Exercise 1.4

Question 1:

Find the union of each of the following pairs of sets:

(i) $X \{1, 3, 5\} Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\} B = \{a, b, c\}$

(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\}$

$B = \{x : x \text{ is a natural number less than } 6\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$

$B = \{x : x \text{ is a natural number and } 6 < x < 10\}$

$$A = \{a, e, i, o, u\} \quad B = \{a, b, c\}$$

(v) $A = \{1, 2, 3\}, B = \varnothing$

Solution 1:

(i) $X = \{1, 3, 5\} \quad Y = \{1, 2, 3\}$

$X \cup Y = \{1, 2, 3, 5\}$

(ii)

$A \cup B = \{a, b, c, e, i, o, u\}$

(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9 \ldots\}$

As $B = \{x : x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5, 6\}$

$A \cup B = \{1, 2, 4, 5, 3, 6, 9, 12 \ldots\}$

$\therefore A \cup B = \{x : x = 1, 2, 4, 5 \text{ or a multiple of } 3\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x : x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$

$A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore A \cup B = \{x : x \in N \text{ and } 1 < x < 10\}$

(v) $A = \{1, 2, 3\}, \ B = \varnothing$

$A \cup B = \{1, 2, 3\}$

Question 2:

Let $A = \{a, b\}, B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution 2:

Here, $A = \{a, b\}$ and $B = \{a, b, c\}$

Yes, $A \subset B$

$A \cup B = \{a, b, c\} = B$

Question 3:

If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution 3:

If A and B are two sets such that $A \subset B,$ then $A \cup B = B$.

Question 4:

If $A = \{1,2,3,4\}, B = \{3,4,5,6\}, C = \{5,6,7,8\}$ and $D = \{7,8,9,10\}$; find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $B \cup D$

(v) $A \cup B \cup C$

(vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$

Solution 4:

$A = \{1,2,3,4], B = \{3,4,5,6\}, C = \{5,6,7,8\}$ and $D = \{7,8,9,10\}$

(i) $A \cup B = \{1,2,3,4,5,6\}$

(ii) $A \cup C = \{1,2,3,4,5,6,7,8\}$

(iii) $B \cup C = \{3,4,5,6,7,8\}$

(iv) $B \cup D = \{3,4,5,6,7,8,9,10\}$

(v) $A \cup B \cup C = \{1,2,3,4,5,6,7,8\}$

(vi) $A \cup B \cup D = \{1,2,3,4,5,6,7,8,9,10\}$

(vii) $B \cup C \cup D = \{3,4,5,6,7,8,9,10\}$

Question 5:

Find the intersection of each pair of sets:

(i) $X = \{1,3,5\} Y = \{1,2,3\}$

(ii)

(iii)

$B = \{x : x \text{ is a natural number less than } 6\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\}$

(v) $A = \{1, 2, 3\}, B = \varnothing$

Solution 5:

(i) $X = \{1, 3, 5\}, Y = \{1, 2, 3\}$

$X \cap Y = \{1, 3\}$

(ii) $A = \{a, e, i, o, u\}, B = \{a, b, c\}$

$A \cap B = \{a\}$

(iii) $A = \{x : x \text{ is a natural number and multiple of } 3\} = \{3, 6, 9...\}$

$B = \{x : x \text{ is a natural number less than } 6\} = \{1, 2, 3, 4, 5\}$

$\therefore A \cap B = \{3\}$

(iv) $A = \{x : x \text{ is a natural number and } 1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x : x \text{ is a natural number and } 6 < x < 10\} = \{7, 8, 9\}$

$A \cap B = \varnothing$

(v) $A = \{1, 2, 3\}, B = \varnothing$

$A \cap B = \varnothing$

Question 6:

If $A = \{3, 5, 7, 9, 11\}, B = \{7, 9, 11, 13\}, C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$

(ii) $B \cap C$

(iii) $A \cap C \cap D$

(iv) $A \cap C$

(v) $B \cap D$

(vi) $A \cap (B \cup C)$

(vii) $A \cap D$

(viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cup C)$

(x) $(A \cup D) \cap (B \cup C)$

Solution 6:

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = \{A \cap C\} \cap D = \{11\} \cap \{15, 17\} = \varnothing$

(iv) $A \cap C \{11\}$

(v) $B \cap D = \varnothing$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$= \{7, 9, 11\} \cup \{11\} = \{7, 9, 11\}$

(vii) $A \cap D = \varnothing$

(viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

$= \{7, 9, 11\} \cup \varnothing = \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$

(x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11, 15\}$

Question 7:

If $A = \{x : x \text{ is a natural number}\}$, $B = \{x : x \text{ is an even natural number}\}$

$C = \{x : x \text{ is an odd natural number}\}$ and $D = \{x : x \text{ is a prime number}\}$, find

(i) $A \cap B$

(ii) $A \cap C$

(iii) $A \cap D$

(iv) $B\bigcap C$

(v) $B\bigcap D$

(vi) $C\bigcap D$

Solution 7:

$A = \{x : x \text{ is a natural number}\} = \{1,2,3,4,5...\}$

$B = \{x : x \text{ is an even natural number}\} = \{2,4,6,8...\}$

$C = \{x : x \text{ is an odd natural number}\} = \{1,3,5,7,9...\}$

$D = \{x : x \text{ is a prime number}\} = \{2,3,5,7...\}$

(i) $A\bigcap B = \{x : x \text{ is a even natural number}\} = B$

(ii) $A\bigcap C = \{x : x \text{ is an odd natural number}\} = C$

(iii) $A\bigcap D = \{x : x \text{ is a prime number}\} = D$

(iv) $B\bigcap C = \varnothing$

(v) $B\bigcap D = \{2\}$

(vi) $C\bigcap D = \{x : x \text{ is odd prime number}\}$

Question 8:

Which of the following pairs of sets are disjoint

(i) $\{1,2,3,4\}$ and $\{x : x \text{ is a natural number and } 4 \le x \le 6\}$

(ii) $\{a,e,i,o,u\}$ and $\{c,d,e,f\}$

(iii) $\{x : x \text{ is an even integer}\}$ and $\{x : x \text{ is an odd integer}\}$

Solution 8:

(i) $\{1,2,3,4\}$

$\{x : x \text{ is a natural number and } 4 \le x \le 6\} = \{4,5,6\}$

Now, $\{1,2,3,4\}\bigcap\{4,5,6\} = \{4\}$

Therefore, this pair of sets is not disjoint.

(ii) $\{a,e,i,o,u\} \bigcap \{c,d,e,f\} = \{e\}$

Therefore, $\{a,e,i,o,u\}$ and $\{c,d,e,f\}$ are not disjoint.

(iii) $\{x:x \text{ is an even integer}\} \bigcap \{x:x \text{ is an odd integer}\} = \varnothing$

Therefore, this pair of sets is disjoint.

Question 9:

If $A = \{3,6,9,12,15,18,21\}, B = \{4,8,12,16,20\}$,
$C = \{2,4,6,8,10,12,14,16\}, D = \{5,10,15,20\}$; find

(i) $A - B$

(ii) $A - C$

(iii) $A - D$

(iv) $B - A$

(v) $C - A$

(vi) $D - A$

(vii) $B - C$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

Solution 9:

(i) $A - B = \{3,6,9,15,18,21\}$

(ii) $A - C = \{3,9,15,18,21\}$

(iii) $A - D = \{3,6,9,12,18,21\}$

(iv) $B - A = \{4,8,16,20\}$

(v) $C - A = \{2,4,8,10,14,16\}$

(vi) $D - A = \{5,10,20\}$

(vii) $B - C = \{20\}$

(viii) $B - D = \{4, 8, 12, 16\}$

(ix) $C - B = \{2, 6, 10, 14\}$

(x) $D - B = \{5, 10, 15\}$

(xi) $C - D = \{2, 4, 6, 8, 12, 14, 16\}$

(xii) $D - C = \{5, 15, 20\}$

Question 10:

If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find

(i) $X - Y$

(ii) $Y - X$

(iii) $X \cap Y$

Solution 10:

(i) $X - Y = \{a, c\}$

(ii) $Y - X = \{f, g\}$

(iii) $X \cap Y = \{b, d\}$

Question 11:

IF R is the set real numbers and Q is the set of rational numbers, then what is R – Q?

Solution 11:

R: Set of real numbers

Q: Set of rational numbers

Therefore, R – Q is a set of irrational number.

Question 12:

State whether each of the following statement is true or false. Justify you answer.

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

(iii) $\{2,6,10,14\}$ and $\{3,7,11,15\}$ are disjoint sets.

(iv) $\{2,6,10\}$ and $\{3,7,11\}$ are disjoint sets.

Solution 12:

(i) False

As $3 \in \{2,3,4,5\}, 3 \in \{3,6\}$

$\Rightarrow \{2,3,4,5\} \cap \{3,6\} = \{3\}$

(ii) False

As $a \in \{a,e,i,o,u\}, a \in \{a,b,c,d\}$

$\Rightarrow \{a,e,i,o,u\} \cap \{a,b,c,d\} = \{a\}$

(iii) True

As $\{2,6,10,14\} \cap \{3,7,11,15\} = \varnothing$.

(iv) True

As $\{2,6,10\} \cap \{3,7,11\} = \varnothing$

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