Exercise 13.3

1. **An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red?**

Sol. Given: Urn contains 5 red and 5 black balls

($\Rightarrow$ Total balls = 5 + 5 = 10)

Let E_1: first draw gives a red ball

E_2: first draw gives a black ball

(then E_1 and E_2 are mutually exclusive and exhaustive events.)

$$\therefore \quad P(E_1) = \frac{5}{10} = \frac{1}{2} \quad \text{and} \quad P(E_2) = \frac{5}{10} = \frac{1}{2}$$

When the first draw gives a red ball, two additional red balls are put in the urn so that its contents are 7 (= 5 + 2) red and 5 black balls. When the first draw gives a black ball, two additional black balls are put in the urn so that its contents are 5 red and 7 (= 5 + 2) black balls.

Let A: second draw gives a red ball

Required probability = P(A)

= Probability that first ball is red and then second ball drawn after two red are added in the urn is also red + Probability that first ball is black and second is red.

$$= \frac{1}{2} \times \frac{7}{12} + \frac{1}{2} \times \frac{5}{12} = \frac{7}{24} + \frac{5}{24} = \frac{12}{24} = \frac{1}{2}.$$

2. **A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.**

Sol. Let event E_1: first bag is selected

E_2: second bag is selected

(then E_1 and E_2 are mutually exclusive and exhaustive events.)

$$\therefore \quad P(E_1) = P(E_2) = \frac{1}{2}$$

Let A : ball drawn is red.

Then $P(A / E_1)$ = Probability that a red ball is chosen from bag first $= \frac{4}{4+4} = \frac{4}{8}$ and Similarly $P(A / E_2) = \frac{2}{8}$

We have to find $P(E_1 / A)$.

i.e., Probability (that the ball is from bag I given that it is red).

We know that $P(E_1/A) = \dfrac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$

(By Baye's Theorem)

Putting values, $= \dfrac{\frac{1}{2} \times \frac{4}{8}}{\frac{1}{2} \times \frac{4}{8} + \frac{1}{2} \times \frac{2}{8}}$

Multiplying by L.C.M. = 8 $= \dfrac{4}{4+2} = \dfrac{4}{6} = \dfrac{2}{3}$.

3. Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Sol. Let event E_1: a student is residing in hostel

E_2: a student is a day scholar (*i.e.,* not residing in hostel)

then E_1 and E_2 are mutually exclusive and exhaustive events.

Given: 60% students reside in hostel and 40% don't reside in hostel (*i.e.,* are day scholars)

$\therefore \quad P(E_1) = \dfrac{60}{100} = \dfrac{3}{5}$ and $P(E_2) = \dfrac{40}{100} = \dfrac{2}{5}$

Let event E: a student attains A grade

Given: 30% of hostelers get A grade and 20% day scholars get A grade.

$\therefore$ $P(E/E_1)$ *i.e.,* probability that a hostlier gets A grade

$= \dfrac{30}{100} = \dfrac{3}{10}$ and $P(E/E_2) = \dfrac{20}{100} = \dfrac{2}{10}$

We have to find $P(E_1/E)$. *i.e.,* P(a student getting A grade resides in hostel)

We know that

$P(E_1/E) = \dfrac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)}$ (By Baye's Theorem)

$= \dfrac{\frac{3}{5} \times \frac{3}{10}}{\frac{3}{5} \times \frac{3}{10} + \frac{2}{5} \times \frac{2}{10}}$

Multiply every term by 50, $= \dfrac{9}{9+4} = \dfrac{9}{13}$.

4. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the

probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

Sol. Let event E_1: the student knows the answer

E_2: the student guesses the answer

then E_1 and E_2 are mutually exclusive and exhaustive events.

Given: $P(E_1) = \frac{3}{4}$ and $P(E_2) = \frac{1}{4}$

Let event A: the student answered correctly.

Then $P(A / E_1) = 1$ ($\because$ When he knows the answer, he answers correctly is a **sure event**)

and $P(A / E_2)$ *i.e.,* P(he answers correctly when he guesses the answer) $= \frac{1}{4}$ (given)

We have to find $P(E_1 / A)$.

Required probability = Probability that the student knows the answer given that he answered it correctly

$$P(E_1 / A) = \frac{P(E_1)\, P(A / E_1)}{P(E_1)\, P(A / E_1) + P(E_2)\, P(A / E_2)} \quad \text{(By Baye's Theorem)}$$

Putting values

$$= \frac{\frac{3}{4} \times 1}{\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{1}{4}} = \frac{\frac{3}{4}}{\frac{3}{4} + \frac{1}{16}} = \frac{\frac{3}{4}}{\frac{12+1}{16}} = \frac{\frac{3}{4}}{\frac{13}{16}} = \frac{3}{4} \times \frac{16}{13} = \frac{12}{13}.$$

5. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (*i.e.,* if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 per cent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

Sol. Let event E_1: the person has the disease

and $E_2 = E_1'$: the person is healthy

then E_1 and E_2 are mutually exclusive and exhaustive events.

Given: 0.1 per cent (*i.e.,* 0.1%) of the population actually has the disease.

$$\therefore \quad P(E_1) = \frac{0.1}{100} = \frac{1}{1000},$$

and therefore $P(E_2) = P(E_1') = 1 - P(E_1) = \frac{999}{1000}$

Let A : test result is positive

Given: $P(A / E_1)$ = P(Test result of a person having disease is positive) = 99% = $\frac{99}{100}$ and $P(A/E_2) = 0.5\% = \frac{0.5}{100} = \frac{5}{1000}$.

We have to find $P(E_1/A)$.

i.e., probability that a person has the disease given that his test result is positive.

We know that $P(E_1/A) = \dfrac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2)}$

(By Baye's Theorem)

$$= \frac{\frac{1}{1000} \times \frac{99}{100}}{\frac{1}{1000} \times \frac{99}{100} + \frac{999}{1000} \times \frac{5}{1000}}$$

Multiplying every term by 1000 × 1000,

$$= \frac{990}{990 + 4995} = \frac{990}{5985} = \frac{198}{1197} = \frac{22}{133}.$$

6. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

Sol. Let event E_1: the chosen coin is two headed

E_2: the chosen coin is biased

E_3: the chosen coin is unbiased

then E_1, E_2, E_3 are mutually exclusive and exhaustive events.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

[$\because$ Each of the **three** coins has equal chance of being chosen]

Let event A: the tossed coin shows head. Then $P(A/E_1)$ *i.e.,* P(A coin having head on both faces shows head)

$$= \frac{2}{2} = 1,$$

Given: $P(A/E_2) = \frac{75}{100} = \frac{3}{4}$, $P(A/E_3) = \frac{1}{2}$.

($\because$ Third coin is unbiased *i.e.,* true coin)

We have to find $P(E_1/A)$. *i.e.,* probability (that the coin chosen is two headed coin given that the coin shows heads)

We know that

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

(By Baye's Theorem)

Putting values, $= \dfrac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}}$

Multiplying every term by 3, $= \dfrac{1}{1+\frac{3}{4}+\frac{1}{2}} = \dfrac{1}{\frac{9}{4}} = \dfrac{4}{9}$.

7. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Sol. Let event E_1: the insured person is a scooter driver

E_2: the insured person is a car driver

E_3: the insured person is a truck driver

then E_1, E_2, E_3 are mutually exclusive and exhaustive events.

Total number of insured vehicles

$$= 2000 + 4000 + 6000 = 12000$$

$\therefore \quad P(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{2000}{12000} = \dfrac{1}{6}, \quad P(E_2) = \dfrac{4000}{12000} = \dfrac{1}{3},$

$P(E_3) = \dfrac{6000}{12000} = \dfrac{1}{2}$

Let event A: insured person meets with an accident.

Given: $P(A/E_1)$ *i.e.,* P(An insured scooter driver meets with an accident) $= 0.01 = \dfrac{1}{100}$, $P(A/E_2) = 0.03 = \dfrac{3}{100}$,

and $\quad P(A/E_3) = 0.15 = \dfrac{15}{100}$

We have to find $P(E_1/A)$. *i.e.,* P(The person is a scooter driver given that an insured person has met with an accident).

We know that

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)}$$

(By Baye's Theorem)

$$= \frac{\frac{1}{6}\times\frac{1}{100}}{\frac{1}{6}\times\frac{1}{100}+\frac{1}{3}\times\frac{3}{100}+\frac{1}{2}\times\frac{15}{100}}$$

Multiplying every term by 600, $= \dfrac{1}{1+6+45} = \dfrac{1}{52}$.

8. A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

Sol. Let event E_1: the item is produced by machine A

and E_2: the item is produced by machine B

then E_1, E_2 are mutually exclusive and exhaustive events.

Given: $P(E_1) = 60\% = \frac{60}{100} = \frac{3}{5}$, $P(E_2) = 40\% = \frac{40}{100} = \frac{2}{5}$

Let D: the chosen item is defective

Given: $P(D/E_1)$ *i.e.,* P(an item produced by machine A is defective) $= \frac{2}{100}$, $P(D/E_2) = \frac{1}{100}$

We have to find $P(E_2/D)$ = P(An item is produced by machine B given that it is defective)

We know that

$$P(E_2/D) = \frac{P(E_2)\,P(D/E_2)}{P(E_1)\,P(D/E_1) + P(E_2)\,P(D/E_2)}$$

(By Baye's Theorem)

$$= \frac{\frac{2}{5} \times \frac{1}{100}}{\frac{3}{5} \times \frac{2}{100} + \frac{2}{5} \times \frac{1}{100}}$$

Multiplying every term by 500, $= \frac{2}{6+2} = \frac{2}{8} = \frac{1}{4}$.

9. Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3, if the second group wins. Find the probability that the new product introduced was by the second group.

Sol. Let event E_1: first group wins

and event E_2: second group wins

then E_1, E_2 are mutually exclusive and exhaustive events.

Given: $P(E_1) = 0.6 = \frac{6}{10}$, $P(E_2) = 0.4 = \frac{4}{10}$

Let A: the new product is introduced

Given: $P(A/E_1)$ = P(New product being introduced if first group wins) $= 0.7 = \frac{7}{10}$, and $P(A/E_2) = 0.3 = \frac{3}{10}$.

We have to find $P(E_2/A)$. (*i.e.,* probability that second group wins given that the new product was introduced)

We know that

$$P(E_2/A) = \frac{P(E_2)\,P(A/E_2)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$ (By Baye's Theorem)

$$= \frac{\frac{4}{10} \times \frac{3}{10}}{\frac{6}{10} \times \frac{7}{10} + \frac{4}{10} \times \frac{3}{10}}$$

Multiplying every term by 100,

$$= \frac{12}{42 + 12} = \frac{12}{54} = \frac{2}{9}.$$

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

Sol. Let event E_1: the die shows 1, 2, 3 or 4 $\Rightarrow$ $n(E_1) = 4$

and E_2: the die shows 5 or 6 $\Rightarrow$ $n(E_2) = 2$

then E_1, E_2 are mutually exclusive and exhaustive events.

$$\therefore \quad P(E_1) = \frac{4}{6} = \frac{2}{3},\ P(E_2) = \frac{2}{6} = \frac{1}{3}$$

[$\because$ We know that sample space on tossing a dice is = {1, 2, 3, 4, 5, 6} and has 6 points]

Let A: the girl obtained exactly one head

then $P(A/E_1) = P(\text{exactly one head when a coin is tossed once}) = \frac{1}{2}$

and $P(A/E_2) = P(\text{exactly one head when a coin is tossed three times}) = \frac{3}{8}$

[$\because$ the sample space when a coin is tossed three times is S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} Let event E: exactly one head appears, then E = {HTT, THT, TTH}

and $P(E) = (P(A/E_2)$ here in this question$) = \frac{3}{8}$]

We have to find $P(E_1/A)$. *i.e.,* P(A dice shows 1, 2, 3, 4 given that she gets exactly one head).

We know that

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)} \quad \text{(By Baye's Theorem)}$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{3}{8}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{8}}$$

Multiplying by L.C.M. = 24, $= \frac{8}{8 + 3} = \frac{8}{11}.$

11. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, where as

the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by A?

Sol. Let event E_1: operator A is on job

E_2: operator B is on job

E_3: operator C is on job

then E_1, E_2, E_3 are mutually exclusive and exhaustive.

Given: $P(E_1) = 50\% = \frac{50}{100}$, $P(E_2) = \frac{30}{100}$, $P(E_3) = \frac{20}{100}$

Let D: a defective item is produced

Given: $P(D/E_1)$ = P(an item produced by operator A on the job is defective) $= \frac{1}{100}$, $P(D/E_2) = \frac{5}{100}$, $P(D/E_3) = \frac{7}{100}$

We have to find $P(E_1/D)$. = P(An item was produced by A given that it is defective)

We know that

$$P(E_1/D) = \frac{P(E_1)\,P(D/E_1)}{P(E_1)\,P(D/E_1) + P(E_2)\,P(D/E_2) + P(E_3)\,P(D/E_3)}$$

(By Baye's Theorem)

$$\text{Putting values, } = \frac{\frac{50}{100}\times\frac{1}{100}}{\frac{50}{100}\times\frac{1}{100}+\frac{30}{100}\times\frac{5}{100}+\frac{20}{100}\times\frac{7}{100}}$$

Multiplying every term by 100 × 100

$$= \frac{50}{50+150+140} = \frac{50}{340} = \frac{5}{34}.$$

12. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

Sol. Let E_1: the lost card is a diamond

and $E_2 = E_1'$: the lost card is not a diamond

then E_1, E_2 are mutually exclusive and exhaustive.

We know that there are 13 diamond cards in a pack of 52 cards.

$\therefore$ $P(E_1) = \frac{13}{52} = \frac{1}{4}$, $P(E_2) = P(E_1') = 1 - P(E_1) = 1 - \frac{1}{4} = \frac{3}{4}$

Let event A: two cards drawn from the remaining pack are diamonds

then $P(A/E_1)$ = P(drawing two diamond cards when the lost card is a diamond card) $= \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{\frac{12\times 11}{2\times 1}}{\frac{51\times 50}{2\times 1}} = \frac{132}{2550}$

[$\because$ The lost card is a diamond, therefore, there are 12 diamond cards in the remaining pack of 51 cards]

and $\quad P(A/E_2) = \dfrac{^{13}C_2}{^{51}C_2} = \dfrac{\dfrac{13\times 12}{2\times 1}}{\dfrac{51\times 50}{2\times 1}} = \dfrac{156}{2550}$

[$\because$ The lost card is not a diamond, therefore, there are 13 diamond cards in the remaining pack of 51 cards]

We have to find $P(E_1/A)$ *i.e.,* P(lost card is a diamond card given that the two cards drawn from the remaining pack of 51 cards are diamonds)

We know that

$$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)} \text{ (By Baye's Theorem)}$$

$$= \frac{\dfrac{1}{4}\times\dfrac{132}{2550}}{\dfrac{1}{4}\times\dfrac{132}{2550}+\dfrac{3}{4}\times\dfrac{156}{2550}}$$

Multiplying every term by 4 × 2550, $= \dfrac{132}{132+468} = \dfrac{132}{600} = \dfrac{11}{50}$.

13. Probability that A speaks truth is $\dfrac{4}{5}$. A coin is tossed. A reports that a head appears. The probability that actually there was head is

(A) $\dfrac{4}{5}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{2}{5}$.

Sol. Let event E_1: a head appears on a coin.

and $E_2 = E_1'$: a head does not appear

then E_1, E_2 are mutually exclusive and exhaustive events

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Let event H: (Person) A **reports** that a head appears

Given: $P(H/E_1)$ = P(Person A reports that a head appears when actually there is head) = P(A speaks truth) = $\dfrac{4}{5}$

and hence $P(H/E_2)$ = P(A tells a lie) = $1 - \dfrac{4}{5} = \dfrac{1}{5}$

we have to find $P(E_1/H)$ = P(A head (actually) appears; reported that a head has appeared)

We know that

$$P(E_1/H) = \frac{P(E_1)\,P(H/E_1)}{P(E_1)\,P(H/E_1) + P(E_2)\,P(H/E_2)} \quad \text{(By Baye's Theorem)}$$

Putting values $= \dfrac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{1}{2} \times \frac{1}{5}}$

Multiplying by L.C.M. = 10, $= \dfrac{4}{4+1} = \dfrac{4}{5}$.

$\therefore$ The correct option is (A).

14. If A and B are two events such that $A \subset B$ and $P(B) \neq 0$, then which of the following is correct?

(A) $P(A/B) = \dfrac{P(B)}{P(A)}$ **(B) $P(A/B) < P(A)$**

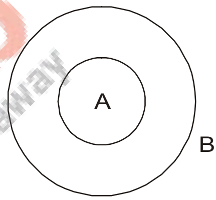
(C) $P(A/B) \geq P(A)$ **(D) None of these.**

Sol. $A \subset B \Rightarrow A \cap B = A$

$\therefore$ $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$ ...(*i*)

Since, $P(B) \neq 0$,

$\therefore$ $0 < P(B) \leq 1 \Rightarrow \dfrac{1}{P(B)} \geq 1$



$\therefore$ From (*i*), $P(A/B) = P(A) \times \dfrac{1}{P(B)} \geq P(A)$

Hence, the correct option is (C).