NCERT Class 12 Maths

Solutions

Chapter - 13

Exercise 13.1

1. Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find P(E / F) and P(F / E).

Sol.

Given: P(E) = 0.6, P(F) = 0.3, P(E ∩ F) = 0.2

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$\text{and } P(F/E) = \frac{P(F \cap E)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}.$$

2. Compute P(A / B), if P(B) = 0.5 and P(A ∩ B) = 0.32.

Sol.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}.$$

3. If P(A) = 0.8, P(B) = 0.5 and P(B / A) = 0.4, find

(*i*) P(A ∩ B) (*ii*) P(A / B) (*iii*) P(A ∪ B).

Sol.

(*i*) **Given:** P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4

Now P(B / A) = 0.4 (given)

$$\Rightarrow \frac{P(B \cap A)}{P(A)} = 0.4 \quad \Rightarrow \quad \frac{P(A \cap B)}{0.8} = 0.4$$

$$\Rightarrow \quad P(A \cap B) = 0.8 \times 0.4 = 0.32.$$

(*ii*) $$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{100} \times \frac{10}{5} = \frac{64}{100} = 0.64$$

(*iii*) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.5 - 0.32 = 1.3 - 0.32 = 0.98.$

4. Evaluate P(A ∪ B), if 2P(A) = P(B) = $\frac{5}{13}$ and P(A / B) = $\frac{2}{5}$.

Sol. Given: $2P(A) = P(B) = \frac{5}{13} \quad \Rightarrow \quad P(A) = \frac{5}{26}, P(B) = \frac{5}{13}$

Now, $P(A / B) = \frac{2}{5}$ (given) $\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{2}{5}$

$$\Rightarrow \quad P(A \cap B) = \frac{2}{5} P(B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{5 + 10 - 4}{26} = \frac{11}{26}.$$

5. If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{7}{11}$, find

(*i*) P(A ∩ B) (*ii*) P(A / B) (*iii*) P(B / A).

Sol. (*i*) **Given:** $P(A \cup B) = \frac{7}{11}$

$$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = \frac{7}{11}$$

$$\Rightarrow \quad \frac{6}{11} + \frac{5}{11} - P(A \cap B) = \frac{7}{11}$$

$$\Rightarrow \quad P(A \cap B) = 1 - \frac{7}{11} = \frac{4}{11}.$$

(*ii*) $$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}.$$

(*iii*) $$P(B / A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{11}}{\frac{6}{11}} = \frac{2}{3}.$$

Determine P(E / F) in Exercises 6 to 9.

6. A coin is tossed three times, where

(*i*) E: head on third toss,
F: heads on first two tosses

(*ii*) E: at least two heads, F: at most two heads

(*iii*) E: at most two tails, F: at least one tail.

Sol. We know that the sample space for the random experiment 'a coin is tossed three times' is

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore$ n(S) = 8

(*i*) E: head on third toss

$\Rightarrow$ E = {HHH, HTH, THH, TTH} $\therefore$ n(E) = 4

F: heads on first two tosses

$\Rightarrow$ F = {HHH, HHT} $\therefore$ n(F) = 2

$\therefore$ E $\cap$ F = {HHH} $\Rightarrow$ n(E $\cap$ F) = 1

$$\text{Hence, } P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2},$$

$$P(F) = \frac{2}{8} = \frac{1}{4}, \quad P(E \cap F) = \frac{1}{8}$$

$$\text{and hence } P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{2}.$$

(*ii*) E : at least two heads

$\Rightarrow$ E = {HHH, HHT, HTH, THH} $\therefore$ n(E) = 4

F : at most two heads

$\Rightarrow$ F = {HHT, HTH, THH, HTT, THT, TTH, TTT}

$\therefore$ n(F) = 7

$\therefore$ E $\cap$ F = {HHT, HTH, THH} $\Rightarrow$ n(E $\cap$ F) = 3

$$\text{Hence, } P(E) = \frac{4}{8} = \frac{1}{2}, \; P(F) = \frac{7}{8},$$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{3}{8}$$

$$\text{and hence } P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{8}}{\frac{7}{8}} = \frac{3}{7}.$$

(*iii*) E : at most two tails

$\Rightarrow$ E = {TTH, THT, HTT, THH, HTH, HHT, HHH}

$\therefore$ n(E) = 7

F : at least one tail

$\Rightarrow$ F = {THH, HTH, HHT, TTH, THT, HTT, TTT}

$\therefore$ n(F) = 7

$\therefore$ E $\cap$ F = {TTH, THT, HTT, THH, HTH, HHT}

$\Rightarrow$ n(E $\cap$ F) = 6

Hence, $P(E) = \frac{7}{8}$, $P(F) = \frac{n(F)}{n(S)} = \frac{7}{8}$, $P(E \cap F) = \frac{6}{8} = \frac{3}{4}$

and hence $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{3}{4}}{\frac{7}{8}} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$.

7. Two cons are tossed once, where

(*i*) E: tail appears on one coin

F: one coin shows head

(*ii*) E: no tail appears,

F: no head appears.

Sol. The sample space for the random experiment 'two coins are tossed once' is

S = {HH, HT, TH, TT} $\therefore$ $n(S) = 4$

(*i*) E : tail appears on one coin

$\Rightarrow$ E = {HT, TH} $\therefore$ $n(E) = 2$

F : one coin shows head

$\Rightarrow$ F = {HT, TH} $\therefore$ $n(F) = 2$

$\therefore$ $E \cap F$ = {HT, TH} $\Rightarrow n(E \cap F) = 2$

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$,

$P(F) = \frac{2}{4} = \frac{1}{2}$, $P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

and hence $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

(*ii*) E : no tail appears

$\Rightarrow$ E = {HH} $\therefore$ $n(E) = 1$

F : no head appears

$\Rightarrow$ F = {TT} $\therefore$ $n(F) = 1$

$\therefore$ $E \cap F = \phi$ $\therefore n(E \cap F) = 0$

Hence, $P(E) = \frac{1}{4}$, $P(F) = \frac{n(F)}{n(S)} = \frac{1}{4}$, $P(E \cap F) = 0$

and hence $P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{\frac{1}{4}} = 0$.

8. A die is thrown three times,

E: 4 appears on the third toss,

F: 6 and 5 appear respectively on first two tosses

Sol. The sample space for the random experiment that a die is thrown three times has $6 \times 6 \times 6 = 6\text{-}^3 = 216$ points *i.e.*, $n(S) = 216$.

Now, E: 4 appears on third toss

= {(1, 1, 4) (1, 2, 4) ... (1, 6, 4) (2, 1, 4) (2, 2, 4) ... (2, 6, 4)
(3, 1, 4) (3, 2, 4) ... (3 6 4) (4, 1, 4) (4, 2, 4) ... (4, 6, 4)
(5, 1, 4) (5, 2, 4) ... (5, 6, 4) (6, 1, 4) (6, 2, 4) ... (6, 6, 4)}

F: 6 and 5 appear respectively on first two tossess
= {(6, 5, 1) (6, 5, 2) (6, 5, 3) (6, 5, 4) (6, 5, 5) (6, 5, 6)}

$\therefore$ $E \cap F = \{(6, 5, 4)\}$

Therefore, $P(F) = \frac{6}{216}$ and $P(E \cap F) = \frac{1}{216}$

Then, $P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$.

9. Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

Sol. Let m, f and s denote the mother, father and son respectively. The sample space is

$S = \{mfs, msf, fms, fsm, smf, sfm\}$

$\therefore$ $n(S) = 6$

E: son on one end $\Rightarrow$ $E = \{mfs, fms, smf, sfm\}$

$\Rightarrow$ $n(E) = 4$ $\Rightarrow$ $P(E) = \frac{4}{6}$

F: father in middle $\Rightarrow$ $F = \{mfs, sfm\}$

$\Rightarrow$ $n(F) = 2$ $\Rightarrow$ $P(F) = \frac{2}{6}$

$\therefore$ $E \cap F = \{mfs, sfm\}$ $\Rightarrow$ $n(E \cap F) = 2$ $\Rightarrow$ $P(E \cap F) = \frac{2}{6}$

Then, $P(E / F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{6}}{\frac{2}{6}} = 1$.

10. A black and a red dice are rolled.

(*a*) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(*b*) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Sol. Let x denote the outcome on black die and y denote the outcome on red die. The sample space is

$S = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$ $\Rightarrow$ $n(S) = 6 \times 6 = 36$

(*a*) Let E : sum $x + y > 9$ $\Rightarrow$ $x + y = 10, 11, 12$

$\Rightarrow$ $E = \{(6, 4), (6, 5), (6, 6), (5, 5), (5, 6), (4, 6)\}$

F : black die resulted in a 5.

$\Rightarrow$ $F = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

$\therefore$ $n(F) = 6$ $\Rightarrow$ $P(F) = \frac{6}{36}$

$E \cap F = \{(5, 5), (5, 6)\}$

$\therefore \quad n(E \cap F) = 2 \quad \Rightarrow \quad P(E \cap F) = \frac{2}{36}$

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}.$$

(*b*) Let E : sum $x + y = 8$

$\Rightarrow \quad E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

F : red die resulted in a number less than 4

$\Rightarrow \quad F = \{(x, y) : x \in \{1, 2, 3, 4, 5, 6\}$ and $y \in \{1, 2, 3\}\}$

$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

$\therefore \quad n(F) = 6 \times 3 = 18 \quad \Rightarrow \quad P(F) = \frac{18}{36}$

$E \cap F = \{(5, 3), (6, 2)\}$

$\therefore \quad n(E \cap F) = 2 \quad \Rightarrow \quad P(E \cap F) = \frac{2}{36}$

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{1}{9}.$$

11. A fair die is rolled. Consider events E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5}. Find

(*i*) P(E / F) and P(F / E)　　(*ii*) P(E / G) and P(G / E)

(*iii*) P((E ∪ F) / G) and P((E ∩ F) / G).

Sol. Sample space $S = \{1, 2, 3, 4, 5, 6\} \quad \Rightarrow \quad n(S) = 6$

Given: Event $E = \{1, 3, 5\}$, $F = \{2, 3\}$, $G = \{2, 3, 4, 5\}$

(*i*) $\therefore \quad E \cap F = \{3\}$

$n(E) = 3$, $n(F) = 2$, $n(G) = 4$, $n(E \cap F) = 1$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{3}{6}$, $P(F) = \frac{2}{6}$, $P(G) = \frac{4}{6}$, $P(E \cap F) = \frac{1}{6}$

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{2}{6}} = \frac{1}{2}$$

$$\text{and } P(F/E) = \frac{P(F \cap E)}{P(E)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

(*ii*) Again we can observe that $E \cap G = \{3, 5\}$

$\Rightarrow \quad n(E \cap G) = 2 \quad \Rightarrow \quad P(E \cap G) = \frac{2}{6}$

$$\therefore \quad P(E/G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{1}{2}$$

$$\text{and } P(G/E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}.$$

(*iii*) We can see that $E \cup F = \{1, 2, 3, 5\}$, $E \cap F = \{3\}$

$(E \cup F) \cap G = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\} = \{2, 3, 5\}$

$$\Rightarrow \quad n((E \cup F) \cap G) = 3 \quad \Rightarrow \quad P((E \cup F) \cap G) = \frac{3}{6}$$

$(E \cap F) \cap G = \{3\} \cap \{2, 3, 4, 5\} = \{3\}$

$$\Rightarrow \quad n((E \cap F) \cap G) = 1 \quad \Rightarrow \quad P((E \cap F) \cap G) = \frac{1}{6}$$

$$\therefore \quad P((E \cup F)/G) = \frac{P((E \cup F) \cap G)}{P(G)} = \frac{\frac{3}{6}}{\frac{4}{6}} = \frac{3}{4}$$

$$\text{and } \quad P((E \cap F)/G) = \frac{P((E \cap F) \cap G)}{P(G)} = \frac{\frac{1}{6}}{\frac{4}{6}} = \frac{1}{4}.$$

12. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that
(*i*) the youngest is a girl, (*ii*) at least one is a girl?

Sol. Let the first (elder) child be denoted by capital letter and the second (younger) by a small letter. The sample space is

$S = \{Bb, Bg, Gb, Gg\}$ $\quad \therefore \quad n(S) = 4$

Let E: both children are girls, then $E = \{Gg\}$

$$\Rightarrow \quad n(E) = 1 \quad \Rightarrow \quad P(E) = \frac{1}{4}$$

(*i*) Let F: the youngest (second) child is a girl, then

$$F = \{Bg, Gg\} \quad \therefore \quad n(F) = 2 \quad \Rightarrow \quad P(F) = \frac{n(F)}{n(S)} = \frac{2}{4}$$

$$E \cap F = \{Gg\} \quad \therefore \quad n(E \cap F) = 1 \quad \Rightarrow \quad P(E \cap F) = \frac{1}{4}$$

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}.$$

(*ii*) Let F : at least one (child) is a girl.
then $F = \{Bg, Gb, Gg\}$

$\therefore \quad n(\text{F}) = 3 \quad \Rightarrow \quad \text{P(F)} = \dfrac{n(\text{F})}{n(\text{S})} = \dfrac{3}{4}$

$\text{E} \cap \text{F} = \{\text{G}g\} \quad \therefore \quad n(\text{E} \cap \text{F}) = 1 \quad \Rightarrow \quad \text{P}(\text{E} \cap \text{F}) = \dfrac{1}{4}$

$$\therefore \quad \text{P(E / F)} = \frac{\text{P}(\text{E} \cap \text{F})}{\text{P(F)}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$$

13. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Sol. Total number of questions = 300 + 200 + 500 + 400 = 1400

$\therefore \quad n(\text{S}) = 1400$

Let E : selected question is easy

and F : selected question is a multiple choice question

then E ∩ F : selected question is an easy multiple choice question

$n(\text{E} \cap \text{F}) = 500, \quad n(\text{F}) = 500 + 400 = 900$

$$\therefore \quad \text{P}(\text{E} \cap \text{F}) = \frac{n(\text{E} \cap \text{F})}{n(\text{S})} = \frac{500}{1400}$$

$$\text{and} \quad \text{P(F)} = \frac{n(\text{F})}{n(\text{S})} = \frac{900}{1400}$$

$$\therefore \quad \text{Required probability} = \text{P(E / F)} = \frac{\text{P}(\text{E} \cap \text{F})}{\text{P(F)}} = \frac{\frac{500}{1400}}{\frac{900}{1400}} = \frac{5}{9}.$$

14. Given that the two numbers appearing on throwing two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Sol. Sample space for the random experiment of throwing two dice is

$\text{S} = \{(x, y) : x, y \in \{1, 2, 3, 4, 5, 6\}\}$

$n(\text{S}) = 6 \times 6 = 36$

Let E : the sum of numbers on the dice is 4.

$\Rightarrow \quad \text{E} = \{(1, 3), (2, 2), (3, 1)\}$

$\Rightarrow \quad n(\text{E}) = 3 \quad \Rightarrow \quad \text{P(E)} = \dfrac{n(\text{E})}{n(\text{S})} = \dfrac{3}{36}$

Let F : numbers appearing on the dice are different

$\Rightarrow \quad \text{F} = \text{S} - \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore \quad n(\text{F}) = 36 - 6 = 30 \quad \Rightarrow \quad \text{P(F)} = \dfrac{30}{36}$

Also, $E \cap F = \{(1, 3), (3, 1)\}$ $\therefore$ $n(E \cap F) = 2$

$$\therefore \quad P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{2}{36}$$

$$\therefore \quad \text{Required probability} = P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{30}{36}} = \frac{1}{15}.$$

15. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Sol. The sample space is given by

$S = \{(1, H), (1, T), (2, H), (2, T), (3, 1), (3, 2), (3, 3), (3, 4)$
$(3, 5), (3, 6), (4, H), (4, T), (5, H), (5, T), (6, 1), (6, 2),$
$(6, 3), (6, 4), (6, 5), (6, 6)\}$

$\Rightarrow$ $n(S) = 20$

Let E: the coin shows a tail

$\Rightarrow$ $E = \{(1, T), (2, T), (4, T), (5, T)\}$ $\Rightarrow$ $n(E) = 4$

Let F: at least one die shows a 3

$\Rightarrow$ $F = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (6, 3)\}$

$\Rightarrow$ $n(F) = 7$

$E \cap F = \phi$

$$\Rightarrow \quad n(E \cap F) = 0 \Rightarrow P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{0}{20} = 0.$$

$$\therefore \quad \text{Required probability} = P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0.$$

In each of the Exercises 16 and 17 choose the correct answer:

16. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A/B)$ is

(A) 0 **(B) $\frac{1}{2}$**

(C) not defined **(D) 1.**

Sol. $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A \cap B)}{0}$ which is not defined

$\therefore$ The correct option is (C).

17. If A and B are events such that $P(A/B) = P(B/A)$, then

(A) $A \subset B$ but $A \neq B$ **(B) $A = B$**

(C) $A \cap B = \phi$ **(D) $P(A) = P(B)$.**

Sol. Given: $P(A/B) = P(B/A)$

$$\Rightarrow \quad \frac{P(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)} \Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}$$

$[\because \ A \cap B = B \cap A \ \therefore \ P(A \cap B) = P(B \cap A)]$

Dividing both sides by $P(A \cap B)$, $\frac{1}{P(B)} = \frac{1}{P(A)}$

Cross-multiplying, $P(A) = P(B)$ $\quad \therefore$ The correct option is (D).