Exercise 3.1

Question 1:

Find the radian measures corresponding to the following degree measures:

(i) $25°$ (ii) $-47°30'$ (iii) $240°$ (iv) $520°$

Solution 1:

(i) $25°$

We know that $180° = \pi$ radian

$\therefore 25° = \dfrac{\pi}{180} \times 25 \text{ radian} = \dfrac{5\pi}{36} \text{ radian}$

(ii) $-47°30'$

$-47°30' - 47\dfrac{1}{2}$

$= \dfrac{-95}{2}$ degree

Since $180° = \pi$ radian

$\dfrac{-95}{2} \text{degree} = \dfrac{\pi}{180} \times \left(\dfrac{-95}{2}\right) \text{radian} = \left(\dfrac{-19}{36 \times 2}\right) \pi \text{ radian} = \dfrac{-19}{72} \pi \text{ radian}$

$\therefore -47°30' = \dfrac{-19}{72} \pi$ radian

(iii) $240°$

We know that $180° = \pi$ radian

$\therefore 240° = \dfrac{\pi}{180} \times 240 \text{ radian} = \dfrac{4}{3} \pi \text{ radian}$

(iv) $520°$

We know that $180° = \pi$ radian

$\therefore 520° = \dfrac{\pi}{180} \times 520 \text{ radian} = \dfrac{26\pi}{9} \text{ radian}$

Question 2:

Find the degree measures corresponding to the following radian measures

$\left(\text{Use } \pi = \dfrac{22}{7}\right)$

(i) $\dfrac{11}{16}$ (ii) -4 (iii) $\dfrac{5\pi}{3}$ (iv) $\dfrac{7\pi}{6}$

Solution 2:

(i) $\dfrac{11}{16}$

We know that π radian $= 180°$

$\therefore \dfrac{11}{16} \text{radian} = \dfrac{180}{\pi} \times \dfrac{11}{16} \text{degree} = \dfrac{45 \times 11}{\pi \times 4} \text{degree}$

$= \dfrac{45\times11\times7}{22\times4}\text{degree} = \dfrac{315}{8}\text{degree}$

$= 36\dfrac{3}{8}\text{degree}$

$= 39^\circ + \dfrac{3\times60}{8}\text{minutes} \qquad [1^\circ = 60']$

$= 39^\circ + 22' + \dfrac{1}{2}\text{minutes}$

$= 39^\circ 22' 30'' \qquad [1' = 60'']$

(ii) -4

We know that π radian $= 180^\circ$

$-4\,\text{radian} = \dfrac{180}{\pi}\times(-4)\,\text{degree} = \dfrac{180\times7(-4)}{22}\text{degree}$

$= \dfrac{-2520}{11}\text{degree} = -229\dfrac{1}{11}\text{degree}$

$= -229^\circ + \dfrac{1\times60}{11}\text{minutes} \qquad [1^\circ = 60']$

$= -229^\circ + 5' + \dfrac{5}{11}\text{minutes}$

$= -229^\circ 5' 27'' \qquad [1' = 60'']$

(iii) $\dfrac{5\pi}{3}$

We know that π radian $= 180^\circ$

$\therefore \dfrac{5\pi}{3}\text{radian} = \dfrac{180}{\pi}\times\dfrac{5\pi}{3}\text{degree} = 300^\circ$

(iv) $\dfrac{7\pi}{6}$

We know that π radian $= 180^\circ$

$\therefore \dfrac{7\pi}{6}\text{radian} = \dfrac{180}{\pi}\times\dfrac{7\pi}{6} = 210^\circ$

Question 3:

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Solution 3:

Number of revolutions made by the wheel in 1 minute = 360

$\therefore$ Number of revolutions made by the wheel in 1 second $= \dfrac{360}{60} = 6$

In one complete revolution, the wheel turns an angle of 2π radian.

Hence, in 6 complete revolutions, it will turn an angle of $6\times2\pi$ radian,

i.e., 12π radian

Thus, in one second, the wheel turns an angle of 12π radian.

Question 4:

Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

$$\left(\text{Use}\,\pi = \frac{22}{7}\right)$$

Solution 4:

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre, then

$$\theta = \frac{1}{r}$$

Therefore, for $r = 100\,cm,\ l = 22\,cm,$ we have

$$\theta = \frac{22}{100}\text{radian} = \frac{180}{\pi} \times \frac{22}{100}\text{degree} = \frac{180 \times 7 \times 22}{22 \times 100}\text{degree}$$

$$= \frac{126}{10}\text{degree} = 12\frac{3}{5}\text{degree} = 12°36' \qquad [1° = 60']$$

Thus, the required angle is $12°36'$.

Question 5:

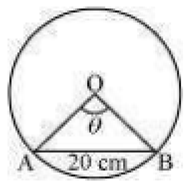
In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Solution 5:

Diameter of the circle = 40 cm

$\therefore$ Radius (r) of the circle $= \frac{40}{2}cm = 20\,cm$

Let AB be a chord $(\text{length} = 20\,cm)$ of the circle.



In $\Delta OAB, OA = OB = \text{Radius of circle} = 20\,cm$

Also, AB = 20 cm

Thus, ΔOAB is an equilateral triangle.

$$\therefore \theta = 60° = \frac{\pi}{3}\text{radian}$$

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ radian at the centre then

$$\theta = \frac{l}{r}$$

$\frac{\pi}{3} = \frac{\widehat{AB}}{20} \Rightarrow \widehat{AB} = \frac{20\pi}{3}$ cm

Thus, the length of the minor arc of the chord is $\frac{20\pi}{3} cm$.

Question 6:

If in two circles, arcs of the same length subtend angles $60°$ and $75°$ at the centre, find the ratio of their radii.

Solution 6:

Let the radii of the two circles be r_1 and r_2. Let an arc of length l subtend an angle of $60°$ at the centre of the circle of radius r_1, while let an arc of length/subtend an angle of $75°$ at the centre of the circle of radius r_2.

Now, $60° = \frac{\pi}{3}$ radian and $75° = \frac{5\pi}{12}$ radian

We know that in a circle of radius r unit, if an arc of length l unit subtends an angle θ. radian at the centre then

$\theta = \frac{l}{r}$ or $l = r\theta$

$\therefore l = \frac{r_1\pi}{3}$ and $l = \frac{r_2 5\pi}{12}$

$\Rightarrow \frac{r_1\pi}{3} = \frac{r_2 5\pi}{12}$

$\Rightarrow r_1 = \frac{r_2 5}{4}$

$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4}$

Thus, the ratio of the radii is 5:4.

Question 7:

Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length.

(i) 10 cm (ii) 15 cm (iii) 21 cm

Solution 7:

We know that in a circle of radius r unit, if an arc of length l unit subtends

An angle θ radian at the centre, then $\theta = \frac{l}{r}$

It is given that $r = 75 cm$

(i) Here, $l = 10 cm$

$\theta = \frac{10}{75}$ radian $= \frac{2}{15}$ radian

(ii) Here, $l = 15cm$

$$\theta = \frac{15}{75}\text{radian} = \frac{1}{5}\text{radian}$$

(iii) Here, $l = 21\text{cm}$

$$\theta = \frac{21}{75}\text{radian} = \frac{7}{25}\text{radian}$$