Question 1:

Find the values of other five trigonometric functions if $\cos x = -\dfrac{1}{2}$, x lies in third quadrant.

Solution 1:

$\cos x = -\dfrac{1}{2}$

$\therefore \sec x = \dfrac{1}{\cos x} = \dfrac{1}{\left(-\dfrac{1}{2}\right)} = -2$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \sin^2 x = 1 - \left(-\dfrac{1}{2}\right)^2$

$\Rightarrow \sin^2 x = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

$\Rightarrow \sin x = \pm \dfrac{\sqrt{3}}{2}$

Since x lies in the 3rd quadrant, the value of $\sin x$ will be negative.

$\therefore \sin x = -\dfrac{\sqrt{3}}{2}$

$\operatorname{cosec} x = \dfrac{1}{\sin x} = \dfrac{1}{\left(-\dfrac{\sqrt{3}}{2}\right)} = -\dfrac{2}{\sqrt{3}}$

$\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\left(-\dfrac{\sqrt{3}}{2}\right)}{\left(-\dfrac{1}{2}\right)} = \sqrt{3}$

$\cot x = \dfrac{1}{\tan x} = \dfrac{1}{\sqrt{3}}$.

Question 2:

Find the values of other five trigonometric functions if $\sin x = \dfrac{3}{5}$, x lies in second quadrant.

Solution 2:

$$\sin x = \frac{3}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\dfrac{3}{5}\right)} = \frac{5}{3}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm \frac{4}{5}$$

Since x lies in the 2nd quadrant, the value of $\cos x$ will be negative

$$\therefore \cos x = -\frac{4}{5}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\left(-\dfrac{4}{5}\right)} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\dfrac{3}{5}\right)}{\left(-\dfrac{4}{5}\right)} = -\frac{3}{4}$$

$$\cot x = \frac{1}{\tan x} = -\frac{4}{3}.$$

Question 3:

Find the values of other five trigonometric functions if $\cot x = \dfrac{3}{4}$, x lies in third quadrant.

Solution 3:

$$\cot x = \frac{3}{4}$$

$$\tan x = \frac{1}{\cot x} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2 x$$

$$\Rightarrow \frac{25}{9} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{5}{3}$$

Since x lies in the 3rd quadrant, the value of $\sec x$ will be negative.

$$\therefore \sec x = -\frac{5}{3}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\frac{5}{3}\right)} = -\frac{3}{5}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \frac{4}{3} = \frac{\sin x}{\left(\frac{-3}{5}\right)}$$

$$\Rightarrow \sin x = \left(\frac{4}{3}\right) \times \left(\frac{-3}{5}\right) = -\frac{4}{5}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$$

Question 4:

Find the values of other five trigonometric functions if $\sec x = \frac{13}{5}$, x lies in fourth quadrant.

Solution 4:

$$\sec x = \frac{13}{5}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(\frac{13}{5}\right)} = \frac{5}{13}$$

$$\sin^2 x + \cos^2 = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{5}{13}\right)^2$$

$$\Rightarrow \sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\Rightarrow \sin x = \pm \frac{12}{13}$$

Since x lies in the 4th quadrant, the value of sin x will be negative.

$$\therefore \sin x = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(-\dfrac{12}{13}\right)} = -\frac{13}{12}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\left(\dfrac{-12}{13}\right)}{\left(\dfrac{5}{13}\right)} = -\frac{12}{5}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\dfrac{12}{5}\right)} = -\frac{5}{12}.$$

Question 5:

Find the values of other five trigonometric functions if $\tan x = -\dfrac{5}{12}$, x lies in second quadrant.

Solution 5:

$$\tan x = -\frac{5}{12}$$

$$\cot x = \frac{1}{\tan x} = \frac{1}{\left(-\dfrac{5}{12}\right)} = -\frac{12}{5}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \left(-\frac{5}{12}\right)^2 = \sec^2 x$$

$$\Rightarrow 1 + \frac{25}{144} = \sec^2 x$$

$$\Rightarrow \frac{169}{144} = \sec^2 x$$

$$\Rightarrow \sec x = \pm \frac{13}{12}$$

Since x lies in the 2nd quadrant, the value of sec x will be negative.

$$\therefore \sec x = -\frac{13}{12}$$

$$\cos x = \frac{1}{\sec x} = \frac{1}{\left(-\dfrac{13}{12}\right)} = -\frac{12}{13}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow -\frac{5}{12} = \frac{\sin x}{\left(-\dfrac{12}{13}\right)}$$

$$\Rightarrow \sin x = \left(-\frac{5}{12}\right) \times \left(-\frac{12}{13}\right) = \frac{5}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\left(\dfrac{5}{13}\right)} = \frac{13}{5}.$$

Question 6:
Find the value of the trigonometric function $\sin 765°$.

Solution 6:
It is known that the values of $\sin x$ repeat after an interval of $2n$ or $360°$.

$$\therefore \sin 765° = \sin\left(2 \times 360° + 45°\right) = \sin 45° = \frac{1}{\sqrt{2}}.$$

Question 7:
Find the value of the trigonometric function $\operatorname{cosec}\left(-1410°\right)$

Solution 7:
It is known that the values of $\operatorname{cosec} x$ repeat after an interval of $2n$ or $360°$.

$$\therefore \operatorname{cosec}\left(-1410°\right) = \operatorname{cosec}\left(-1410° + 4 \times 360°\right)$$
$$= \operatorname{cosec}\left(-1410° + 1440°\right)$$
$$= \operatorname{cosec} 30° = 2.$$

Question 8:
Find the value of the trigonometric function $\tan \dfrac{19\pi}{3}$.

Solution 8:
It is known that the values of $\tan x$ repeat after an interval of n or $180°$.

$$\therefore \tan \frac{19\pi}{3} = \tan 6\frac{1}{3}\pi = \tan\left(6\pi + \frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \tan 60° = \sqrt{3}.$$

Question 9:

Find the value of the trigonometric function $\sin\left(-\dfrac{11\pi}{3}\right)$

Solution 9:

It is known that the values of $\sin x$ repeat after an interval of $2n$ or $360°$.

$\therefore \sin\left(-\dfrac{11\pi}{3}\right) = \sin\left(-\dfrac{11\pi}{3} + 2 \times 2\pi\right) = \sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$.

Question 10:

Find the value of the trigonometric function $\cot\left(-\dfrac{15\pi}{4}\right)$

Solution 10:

It is known that the values of cot x repeat after an interval of n or $180°$.

$\therefore \cot\left(-\dfrac{15\pi}{4}\right) = \cot\left(-\dfrac{15\pi}{4} + 4\pi\right) = \cot\dfrac{\pi}{4} = 1$.