Exercise 1.3

Question 1:

Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces:

(i) $\{2,3,4\}\ldots\{1,2,3,4,5\}$

(ii) $\{a,b,c\}\ldots\{b,c,d\}$

(iii) $\{x : x \text{ is a student of class XI of your school}\}\ldots\{x : x \text{ student of your school}\}$

(iv) $\{x : x \text{ is a circle in the plane}\}\ldots\{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

(v) $\{x : x \text{ is a triangle in a plane}\}\ldots\{x : x \text{ is a rectangle in the plane}\}$

(vi) $\{x : x \text{ is an equilateral triangle in a plane}\}\ldots\{x : x \text{ is a triangle in the same plane}\}$

(vii) $\{x : x \text{ is an even natural number}\}\ldots\{x : x \text{ is an integer}\}$

Solution 1:

(i) $\{2,3,4\} \subset \{1,2,3,4,5\}$

(ii) $\{a,b,c\} \not\subset \{b,c,d\}$

(iii) $\{x : x \text{ is a student of class XI of your school}\} \subset \{x : x \text{ is student of your school}\}$

(iv) $\{x : x \text{ is a circle in the plane}\} \not\subset \{x : x \text{ is a circle in the same plane with radius 1 unit}\}$

(v) $\{x : x \text{ is a triangle in a plane}\} \not\subset \{x : x \text{ is a rectangle in the plane}\}$

(vi) $\{x : x \text{ is an equilateral triangle in a plane}\} \subset \{x : x \text{ in a triangle in the same plane}\}$

(vii) $\{x : x \text{ is an even natural number}\} \subset \{x : x \text{ is an integer}\}$

Question 2:

Examine whether the following statements are true or false:

(i) $\{a,b\} \not\subset \{b,c,a\}$

(ii) $\{a,e\} \subset \{x : x \text{ is a vowel in the English alphabet}\}$

(iii) $\{1,2,3\} \subset \{1,3,5\}$

(iv) $\{a\} \subset \{a,b,c\}$

(v) $\{a\} \in (a,b,c)$

(vi) $\{x : x \text{ is an even natural number less than } 6\} \subset \{x : x \text{ is a natural number which divide } 36\}$

Solution 2:

(i) False. Each element of $\{a,b\}$ is also an element of $\{b,c,a\}$.

(ii) True, a,e are two vowels of the English alphabet.

(iii) False. $2 \in \{1,2,3\}$; however, $2 \notin \{1,3,5\}$

(iv) True. Each element of {a} is also an element of {a, b, c}.

(v) False. The element of {a, b, c} are a, b, c. Therefore, $\{a\} \subset \{a,b,c\}$

(vi) True. $\{x : x \text{ is an even natural number less than } 6\} = \{2,4\}$

$\{x : x \text{ is a natural number which divides } 36\} = \{1,2,3,4,6,9,12,18,36\}$

Question 3:

Let $A = \{1,2,\{3,4\},5\}$. Which of the following statements are incorrect and why?

(i) $\{3,4\} \subset A$

(ii) $\{3,4\} \in A$

(iii) $\{\{3,4\}\} \subset A$

(iv) $1 \in A$

(v) $1 \subset A$

(vi) $\{1,2,5\} \subset A$

(vii) $\{1,2,5\}\in A$

(viii) $\{1,2,3\}\subset A$

(ix) $\varnothing\in A$

(x) $\varnothing\subset A$

(xi) $\{\varnothing\}\subset A$

Solution 3:

$A=\{1,2,\{3,4\},5\}$

(i) The statement $\{3,4\}\subset A$ is incorrect because $3\in\{3,4\}$; however, $3\notin A$.

(ii) The statement $\{3,4\}\in A$ is correct because $\{3,4\}$ is an element of A.

(iii) The statement $\{\{3,4\}\}\subset A$ is correct because $\{3,4\}\in\{\{3,4\}\}$ and $\{3,4\}\in A$.

(iv) The statement $1\in A$ is correct because 1 is an element of A.

(v) The statement $1\subset A$ is incorrect because an element of a set can never be a subset of itself.

(vi) The statement $\{1,2,5\}\subset A$ is correct because each element of $\{1,2,5\}$ is also an element of A.

(vii) The statement $\{1,2,5\}\in A$ is incorrect because $\{1,2,5\}$ is not an element of A.

(viii) The statement $\{1,2,5\}\subset A$ is incorrect because $3\in\{1,2,3\}$; however, $3\notin A$.

(ix) The statement $\varnothing\in A$ is incorrect because $\varnothing$ is not an element of A.

(x) The statement $\varnothing\subset A$ is correct because $\varnothing$ is a subset of every set.

(xi) The statement $\{\varnothing\}\subset A$ is incorrect because, $\varnothing$ is a subset of A and it is not an element of A.

Question 4:

Write down all the subsets of the following sets:

(i) $\{a\}$

(ii) $\{a,b\}$

(iii) $\{1,2,3\}$

(iv) $\varnothing$

Solution 4:

(i) The subsets of $\{a\}$ are $\varnothing$ and $\{a\}$.

(ii) The subsets $\{a,b\}$ are $\varnothing, \{a\}, \{b\}$, and $\{a,b\}$.

(iii) The subsets of $\{1,2,3\}$ are $\varnothing$, $\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}$ and $\{1,2,3\}$.

(iv) The only subset of $\varnothing$ is $\varnothing$.

Question 5:

How many elements has $P(A), if\ A=\varnothing$?

Solution 5:

We know that if A is a set with m elements i.e., $n(A)=m$, then $n[p(A)]=2^m$.

If $A=\varnothing$, then $n(A)=0$.

$\therefore n[P(A)]=2^0=1$

Hence, $P(A)$ has one element.

Question 6:

Write the following as intervals:

(i) $\{x : x \in R, -4 < x \le 6\}$

(ii) $\{x : x \in R, -12 < x < -10\}$

(iii) $\{x : x \in R, 0 \le x < 7\}$

(iv) $\{x : x \in R, 3 \le x \le 4\}$

Solution 6:

(i) $\{x : x \in R, -4 < x \le 6\} = (-4, 6]$

(ii) $\{x : x \in R, -12 < x < -10\} = (-12, -10)$

(iii) $\{x : x \in R,\ 0 \le x < 7\} = [0, 7)$

(iv) $\{x : x \in R,\ 3 \le x \le 4\} = [3, 4]$

Question 7:

Write the following intervals in set-builder form:

(i) $(-3,0)$

(ii) $[6,12]$

(iii) $(6,12]$

(iv) $[-23,5)$

Solution 7:

(i) $(-3,0)=\{x:x\in R,-3<x<0\}$

(ii) $[6,12]=\{x:x\in R,6\le x\le 12\}$

(iii) $(6,12]=\{x:x\in R,6<x\le 12\}$

(iv) $[-23,5)=\{x:x\in R,-23\le x<5\}$

Question 8:

What universal set (s) would you propose for each of the following:

(i) The set of right triangles

(ii) The set of isosceles triangles

Solution 8:

(i) For the set of right triangles, the universal set can be the set of triangles or the set of polygons.

(ii) For the set of isosceles triangles, the universal set can be the set of triangles or the set of polygons or the set of two-dimensional figures.

Question 9:

Given the sets $A=\{1,3,5\},B=\{2,4,6\}$ and $C=\{0,2,4,6,8\}$, which of the following may be considered as universals set (s) for all the three sets A, B and C

(i) $\{0,1,2,3,4,5,6\}$

(ii) $\varnothing$

(iii) $\{0,1,2,3,4,5,6,7,8,9,10\}$

(iv) $\{1,2,3,4,5,6,7,8\}$

Solution 9:

(i) It can be seen that $A \subset \{0,1,2,3,4,5,6\}$

$B \subset \{0,1,2,3,4,5,6\}$

However, $C \not\subset \{0,1,2,3,4,5,6\}$

Therefore, the set $\{0,1,2,3,4,5,6\}$ cannot be the universal set for the sets A, B, and C.

(ii) $A \not\subset \varnothing, B \not\subset \varnothing, C \not\subset \varnothing$

Therefore, $\varnothing$ cannot be the universal set for the sets A, B and C.

(iii) $A \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$B \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

$C \subset \{0,1,2,3,4,5,6,7,8,9,10\}$

Therefore, the set $\{0,1,2,3,4,5,6,7,8,9,10\}$ is the universal set for the sets A, B, and C.

(iv) $A \subset \{1,2,3,4,5,6,7,8\}$

$B \subset \{1,2,3,4,5,6,7,8\}$

However, $C \not\subset \{1,2,3,4,5,6,7,8\}$

Therefore, the set $\{1,2,3,4,5,6,7,8\}$ cannot be the universal set for the sets A, B and C.