \leq \frac{11}{3}$$

Thus, the solution set for the given inequality is $\left[1, \frac{11}{3}\right]$.

Question 7:

Solve the inequalities and represent the solution graphically on number line:

$$5x+1 > -24, 5x-1 < 24$$

Solution 7:

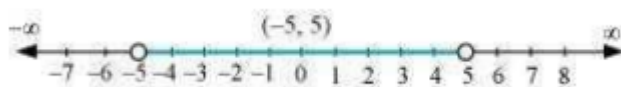
$$5x+1 > -24 \Rightarrow 5x > -25$$

$$\Rightarrow x > -5 \dots\dots(1)$$

$$5x-1 < 24 \Rightarrow 5x < 25$$

$$\Rightarrow x < 5 \dots\dots(2)$$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(-5, 5)$. The solution of the given system of inequalities can be represented on number line as



Question 8:

Solve the inequalities and represent the solution graphically on number line:

$$2(x-1) < x+5, \quad 3(x+2) > 2-x$$

Solution 8:

$$2(x-1) < x+5 \Rightarrow 2x-2 < x+5 \Rightarrow 2x-x < 5+2$$

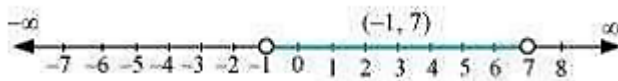
$$\Rightarrow x < 7 \dots\dots(1)$$

$$3(x+2) > 2-x \Rightarrow 3x+6 > 2-x \Rightarrow 3x+x > 2-6$$

$$\Rightarrow 4x > -4$$

$$\Rightarrow x > -1 \dots\dots(2)$$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(-1, 7)$. The solution of the given system of inequalities can be represented on number line as

**Question 9:**

Solve the following inequalities and represent the solution graphically on number line:

$$3x-7 > 2(x-6), \quad 6-x > 11-2x$$

Solution 9:

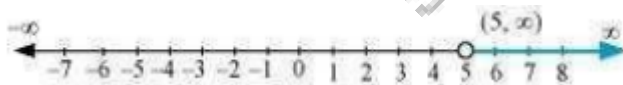
$$3x-7 > 2(x-6) \Rightarrow 3x-7 > 2x-12 \Rightarrow 3x-2x > -12+7$$

$$\Rightarrow x > -5 \dots\dots(1)$$

$$6-x > 11-2x \Rightarrow -x+2x > 11-6$$

$$\Rightarrow x > 5 \dots\dots(2)$$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $(5, \infty)$. The solution of the given system of inequalities can be represented on number line as

**Question 10:**

Solve the inequalities and represent the solution graphically on number line:

$$5(2x-7)-3(2x+3) \leq 0, \quad 2x+19 \leq 6x+47$$

Solution 10:

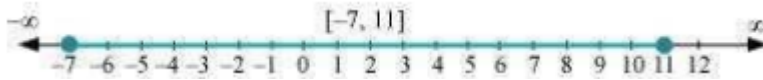
$$5(2x-7)-3(2x+3) \leq 0 \Rightarrow 10x-35-6x-9 \leq 0 \Rightarrow 4x-44 \leq 0 \Rightarrow 4x \leq 44$$

$$\Rightarrow x \leq 11 \dots\dots(1)$$

$$2x+19 \leq 6x+47 \Rightarrow 19-47 \leq 6x-2x \Rightarrow -28 \leq 4x$$

$$\Rightarrow -7 \leq x \dots\dots(2)$$

From (1) and (2), it can be concluded that the solution set for the given system of inequalities is $[-7, 11]$. The solution of the given system of inequalities can be represented on number line as



Question 11:

A solution is to be kept between $68^{\circ}F$ and $77^{\circ}F$. What is the range in temperature in degree Celsius (C) if the Celsius/Fahrenheit (F) conversion formula is given by

$$F = \frac{9}{5}C + 32 ?$$

Solution 11:

Since the solution is to be kept between $68^{\circ}F$ and $77^{\circ}F$, $68 < F < 77$

Putting $F = \frac{9}{5}C + 32$, we obtain

$$68 < \frac{9}{5}C + 32 < 77$$

$$\Rightarrow 68 - 32 < \frac{9}{5}C < 77 - 32$$

$$\Rightarrow 36 < \frac{9}{5}C < 45$$

$$\Rightarrow 36 \times \frac{5}{9} < C < 45 \times \frac{5}{9}$$

$$\Rightarrow 20 < C < 25$$

Thus, the required range of temperature in degree Celsius is between $20^{\circ}C$ and $25^{\circ}C$.

Question 12:

A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Solution 12:

Let x litres of 2% boric acid solution is required to be added.

Then, total mixture = $(x + 640)$ litres

This resulting mixture is to be more than 4% but less than 6% boric acid.

$$\therefore 2\%x + 8\% \text{ of } 640 > 4\% \text{ of } (x + 640) \text{ and } x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640)$$

$$2\%x + 8\% \text{ of } 640 > 4\% \text{ of } (x + 640)$$

$$\Rightarrow \frac{2}{100}x + \frac{8}{100}(640) > \frac{4}{100}(x + 640)$$

$$\Rightarrow 2x + 5120 > 4x + 2560$$

$$\Rightarrow 5120 - 2560 > 4x - 2x$$

$$\Rightarrow 5120 - 2560 > 2x$$

$$\Rightarrow 2560 > 2x$$

$$\Rightarrow 1280 > x$$

$$2\%x + 8\% \text{ of } 640 < 6\% \text{ of } (x + 640)$$

$$\frac{2}{100}x + \frac{8}{100}(640) < \frac{6}{100}(x + 640)$$

$$\Rightarrow 2x + 5120 < 6x + 3840$$

$$\Rightarrow 5120 - 3840 < 6x - 2x$$

$$\Rightarrow 5120 - 3840 < 6x - 2x$$

$$\Rightarrow 1280 < 4x$$

$$\Rightarrow 320 < x$$

$$\therefore 320 < x < 1280$$

Thus, the number of litres of 2% of boric acid solution that is to be added will have to be more than 320 litres but less than 1280 litres.

Question 13:

How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Solution 13:

Let x litres of water is required to be added.

Then, total mixture = $(x + 1125)$ litres

It is evident that the amount of acid contained in the resulting mixture is 45% of 1125 litres.

This resulting mixture will contain more than 25% but less than 30% acid content.

$$\therefore 30\% \text{ of } (1125 + x) > 45\% \text{ of } 1125$$

$$\text{And, } 25\% \text{ of } (1125 + x) < 45\% \text{ of } 1125$$

$$30\% \text{ of } (1125 + x) > 45\% \text{ of } 1125$$

$$\Rightarrow \frac{30}{100}(1125 + x) > \frac{45}{100} \times 1125$$

$$\Rightarrow 30(1125 + x) > 45 \times 1125$$

$$\Rightarrow 30 \times 1125 + 30x > 45 \times 1125$$

$$\Rightarrow 30 > 45 \times 1125 - 30 \times 1125$$

$$\Rightarrow 30x > (45 - 30) \times 1125$$

$$\Rightarrow x > \frac{15 \times 1125}{30} = 562.5$$

$$25\% \text{ of } (1125 + x) < 45\% \text{ of } 1125$$

$$\Rightarrow \frac{25}{100}(1125 + x) < \frac{45}{100} \times 1125$$

$$\Rightarrow 25(1125 + x) < 45 \times 1125$$

$$\Rightarrow 25 \times 1125 + 25x < 45 \times 1125$$

$$\Rightarrow 25x > 45 \times 1125 - 25 \times 1125$$

$$\Rightarrow 25x > (45 - 25) \times 1125$$

$$\Rightarrow x > \frac{20 \times 1125}{25} = 900$$

$$\therefore 562.5 < x < 900$$

Thus, the required number of litres of water that is to be added will have to be more than 562.5 but less than 900.

Question 14:

IQ of a person is given by the formula $IQ = \frac{MA}{CA} \times 100$,

Where MA is mental age and CA is chronological age. If $80 \leq IQ \leq 140$ for a group of 12 years old children, find the range of their mental age.

Solution 14:

It is given that for a group of 12 years old children,

$$80 \leq IQ \leq 140 \dots\dots (i)$$

For a group of 12 years old children, CA = 12 years

$$IQ = \frac{MA}{12} \times 100$$

Putting this value of IQ in (i), we obtain

$$80 \leq \frac{MA}{12} \times 100 \leq 140$$

$$\Rightarrow 80 \times \frac{12}{100} \leq MA \leq 140 \times \frac{12}{100}$$

$$\Rightarrow 9.6 \leq MA \leq 16.8$$

Thus, the range of mental age of the group of 12 years old children is $\Rightarrow 9.6 \leq MA \leq 16.8$.