Exercise 5.1

Question 1:

Express the given complex number in the form $a+ib$: $(5i)\left(-\frac{3}{5}i\right)$

Solution 1:

$$(5i)\left(\frac{-3}{5}i\right)=-5\times\frac{3}{5}\times i\times i$$
$$=-3i^2$$
$$=-3(-1) \qquad \left[i^2=-1\right]$$
$$= 3$$

Question 2:

Express the given complex number in the form $a+ib$: i^9+i^{19}

Solution 2:

$$i^9+i^{19}=i^{4\times2+1}+i^{4\times4+3}$$
$$=\left(i^4\right)^2\cdot i+\left(i^4\right)^4\cdot i^3$$
$$=1\times i+1\times(-i) \qquad \left[i^4=1,\ i^3=-i\right]$$
$$=i+(-i)$$
$$=0$$

Question 3:

Express the given complex number in the form $a+ib$: i^{-39}

Solution 3:

$$i^{-39}=i^{-4\times9-3}=\left(i^4\right)^{-9}\cdot i^{-3}$$
$$=(1)^{-9}\cdot i^{-3} \qquad \left[i^4=1\right]$$
$$=\frac{1}{i^3}=\frac{1}{-i} \qquad \left[i^3=-i\right]$$
$$=\frac{-1}{i}\times\frac{i}{i}$$
$$=\frac{-i}{i^2}=\frac{-i}{-1}=i \qquad \left[i^2=-1\right]$$

Question 4:

Express the given complex number in the form a $a+ib$:

$$3(7+i7)+i(7+i7)$$

Solution 4:

$3(7+i7)+i(7+i7)=21+21i+7i+7i^2$

$=21+28i+7\times(-1) \qquad [\because i^2=-1]$

$=14+28i$

Question 5:

Express the given complex number in the form $a+ib:(1-i)-(-1+i6)$.

Solution 5:

$(1-i)-(-1+i6)=1-i+1-6i$

$=2-7i$

Question 6:

Express the given complex number in the form $a+ib:\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Solution 6:

$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$

$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$

$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$

$=\frac{-19}{5}-\frac{21}{10}i$

Question 7:

Express the given complex number in the form $a+ib:\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Solution 7:

$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$

$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$

$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$

$=\frac{17}{3}+i\frac{5}{3}$

Question 8:

Express the given complex number in the form $a+ib:(1-i)^4$

Solution 8:

$(1-i)^4=\left[(1-i)^2\right]^2$

$=\left[1^2+i^2-2i\right]^2$

$=[1-1-2i]^2$

$=(2i)^2$

$=(-2i)\times(-2i)$

$=4i^2=-4 \qquad \left[i^2=-1\right]$

Question 9:

Express the given complex number in the form $a+ib:\left(\frac{1}{3}+3i\right)^3$

Solution 9:

$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$

$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$

$=\frac{1}{27}+27(-i)+i+9i^2 \qquad \left[i^3=-i\right]$

$=\frac{1}{27}-27i+i-9 \qquad \left[i^2=-1\right]$

$=\left(\frac{1}{27}-9\right)+i(-27+1)$

$=\frac{-242}{27}-26i$

Question 10:

Express the given complex number in the form $a+ib:\left(-2-\frac{1}{3}i\right)^3$

Solution 10:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$

$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$

$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$

$$=-\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number $4-3i$.

Solution 11:

Let $z=4-3i$

Then,

$\bar{z}=4+3i$ and $|z|=4^2+(-3)^2=16+9=25$

Therefore, the multiplicative inverse of $4-3i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{4+3i}{25}=\frac{4}{25}+\frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Solution 12:

Let $z=\sqrt{5}+3i$

Then, $\bar{z}=\sqrt{5}-3i$ and $|z|^2=\left(\sqrt{5}\right)^2+3^2=5+9=14$

Therefore, the multiplicative inverse of $\sqrt{5}+3i$

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5}-3i}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number $-i$

Solution 13:

Let $z=-i$

Then, $\bar{z}=i$ and $|z|^2=1^2=1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}=i$$

Question 14:

Express the following expression in the form of $a+ib$.

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

Solution 14:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$