NCERT Class 12 Maths

Solutions

Chapter - 13

Exercise 13.4

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer.

(*i*)

X	0	1	2
P(X)	0.4	0.4	0.2

(*ii*)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	– 0.1	0.3

(*iii*)

Y	– 1	0	1
P(Y)	0.6	0.1	0.2

(*iv*)

Z	3	2	1	0	– 1
P(Z)	0.3	0.2	0.4	0.1	0.05

Sol. (*i*) Since $p_i > 0$ and $\Sigma p_i = 0.4 + 0.4 + 0.2 = 1$, therefore, **it is** the probability distribution of a random variable.

(*ii*) Since $P(X = 3) = -0.1 < 0$, therefore **it is not** the probability distribution of a random variable.

(*iii*) Since $\Sigma p_i = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$, therefore **it is not** the probability distribution of a random variable.

(*iv*) Since $\Sigma p_i = 0.3 + 0.2 + 0.4 + 0.1 + 0.05 = 1.05 \neq 1$, therefore, **it is not** the probability distribution of a random variable.

2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable?

Sol. When two balls are drawn from the urn, they may contain no black ball, one black ball or two black balls.

$\therefore$ X, the number of black balls; can assume values 0, 1 and 2.

Since, X is a number whose values are defined on the outcomes of a random experiment, therefore, X is a random variable.

3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Sol. Let h denote the number of heads and t, the number of tails when a coin is tossed 6 times. Then

X = difference between h and t = $| h - t |$

Now,	h	:	0	1	2	3	4	5	6
therefore	t	:	6	5	4	3	2	1	0
and hence	X	:	6	4	2	0	2	4	6

$\therefore$ Possible values of X are 6, 4, 2, 0.

4. Find the probability distribution of

(*i*) number of heads in two tosses of a coin.

(*ii*) number of tails in the simultaneous tosses of three coins.

(*iii*) number of heads in four tosses of a coin.

Sol. (*i*) The sample space of the random experiment 'a coin is tossed twice' is

$$S = \{HH, HT, TH, TT\}. \text{ Therefore } n(S) = 4.$$

Let X denote the random variable 'number of heads', then X can take the values 0, 1 or 2.

$$P(X = 0) = P(\text{no head}) = P(\{T\ T\}) = \frac{1}{4}$$

$$P(X = 1) = P(\text{one head}) = P(\{HT, TH\}) = \frac{2}{4} = \frac{1}{2}$$

$$P(X = 2) = P(\text{two heads}) = P(\{H\ H\}) = \frac{1}{4}$$

$\therefore$ The probability distribution of X is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(*ii*) The sample space of the random experiment 'three coins are tossed simultaneously' is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$\therefore$ $n(S) = 8$

Let X denote the random variable 'number of tails', then X can take the values 0, 1, 2 or 3.

$$P(X = 0) = P(\text{no tail}) = P(\{HHH\}) = \frac{1}{8}$$

$P(X = 1) = P(\text{one tail}) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X = 2) = P(\text{two tails}) = P(\{HTT, THT, TTH\}) = \frac{3}{8}$

$P(X = 3) = P(\text{three tails}) = P(\{TTT\}) = \frac{1}{8}$

∴ The probability distribution of X is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(*iii*) The sample space of the random experiment 'a coin is tossed four times' is

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

$n(S) = 2^4 = 16$

Let X denote the random variable 'number of heads', then X can take the values 0, 1, 2, 3 or 4.

$P(X = 0) = P(\text{no head}) = P(\{TTTT\}) = \frac{1}{16}$

$P(X = 1) = P(\text{one head}) = P(\{HTTT, THTT, TTHT, TTTH\})$

$= \frac{4}{16} = \frac{1}{4}$

$P(X = 2) = P(\text{two heads})$

$= P(\{HHTT, HTHT, HTTH, THHT, THTH, TTHH\})$

$= \frac{6}{16} = \frac{3}{8}$

$P(X = 3) = P(\text{three heads})$

$= P(\{HHHT, HHTH, HTHH, THHH\}) = \frac{4}{16} = \frac{1}{4}$

$P(X = 4) = P(\text{four heads}) = P(\{HHHH\}) = \frac{1}{16}$

∴ The probability distribution of X is

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(*i*) number greater than 4

(*ii*) six appears on at least one die.

Sol. (*i*) The sample space of the random experiment 'a die is tossed' is S = {1, 2, 3, 4, 5, 6}. Therefore, $n(S) = 6$

Let *s* denote success *i.e.*, getting a number greater than 4 on the dice and *f*, the failure

$P(s) = P(\text{a number greater than } 4) = P(\{5, 6\}) = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad P(f) = 1 - P(s) = 1 - \frac{1}{3} = \frac{2}{3}$

Let X denote the number of successes in two tosses of a die, then X can take values 0, 1, 2.

$P(X = 0) = P(\text{no success}) = P(\{ff\}) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(X = 1) = P(\text{one success}) = \mathbf{P(\{sf, fs\})}$

$$= \mathbf{P(s)\,P(f) + P(f)\,P(s)} = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$$

$P(X = 2) = P(\text{two successes}) = P(\{ss\}) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$\therefore$ The probability distribution of X is

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(*ii*) We know that when a dice is tossed, sample space is

$\{1, 2, 3, 4, 5, 6\} \quad \therefore \quad n(S) = 6.$

Let E be the event of getting a 6 on the dice.

$\therefore$ E = {6}. Therefore, $n(E) = 1$ and hence $P(E) = \frac{1}{6}$.

Let s denote success and f, the failure.

$P(s) = P(6 \text{ appears on at least one die})$

$= P(\{6 \text{ appears on one die or 6 appears on both dice}\})$

= P(6 apperas on first dice and does not appear on second dice) + P(6 does not appear on first dice and 6 appears on second dice) + P(6 appears on both the dice)

$$= \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{5}{36} + \frac{5}{36} + \frac{1}{36} = \frac{11}{36}$$

$\therefore \quad P(f) = 1 - P(s) = 1 - \frac{11}{36} = \frac{25}{36}$

Let X denote the number of successes in two tosses of a dice; then X can take values 0 or 1.

$P(X = 0) = P(\text{no success}) = P(f) = \frac{25}{36}$

$P(X = 1) = P(\text{one success}) = P(s) = \frac{11}{36}$

$\therefore$ The probability distribution of X is

X	0	1
P(X)	$\frac{25}{36}$	$\frac{11}{36}$

6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Sol. Let p = P(a defective bulb) = P(D) = $\frac{6}{30} = \frac{1}{5}$

and q = P(not-defective) = P(a good one) = P(G)

$= 1 - p = 1 - \frac{1}{5} = \frac{4}{5}$

Let X denote the number of defective bulbs when a sample of 4 bulbs is drawn, then X can take the values 0, 1, 2, 3 or 4.

P(X = 0) = P(no defective) = P(all good ones)

$= P(GGGG) = P(G)\,P(G)\,P(G)\,P(G) = q^4 = \left(\frac{4}{5}\right)^4 = \frac{256}{625}$

P(X = 1) = P(one defective in a sample of four)

= P(DGGG or GDGG or GGDG or GGGD)

$= 4\,P(D)\,P(GGG) = 4pq^3 = 4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P(X = 2) = P(two defectives in a sample of four)

= P(DDGG, DGDG, DGGD, GDDG, GGDD, GDGD)

$= 6\,P(DD)\,P(GG) = 6p^2q^2 = 6\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P(X = 3) = P(three defectives in a sample of four)

= P(GDDD, DGDD, DDGD, DDDG)

$= 4\,P(DDD)\,P(G) = 4p^3q = 4\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right) = \frac{16}{625}$

P(X = 4) = P(four defectives in a sample of four)

$= P(DDDD) = p^4 = \left(\frac{1}{5}\right)^4 = \frac{1}{625}$

∴ The probability distribution of X is

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Remark: Solution of this Q.N. 6 can be better done by formula of Binomial Distribution given at No. 8 in this chapter under Heading "Lesson at a glance" page 818, *i.e* following the method of applying Binomial distribution given in Exercise 13.5.

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

Sol. A coin is biased (not true) and it is given that "head is 3 times as likely to occur as a tail".

∴ P(H) = 3P (Tail).

Let P(Tail) = p. Therefore, P(Head) = $3p$

We know that P(H) + P(T) = 1

$\therefore\ 3p + p = 1$ or $4p = 1\ \therefore\ p = \frac{1}{4}$ *i.e.,* $P(T) = \frac{1}{4}$

$\therefore\ P(H) = 3p = \frac{3}{4}$.

Let X denote the random variable "Number of Tails" in two tosses of the coin.

$\therefore$ X = 0, 1, 2.

P(X = 0) = P(getting no tail *i.e.,* both heads) = P(H)P(H)

$$= \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

P(X = 1) = P(getting one tail) *i.e.,* P(getting one head and one tail)

= P(HT) + P(TH) = P(H) P(T) + P(T) P(H)

$$= \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{16} + \frac{3}{16} = \frac{6}{16}$$

P(X = 2) = P(getting both tails) = P(TT) = P(T) P(T)

$$= \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$$

$\therefore$ Required probability distribution is

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

8. A random variable X has the following probability distribution:

X	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(X)	**0**	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Determine

(*i*) *k*   (*ii*) P(X < 3)   (*iii*) P(X > 6)   (*iv*) P(0 < X < 3).

Sol. (*i*) We know that for a probability distribution

$$\sum_{i=1}^{n} p_i \text{ or } \Sigma P(X) \text{ i.e., } P(X = 0) + P(X = 1) + P(X = 2) + \ldots + P(X = 7) = 1$$

Putting values from the given table,

$\therefore\ 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

or $10k^2 + 9k - 1 = 0$

or $10k^2 + 10k - k - 1 = 0$ or $10k(k + 1) - 1(k + 1) = 0$

or $(k + 1)(10k - 1) = 0$

$\therefore$ Either $k + 1 = 0$ or $10k - 1 = 0$

i.e., $k = -1$ rejected

[$\because$ P(X = 1) = k (given) and hence must be > 0 by def. of P.D.]

$\therefore\ 10k - 1 = 0$ or $k = \frac{1}{10}$ ...(1)

(*ii*) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0 + k + 2k = 3k = \frac{3}{10}$ $\left[\because k = \frac{1}{10} \text{ by (1)}\right]$

(*iii*) $P(X > 6) = P(X = 7) = 7k^2 + k$

$= 7\left(\frac{1}{10}\right)^2 + \frac{1}{10}$ $\left[\because k = \frac{1}{10} \text{ by (1)}\right]$

$= \frac{7}{100} + \frac{1}{10} = \frac{7+10}{100} = \frac{17}{100}$.

(*iv*) $P(0 < X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k = 3k = \frac{3}{10}$ $\left[\because k = \frac{1}{10} \text{ by (1)}\right]$

9. The random variable X has a probability distribution P(X) of the following form, where *k* is some number:

$$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

(*a*) Determine the value of *k*.

(*b*) Find P(X < 2), P(X ≤ 2), P(X ≥ 2).

Sol. (*a*) Since $\Sigma P(X) = 1$

$\therefore\ P(X = 0) + P(X = 1) + P(X = 2) = 1$

($\because$ Given: $P(X) = 0$ otherwise $\Rightarrow$ $P(X) = 0$ for all $X \geq 3$)

$\Rightarrow\ k + 2k + 3k = 1 \Rightarrow 6k = 1 \quad \therefore\ k = \frac{1}{6}$ ...(*i*)

(*b*) $P(X < 2) = P(X = 0) + P(X = 1) = k + 2k = 3k = 3 \times \frac{1}{6}$ (By (*i*)]

$= \frac{1}{2}$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= k + 2k + 3k = 6k = 6 \times \frac{1}{6}$ [By (*i*)] $= 1$

$P(X \geq 2) = P(X = 2) = 3k = 3 \times \frac{1}{6}$ [By (*i*)] $= \frac{1}{2}$.

10. Find the mean number of heads in three tosses of a fair coin.

Sol. The sample space for three tosses of a fair coin is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore\ n(S) = 8$

Let X denote the number of heads, in **three** tosses of a fair coin;

then X can take values 0, 1, 2 or 3.

$P(X = 0) = P(\text{no head}) = P(\{TTT\}) = \frac{1}{8}$

$P(X = 1) = P(\text{one head}) = P(\{HTT, THT, TTH\}) = \frac{3}{8}$

$P(X = 2) = P(\text{two heads}) = P(\{HHT, HTH, THH\}) = \frac{3}{8}$

$P(X = 3) = P(\text{three heads}) = P(\{HHH\}) = \frac{1}{8}$

∴ The probability distribution of X is

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

∴ **Mean number of heads = $\mu = \Sigma XP(X)$**

$$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

[Sum of products of corresponding entries of first row and second row]

$$= \frac{0+3+6+3}{8} = \frac{12}{8} = \frac{3}{2} = 1.5.$$

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

Sol. Two dice are thrown simultaneously.

∴ S = {(1, 1), (1, 2), ... (1, 6),
(2, 1), (2, 2), ... (2, 6),
⋮
(6, 1), (6, 2), ... (6, 6)}

∴ $n(S) = 6^2 = 36$

Let X denote the number of 'sixes' obtained on tossing two dice.

∴ X can take values 0, 1 or 2.

$P(X = 0) = P(\text{no six}) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

($\because$ Probability of not getting a six on a dice $= 1 - \frac{1}{6} = \frac{5}{6}$)

(Also because 36 – 10 – 1 = 25)

P(X = 1) = P(one six and one non-six)

= P{(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)} = $\frac{10}{36}$

$P(X = 2) = P(\text{six and six}) = \frac{1}{36}$

[$\because$ {(6, 6)} is the event of getting six and six]

∴ Probability distribution of X is

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

We know that

Expectation (or Mean) of X = E(X) = Σ X P(X)

$$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{0 + 10 + 2}{36} = \frac{12}{36} = \frac{1}{3}.$$

12. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

Sol. First six positive integers are 1, 2, 3, 4, 5, 6.

n(S) = Total number of ways of selecting two positive integers from 1 to 6 is $^6C_2 = \frac{6.5}{2.1} = 15$

Since X is the larger of the two selected positive integers and 1 is not larger than any of them, therefore, X can take values 2, 3, 4, 5 or 6.

$P(X = 2) = P(2 \text{ and a number less than } 2) = P(\{2, 1\}) = \frac{n(E)}{n(S)} = \frac{1}{15}$

$P(X = 3) = P(3 \text{ and a number less than } 3) = P(\{3, 1\}, \{3, 2\}) = \frac{2}{15}$

$P(X = 4) = P(4 \text{ and a number less than } 4) = P(\{4, 1\}, \{4, 2\}, \{4, 3\}) = \frac{3}{15}$

$P(X = 5) = P(5 \text{ and a number less than } 5)$

$= P(\{5, 1\}, \{5, 2\}, \{5, 3\}, \{5, 4\}) = \frac{4}{15}$

$P(X = 6) = P(6 \text{ and a number less than } 6)$

$= P(\{6, 1\}, \{6, 2\}, \{6, 3\}, \{6, 4\}, \{6, 5\}) = \frac{5}{15}$

∴ Probability distribution of X is

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

We know that

E(X) = μ = Σ X P(X)

$$= 2 \times \frac{1}{15} + 3 \times \frac{2}{15} + 4 \times \frac{3}{15} + 5 \times \frac{4}{15} + 6 \times \frac{5}{15}$$

$$= \frac{2 + 6 + 12 + 20 + 30}{15} = \frac{70}{15} = \frac{14}{3}.$$

13. Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

Sol. We know that the total number of cases on tossing two dice

$= n(S) = 6^2 = 36$

We know that numbers on a dice are 1, 2, 3, 4, 5, 6.

∴ Numbers on the other dice are also 1, 2, 3, 4, 5, 6.

∴ Sum of the numbers on the two dice is 2, 3, 4, ..., 11, 12.

Let X denote the sum of the numbers obtained on the two dice. Then X can take the values 2, 3, 4, ... , 12.

$P(X = 2) = P(\{(1, 1)\}) = \frac{1}{36}$

$P(X = 3) = P(\{(1, 2), (2, 1)\}) = \frac{2}{36}$

$P(X = 4) = P(\{(1, 3), (3, 1), (2, 2)\}) = \frac{n(E)}{n(S)} = \frac{3}{36}$

$P(X = 5) = P(\{(1, 4), (4, 1), (2, 3), (3, 2)\}) = \frac{4}{36}$

$P(X = 6) = P(\{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)\}) = \frac{5}{36}$

$P(X = 7) = P(\{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}) = \frac{6}{36}$

$P(X = 8) = P(\{(2, 6), (6, 2), (3, 5), (5, 3), (4, 4)\}) = \frac{5}{36}$

$P(X = 9) = P(\{(3, 6), (6, 3), (4, 5), (5, 4)\}) = \frac{4}{36}$

$P(X = 10) = P(\{(4, 6), (6, 4), (5, 5)\}) = \frac{3}{36}$

$P(X = 11) = P(\{(5, 6), (6, 5)\}) = \frac{2}{36}$

$P(X = 12) = P(\{(6, 6)\}) = \frac{1}{36}$.

∴ The probability distribution of X is

X :	2	3	4	5	6	7	8	9	10	11	12
P(X) or p_i :	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$.

X	P(X) or p_i	X P(X)	$X^2P(X) = X . X P(X)$
2	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$
3	$\frac{2}{36}$	$\frac{6}{36}$	$\frac{18}{36}$
4	$\frac{3}{36}$	$\frac{12}{36}$	$\frac{48}{36}$
5	$\frac{4}{36}$	$\frac{20}{36}$	$\frac{100}{36}$
6	$\frac{5}{36}$	$\frac{30}{36}$	$\frac{180}{36}$

7	$\frac{6}{36}$	$\frac{42}{36}$	$\frac{294}{36}$
8	$\frac{5}{36}$	$\frac{40}{36}$	$\frac{320}{36}$
9	$\frac{4}{36}$	$\frac{36}{36}$	$\frac{324}{36}$
10	$\frac{3}{36}$	$\frac{30}{36}$	$\frac{300}{36}$
11	$\frac{2}{36}$	$\frac{22}{36}$	$\frac{242}{36}$
12	$\frac{1}{36}$	$\frac{12}{36}$	$\frac{144}{36}$

$\therefore \quad \Sigma X\ P(X) = \frac{252}{36} = 7$ and $\Sigma X^2\ P(X) = \frac{1974}{36} = \frac{329}{6}$

$\therefore$ Mean μ (= E(X)) = Expectation of X = Σ XP(X) = 7

and **variance σ^2 of X (or of P.D.) = $\Sigma X^2\ P(X) - \mu^2$**

$$= \frac{329}{6} - (7)^2 = \frac{329}{6} - 49 = \frac{329 - 294}{6} = \frac{35}{6} = 5.833$$

$\therefore$ S.D. $\sigma = \sqrt{\text{Variance}} = \sqrt{\frac{35}{6}} = \sqrt{5.833} = 2.41.$

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

Sol. Given: Ages of 15 students are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years.

Out of these given ages:

Age X (given discrete variate) has **distinct** values 14, 15, 16, 17, 18, 19, 20, 21

Age (X)	:	14	15	16	17	18	19	20	21
No. of students	:	2	1	2	3	1	2	3	1 = 15

($\because$ Out of the given 15 ages, 2 students have age 14 years, therefore probability that a student has age 14 years is $\frac{2}{15}\left(= \frac{n(E)}{n(S)}\right)$ and so on)

$\therefore$ The probability distribution of X is

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

We know that

Mean μ = Σ X P(X)

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{14 \times 2 + 15 \times 1 + 16 \times 2 + 17 \times 3 + 18 \times 1 + 19 \times 2 + 20 \times 3 + 21 \times 1}{15}$$

$$= \frac{28 + 15 + 32 + 51 + 18 + 38 + 60 + 21}{15} = \frac{263}{15} = 17.53$$

Also **Var (X) = Σ X² P(X) – μ²**

$$= (14)^2 \times \frac{2}{15} + (15)^2 \times \frac{1}{15} + (16)^2 \times \frac{2}{15} + (17)^2 \times \frac{3}{15} + (18)^2 \times \frac{1}{15} + (19)^2 \times \frac{2}{15} + (20)^2 \times \frac{3}{15} + (21)^2 \times \frac{1}{15} - \left(\frac{263}{15}\right)^2$$

$$\Rightarrow \text{Var. (X)} = \frac{392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441}{15} - \left(\frac{263}{15}\right)^2$$

$$= \frac{4683}{15} - \frac{69169}{225} = \frac{15 \times 4683 - 69169}{225} = \frac{70245 - 69169}{225} = \frac{1076}{225} = 4.78$$

$$\therefore \text{ S.D. (X)} = \sqrt{\text{Var. (X)}} = \sqrt{4.78} = 2.19.$$

15. In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).

Sol. Given: Here X takes values 0 and 1.

Also given $P(X = 0) = 30\% = \frac{30}{100} = \frac{3}{10}$

and $P(X = 1) = 70\% = \frac{70}{100} = \frac{7}{10}$

∴ Probability distribution of X is

X	0	1
P(X)	$\frac{3}{10}$	$\frac{7}{10}$

We know that $E(X) (= \mu) = \Sigma X P(X) = 0 \times \frac{3}{10} + 1 \times \frac{7}{10} = \frac{7}{10}$ = 0.7

and $\text{Var (X)} = \Sigma X^2 P(X) - [E(X)]^2$

$$= \left(0^2 \times \frac{3}{10} + 1^2 \times \frac{7}{10}\right) - (0.7)^2 = 0.7 - 0.49 = 0.21.$$

Choose the correct answer in each of the Exercises 16 and 17:

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B) 2 (C) 5 (D) $\frac{8}{3}$.

Sol. Let X denote the number obtained on throwing the die, then X takes values 1, 2 or 5. (given)

$P(X = 1) = \frac{3}{6} = \frac{1}{2}$ ($\because$ 1 is written on three faces out of six)

$P(X = 2) = \frac{2}{6} = \frac{1}{3}$ ($\because$ 2 is written on two faces out of six)

$P(X = 5) = \frac{1}{6}$ ($\because$ 5 is written on one face out of six)

$\therefore$ Probability distribution of X is

X	1	2	5
P(X)	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

$\therefore$ Mean $= \Sigma X\, P(X) = 1 \times \frac{1}{2} + 2 \times \frac{1}{3} + 5 \times \frac{1}{6}$

$$= \frac{3+4+5}{6} = \frac{12}{6} = 2$$

$\therefore$ The correct option is (B).

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

(A) $\frac{37}{221}$ (B) $\frac{5}{13}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$.

Sol. Here X, the number of aces obtained on drawing two cards from a deck of cards will take values 0, 1 or 2.

P(X = 0) = P(no ace) = P(both non-ace cards)

$= \frac{^{48}C_2}{^{52}C_2}$ [$\because$ $n(S) = {}^{52}C_2$ and $n(E) = {}^{48}C_2$]

[$\because$ We know that deck of (of course 52 cards) has 4 aces and hence (52 – 4 = 48 non-ace cards)]

$$= \frac{48 \times 47}{52 \times 51} = \frac{4 \times 3 \times 4 \times 47}{4 \times 13 \times 3 \times 17} = \frac{188}{221}$$

P(X = 1) = P(one ace and one non-ace card)

$$= \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2} = \frac{4 \times 48}{\frac{52 \times 51}{2 \times 1}} = \frac{32}{221}$$

$$P(X = 2) = P(\text{two aces}) = \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{\frac{4 \times 3}{2 \times 1}}{\frac{52 \times 51}{2 \times 1}} = \frac{4 \times 3}{52 \times 51} = \frac{1}{221}$$

$\therefore$ Probability distribution of X is

X	0	1	2
P(X)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$E(X) = \Sigma X\, P(X) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + 2 \times \frac{1}{221}$$

$$= \frac{32 + 2}{221} = \frac{34}{221} = \frac{17 \times 2}{17 \times 13} = \frac{2}{13}$$

$\therefore$ The correct option is (D).