Exercise 3.3

Question 1:

$$\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$$

Solution 1:

$$\text{L.H.S} =\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2$$

$$=\frac{1}{4}+\frac{1}{4}-1=-\frac{1}{2}$$

= R.H.S.

Question 2:

Prove that $2\sin^2\frac{\pi}{6}+\text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Solution 2:

$$\text{L.H.S.} =2\sin^2\frac{\pi}{6}+\text{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$$

$$=2\left(\frac{1}{2}\right)^2+\text{cosec}^2\left(\pi+\frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$$

$$=2\times\frac{1}{4}+\left(-\text{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

$$=\frac{1}{2}+(-2)^2\left(\frac{1}{4}\right)$$

$$=\frac{1}{2}+\frac{4}{4}=\frac{1}{2}+1=\frac{3}{2}$$

= R.H.S.

Question 3:

Prove that $\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Solution 3:

L.H.S. = $\cot^2\frac{\pi}{6}+\text{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$$=\left(\sqrt{3}\right)^2+\text{cosec}\left(\pi-\frac{\pi}{6}\right)+3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$=3+\text{cosec}\frac{\pi}{6}+3\times\frac{1}{3}$$

$$=3+2+1=6$$

$=$ R.H.S

Question 4:

Prove that $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Solution 4:

L.H.S. $=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$$=2\left\{\sin\left(\pi-\frac{\pi}{4}\right)\right\}^2+2\left(\frac{1}{\sqrt{2}}\right)^2+2(2)^2$$

$$=2\left\{\sin\frac{\pi}{4}\right\}^2+2\times\frac{1}{2}+8$$

$$=1+1+8$$

= 10

= R.H.S

Question 5:

Find the value of :

(i) $\sin 75°$
(ii) $\tan 15°$

Solution 5:

(i) $\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$[\sin(x+y) = \sin x \cos y + \cos x \sin y]$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15° = \tan(45° - 30°)$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$ $\left[\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}\right]$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - (1)^2}$

$= \frac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$

Question 6:

Prove that $\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x+y)$

Solution 6:

$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$

$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)\right]$

$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} + \cos\left\{\left(\frac{\pi}{4} - x\right) - \left(\frac{\pi}{4} - y\right)\right\}\right]$

$+ \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\} - \cos\left\{\frac{\pi}{4} - x\right\} - \left(\frac{\pi}{4} - y\right)\right]$

$$\left[\begin{array}{l}\because\ 2\cos A\cos B=\cos(A+B)+\cos(A-B)\\ -2\sin A\sin B=\cos(A+B)-\cos(A-B)\end{array}\right]$$

$$=2\times\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$$=\cos\left[\frac{\pi}{4}-(x+y)\right]$$

$$=\sin(x+y)$$

= R.H.S.

Question 7:

Prove that $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

Solution 7:

It is known that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ and $(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\text{L.H.S.}=\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\frac{\left(\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}\right)}{\left(\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)}=\frac{\left(\dfrac{1+\tan x}{1-\tan x}\right)}{\left(\dfrac{1-\tan x}{1+\tan x}\right)}=\left(\frac{1+\tan x}{1-\tan x}\right)^2=\text{R.H.S.}$$

Question 8:

Prove that $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}=\cot^2 x$

Solution 8:

$$\text{L.H.S.}=\frac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)}$$

$$=\frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)}$$

$$=\frac{-\cos^2 x}{-\sin^2 x}$$

$=\cot^2 x$
= R.H.S.

Question 9:

$$\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]=1$$

Solution 9:

L.H.S. = $\cos\left(\frac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\frac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$=\sin x\cos x[\tan x+\cot x]$

$=\sin x\cos x\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$

$=(\sin x\cos x)\left[\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right]$

= 1 = R.H.S.

Question 10:

Prove that $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x=\cos x$

Solution 10:

L.H.S. $=\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x$

$=\frac{1}{2}[2\sin(n+1)x\sin(n+2)x+2\cos(n+1)x\cos(n+2)x]$

$=\frac{1}{2}\left[\begin{array}{l}\cos\{(n+1)x-(n+2)x\}-cis\{(n+1)x+(n+2)x\}\\+\cos\{(n+1)x+(n+2)x\}+\cos\{(n+1)x-(n+2)x\}\end{array}\right]$

$\left[\begin{array}{l}\because -2\sin A\sin B=\cos(A+B)-\cos(A-B)\\2\cos A\cos B=\cos(A+B)+\cos(A-B)\end{array}\right]$

$=\frac{1}{2}\times 2\cos\{(n+1)x-(n+2)x\}$

$=\cos(-x)=\cos x=$ R.H.S.

Question 11:

Prove that $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$

Solution 11:

It is known that $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $= \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\left(\frac{3\pi}{4}-x\right)\right)}{2}\right\}.\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$$

$$= -2\sin\left(\frac{3\pi}{4}\right)\sin x$$

$$= -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x$$

$$= -2\sin\frac{\pi}{4}\sin x$$

$$= -2\times\frac{1}{\sqrt{2}}\times\sin x$$

$$= -\sqrt{2}\sin x$$

= R.H.S.

Question 12:

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Solution 12:

It is known that

$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$, $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\therefore$ L.H.S. $= \sin^2 6x - \sin^2 4x$

$$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$$

$$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right).\sin\left(\frac{6x-4x}{2}\right)\right]$$

$$= (2\sin 5x\cos x)(2\cos 5x\sin x) = (2\sin 5x\cos 5x)(2\sin x\cos x)$$

$$= \sin 10x \sin 2x$$

= R.H.S.

Question 13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Solution 13:

It is known that

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),\ \cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\therefore$ L.H.S. $=\cos^2 2x-\cos^2 6x$

$=(\cos 2x+\cos 6x)(\cos 2x-6x)$

$$=\left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]$$

$=[2\cos 4x\cos(-2x)][-2\sin 4x\sin(-2x)]$

$=[2\cos 4x\cos 2x][-2\sin 4x(-\sin 2x)]$

$=(2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

$=\sin 8x\sin 4x=$ R.H.S

Question 14:

Prove that $\sin 2x+2\sin 4x+\sin 6x=4\cos^2 x\sin 4x$

Solution 14:

L.H.S. $=\sin 2x+2\sin 4x+\sin 6x$

$=[\sin 2x+\sin 6x]+2\sin 4x$

$$=\left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]+2\sin 4x$$

$$\left[\because\ \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$=2\sin 4x\cos(-2x)+2\sin 4x$

$=2\sin 4x\cos 2x+2\sin 4x$

$=2\sin 4x(\cos 2x+1)$

$=2\sin 4x(2\cos^2 x-1+1)$

$=2\sin 4x(2\cos^2 x)$

$=4\cos^2 x\sin\ 4x$

$=$ R.H.S.

Question 15:

Prove that $\cot 4x(\sin 5x+\sin 3x)=\cot x(\sin 5x-\sin 3x)$

Solution 15:

L.H.S $=\cot 4x(\sin 5x+\sin 3x)$

$$=\frac{\cot 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=\left(\frac{\cos 4x}{\sin 4x}\right)\left[2\sin 4x\cos x\right]$$

$$=2\cos 4x\cos x$$

R.H.S. $=\cot x(\sin 5x-\sin 3x)$

$$=\frac{\cos x}{\sin x}\left[2\cos\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)\right]$$

$$\left[\because \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$=\frac{\cos x}{\sin x}\left[2\cos 4x\sin x\right]$$

$$=2\cos 4x.\cos x$$

L.H.S. = R.H.S.

Question 16:

Prove that $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\frac{\sin 2x}{\cos 10x}$

Solution 16:

It is known that

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),\ \sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$$

$$=\frac{-2\sin\left(\frac{9x+5x}{2}\right).\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right).\sin\left(\frac{17x-3x}{2}\right)}$$

$$=\frac{-2\sin 7x.\sin 2x}{2\cos 10x.\sin 7x}$$

$$=-\frac{\sin 2x}{\cos 10x}$$

$=$ R.H.S.

Question 17:

Prove that: $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Solution 17:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right).\cos\left(\frac{5x-3x}{2}\right)}$$

$$=\frac{2\sin 4x.\cos x}{2\cos 4x.\cos x}$$

$$=\tan 4x=\text{R.H.S.}$$

Question 18:

Prove that $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\frac{x-y}{2}$

Solution 18:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin x-\sin y}{\cos x+\cos y}$$

$$=\frac{2\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)}$$

$$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$=\tan\left(\frac{x-y}{2}\right)=\text{R.H.S.}$$

Question 19:

Prove that $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Solution 19:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right),$$

$$\cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.}=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$$

$$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$$

$$=\frac{\sin 2x}{\cos 2x}$$

$$=\tan 2x$$

= R.H.S.

Question 20:

Prove that $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=2\sin x$

Solution 20:

It is known that

$$\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right), \cos^2 A-\sin^2 A=\cos 2A$$

$$\therefore \text{L.H.S.}=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$$

$$=\frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$$

$$=\frac{2\cos 2x\sin(-x)}{-\cos 2x}$$

$$=-2\times(-\sin x)$$

$$=2\sin x=\text{R.H.S.}$$

Question 21:

Prove that $\dfrac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Solution 21:

$$\text{L.H.S.} =\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$\cot 3x = \text{R.H.S.}$

Question 22:

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Solution 22:

L.H.S. $= \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$

$= \cot x \cot 2x - \cot 3x(\cot 2x + \cot x)$

$= \cot x \cot 2x - \cot(2x + x)(\cot 2x + \cot x)$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x}\right](\cot 2x + \cot x)$$

$$\left[\because \cot(A+B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}\right]$$

$= \cot x \cot 2x - (\cot 2x \cot x - 1) = 1 = \text{R.H.S.}$

Question 23:

Prove that $\tan 4x = \dfrac{4\tan x(1 - \tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$

Solution 23:

It is known that $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\therefore$ L.H.S. $= \tan 4x = \tan 2(2x)$

$$= \frac{2\tan 2x}{1 - \tan^2(2x)}$$

$$= \frac{2\left(\frac{2\tan x}{1-\tan^2 x}\right)}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[1-\frac{4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{\left(\frac{4\tan x}{1-\tan^2 x}\right)}{\left[\frac{\left(1-\tan^2 x\right)^2-4\tan^2 x}{\left(1-\tan^2 x\right)^2}\right]}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{\left(1-\tan^2 x\right)^2-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1+\tan^4 x-2\tan^2 x-4\tan^2 x}$$

$$= \frac{4\tan x\left(1-\tan^2 x\right)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

Question 24:

Prove that: $\cos 4x = 1-8\sin^2 x\cos^2 x$

Solution 24:

L.H.S. $= \cos 4x$

$= \cos 2(2x)$

$= 1-2\sin^2 2x \left[\cos 2A = 1-2\sin^2 A\right]$

$= 1-2(2\sin x\cos x)^2 \left[\sin 2A = 2\sin A\cos A\right]$

$= 1-8\sin^2 x\cos^2 x$

$=$ R.H.S.

Question 25:

Prove that: $\cos 6x = 32x\cos^6 x-48\cos^4 x+18\cos^2 x-1$

Solution 25:

L.H.S. $= \cos 6x$

$= \cos 3(2x)$

$$= 4\cos^3 2x - 3\cos 2x \left[\cos 3A = 4\cos^3 A - 3\cos A\right]$$
$$= 4\left[\left(2\cos^2 x - 1\right)^3 - 3\left(2\cos^2 x - 1\right)\right]\left[\cos 2x = 2\cos^2 x - 1\right]$$
$$= 4\left[\left(2\cos^2 x\right)^3 - (1)^3 - 3\left(2\cos^2 x\right)^2 + 3\left(2\cos^2 x\right)\right] - 6\cos^2 x + 3$$
$$= 4\left[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x\right] - 6\cos^2 x + 3$$
$$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$$
$$= 32\cos^6 x - 48\cos^4 x + 18\ \cos^2 x - 1$$
$$= \text{R.H.S.}$$