Exercise 2.2

Question 1:

Let $A = \{1, 2, 3 ... 14\}$. Define a relation R from A to A by $R = \{(x, y): 3x - y = 0\}$, where $x, y \in A$. Write down its domain, codomain and range.

Solution 1:

The relation R from A to A is given as $R = \{(x, y): 3x - y = 0, \text{where } x, y \in A\}$

i.e., $R = \{(x, y): 3x = y, \text{where } x, y \in A\}$

$\therefore R = \{(1,3), (2,6), (3,9), (4,12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R = \{1, 2, 3, 4\}$

The whole set A is he codomain of the relation R.

$\therefore$ Codomain of $R = A = \{1, 2, 3....14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R = \{3, 6, 9, 12\}$

Question 2:

Define a relation R on the set **N** of natural numbers by $R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4; x, y \in \mathbf{N}\}$. Depict this relationship using roster form. Write down the domain and the range.

Solution 2:

$R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4, x, y \in \mathbf{N}\}$

The natural numbers less than 4 are 1, 2, and 3.

$\therefore R = \{(1,6), (2,7), (3,8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

$\therefore$ Domain of $R = \{1, 2, 3\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R = \{6, 7, 8\}$

Question 3:

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$. Write R in roster form.

Solution 3:

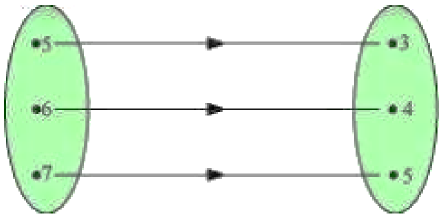
$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$

$R = \{(x, y): \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$

$\therefore R = \{(1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\}$

Question 4:

The given figure shows a relationship between the sets P and Q. Write this relation
(i) in set-builder form
(ii) in roster form.
What is its domain and range?



Solution 4:

According to the given figure, $P=\{5,6,7\}, Q=\{3,4,5\}$

(i) $R=\{(x,y): y=x-2; x\in P\}$ or $R=\{(x,y): y=x-2 \text{ for } x=5,6,7\}$

(ii) $R=\{(5,3),(6,4),(7,5)\}$

Domain of $R=\{5,6,7\}$

Range of $R=\{3,4,5\}$

Question 5:

Let $A=\{1,2,3,4,6\}$. Let R be the relation on A defined by $\{(a,b): a,b\in A, b \text{ is exactly divisible by } a\}$.
(i) Write R in roster form
(ii) Find the domain of R
(iii) Find the range of R.

Solution 5:

$A=\{1,2,3,4,6\}, R=\{(a,b): a,b\in A, b \text{ is exactly divisible by } a\}$

(i) $R=\{(1,1),(1,2),(1,3),(1,4),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(6,6)\}$

(ii) Domain of $R=\{1,2,3,4,6\}$

(iii) Range of $R=\{1,2,3,4,6\}$

Question 6:

Determine the domain and range of the relation R defined by $R=\{(x,x+5): x\in\{0,1,2,3,4,5\}\}$.

Solution 6:

$R=\{(x,x+5): x\in\{0,1,2,3,4,5\}\}$

$\therefore R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

$\therefore$ Domain of $R=\{0,1,2,3,4,5\}$

Range of $R=\{5,6,7,8,9,10\}$

Question 7:
Write the relation $R=\{(x,x^3): x \text{ is a prime number less than } 10\}$ in roster form.

Solution 7:
$R=\{(x,x^3): x \text{ is a prime number less than } 10\}$. The prime numbers less than 10 are 2, 3, 5 and 7.
$\therefore R=\{(2,8),(3,27),(5,125),(7,343)\}$

Question 8:
Let $A=\{x,y,z\}$ and $B=\{1,2\}$. Find the number of relations from A to B.

Solution 8:
It is given that $A=\{x,y,z\}$ and $B=\{1,2\}$.
$\therefore A\times B=\{(x,1),(x,2),(y,1),(y,2),(z,1),(z,2)\}$
Since $n(A\times B)=6$, the number of subsets of $A\times B$ is 2^6.
Therefore, the number of relations from A to B is 2^6.

Question 9:
Let R be the relation on **Z** defined by $R=\{(a,b): a,b\in \mathbf{Z},\ a-b \text{ is an integer}\}$. Find the domain and range of R.

Solution 9:
$R=\{(a,b): a,b\in \mathbf{Z},\ a-b \text{ is an integer}\}$
It is known that the difference between any two integers is always an integer.
$\therefore$ Domain of R = **Z**
Range of R = **Z**
