Exercise 2.1

Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$, find the values of x and y.

Solution 1:

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3},\frac{1}{3}\right)$

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow\frac{x}{3}=\frac{5}{3}-1 \quad y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow\frac{x}{3}=\frac{2}{3} \quad \Rightarrow y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow x=2 \quad \Rightarrow y=1$

$\therefore x=2$ and $y=1$

Question 2:

If the set A has 3 elements and the set B $=\{3,4,5\}$, then find the number of elements in $(A\times B)$?

Solution 2:

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

$\Rightarrow$ Number of elements in set B = 3

Number of elements in $(A\times B)$

= (Number of elements in A) × (Number of elements in B)

$=3\times3=9$

Thus, the number of elements in $(A\times B)$ in 9.

Question 3:

If G = {7, 8} and H = {5, 4, 2}, find $G\times H$ and $H\times G$.

Solution 3:

G = $\{7,8\}$ and $\text{H}=\{5,4,2\}$

We know that the Cartesian product $P\times Q$ of two non-empty sets P and Q is defined as

$P\times Q-\{(p,q): p\in P, q\in Q\}$

$\therefore G\times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

$H\times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$

Question 4:

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If P = {m, n} and Q = {n, m}, then $P\times Q=\{(m,n),(n,m)\}$.

(ii) If A and B are non-empty sets, then $A\times B$ is a non-empty set of ordered pairs (x, y) such that $x\in A$ and $y\in B$.

(iii) If $A=\{1,2\}, B=\{3,4\}$, then $A\times\{B\cap\varnothing\}=\varnothing$.

Solution 4:

(i) False

If $P=\{m,n\}$ and $Q=\{n,m\}$, then

$P\times Q=\{(m,m),(m,n),(n,m),(n,n)\}$

(ii) True

(iii) True

Question 5:

If $A=\{-1,1\}$, find $A\times A\times A$.

Solution 5:

If is known that for any non-empty set $A, A\times A\times A$ is defined as $A\times A\times A=\{(a,b,c):a,b,c\in A\}$

It is given that $A=\{-1,1\}$

$$\therefore A\times A\times A=\begin{Bmatrix}(-1,-1,-1),(-1,-1,1),(-1,1,-1),(-1,1,1),\\(1,-1,-1),(1,-1,1),(1,1,-1),(1,1,1)\end{Bmatrix}$$

Question 6:

If $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$. Find A and B.

Solution 6:

If is given that $A\times B=\{(a,x),(a,y),(b,x),(b,y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as $P\times Q=\{(p,q):p\in P,q\in Q\}$

$\therefore$ A is the set of all first elements and B is the set of all second elements.

Thus, $A=\{a,b\}$ and $B=\{x,y\}$

Question 7:

Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that

(i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) $A\times C$ is a subset of $B\times D$

Solution 7:

(i) To verify: $A\times(B\cap C)=(A\times B)\cap(A\times C)$

We have $B\cap C=\{1,2,3,4\}\cap\{5,6\}=\varnothing$

$\therefore L.H.S.=A\times(B\cap C)=A\times\varnothing=\varnothing$

$A\times B=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$\therefore R.H.S.=(A\times B)\cap(A\times C)=\varnothing$

$\therefore L.H.S.=R.H.S.$

Hence, $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(ii) To verify: $A\times C$ is a subset of $B\times D$

$A\times C=\{(1,5),(1,6),(2,5),(2,6)\}$

$A\times D=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),$
$(3,8),(4,5),(4,6),(4,7),(4,8)\}$

We can observe that all the elements of set $A\times C$ are the elements of set $B\times D$. Therefore, $A\times C$ is a subset of $B\times D$.

Question 8:

Let $A=\{1,2\}$ and $B=\{3,4\}$. Write $A\times B$. How many subsets will $A\times B$ have? List them.

Solution 8:

$A=\{1,2\}$ and $B=\{3,4\}$

$\therefore A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$

$\Rightarrow n(A\times B)=4$

We know that if C is a set with $n(C)=m$, then $n[P(C)]=2^m$.

Therefore, the set $A\times B$ has $2^4=16$ subsets. These are

$\varnothing,\{(1,3)\},\{(1,4)\},\{(2,3)\},\{(2,4)\},\{(1,3)(1,4)\},\{(1,3),(2,3)\},$

$\{(1,3),(2,4)\},\{(1,4),(2,3)\},\{(1,4)(2,4)\},\{(2,3)(2,4)\}$

$\{(1,3),(1,4),(2,3)\},\{(1,3),(1,4),(2,4)\},\{(1,3),(2,3),(2,4)\}$

$\{(1,4),(2,3),(2,4)\},\{(1,3),(1,4),(2,3),(2,4)\}$

Question 9:

Let A and B be two sets such that $n(A)=3$ and $n(B)=2$. If $(x,1),(y,2),(z,1)$ are in $A\times B$, find A and B, where x,y and z are distinct elements.

Solution 9:

It is given that $n(A)=3$ and $n(B)=2$; and $(x,1),(y,2),(z,1)$ are in $A\times B$.

We know that

A = Set of first elements of the ordered pair elements of $A\times B$

B = Set of second elements of the ordered pair elements of $A\times B$.

$\therefore x, y,$ and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A)=3$ and $n(B)=2,$

It is clear that $A=\{x,y,z\}$ and $B=\{1,2\}$.

Question 10:

The Cartesian product $A\times A$ has 9 elements among which are found $(-1,0)$ and (0, 1). Find the set A and the remaining elements of $A\times A$.

Solution 10:

We know that if $n(A)=p$ and $n(B)=q,$ then $n(A\times B)=pq$.

$\therefore n(A\times A)=n(A)\times n(A)$

It is given that $n(A\times A)=9$

$\therefore\ n(A)\times n(A)=9$

$\Rightarrow n(A)=3$

The ordered pairs $(-1,0)$ and (0, 1) are two of the nine elements of $A\times A$.

We know that $A\times A=\{(a,a):a\in A\}$. Therefore, $-1,0,$ and 1 are elements of A.

Since $n(A)=3,$ it is clear that $A=\{-1,0,1\}$.

The remaining elements of set $A\times A$ are $(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),$ and $(1,1)$.
