NCERT Class 12 Maths Solutions

Chapter - 11

Exercise 11.1

1. If a line makes angles 90°, 135°, 45° with the *x*, *y* and *z*-axes respectively, find its direction cosines.

Sol. We know that direction cosines of a line making angles α, β, γ with the *x*, *y* and *z*-axes respectively are **cos α, cos β, cos γ.**

Here α = 90°, β = 135° and γ = 45°.

Therefore, direction cosines of the required line are cos 90°, cos 135° and cos 45° = 0, $\frac{-1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

$$\left[\underset{\text{(II)}}{\cos 135° = \cos (180° - 45°)} = -\cos 45° = \frac{-1}{\sqrt{2}}\right]$$

Result. $\mathbf{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1.}$

2. Find the direction cosines of a line which makes equal angles with the co-ordinate axes.

Sol. Let a line make equal angles α, α, α with the co-ordinate axes.

$\therefore$ Direction cosines of the line are $\cos\alpha, \cos\alpha, \cos\alpha$ ...(*i*)

$\therefore$ $\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$ $[\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$

$\Rightarrow 3\cos^2\alpha = 1 \quad \Rightarrow \quad \cos^2\alpha = \frac{1}{3} \quad \Rightarrow \quad \cos\alpha = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$

Putting $\cos\alpha = \pm\frac{1}{\sqrt{3}}$ in (*i*), direction cosines of the required line making equal angles with the co-ordinate axes are

$$\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}.$$

Very Important Remark. Therefore, direction cosines of a line making equal angles with the co-ordinate axes in the positive (*i.e.,* first) octant are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

3. If a line has direction ratios – 18, 12, – 4, then what are its direction cosines?

Sol. We know that if a, b, c are direction ratios of a line, then direction cosines of the line are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}} \quad ...(i)$$

Here, direction ratios of the line are

$-18, 12, -4 = a, b, c$

$\therefore \sqrt{a^2+b^2+c^2} = \sqrt{(-18)^2+(12)^2+(-4)^2} = \sqrt{324+144+16}$

$= \sqrt{484} = 22$

Putting these values in (*i*), direction cosines of the required line are

$$\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} = \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}.$$

4. Show that the points (2, 3, 4), (– 1, – 2, 1), (5, 8, 7) are collinear.

Sol. The given points are A(2, 3, 4), B(– 1, – 2, 1) and C(5, 8, 7).

$\therefore$ Direction ratios of the line joining A and B are

$-1-2, -2-3, 1-4$ $\quad | \; x_2 - x_1, y_2 - y_1, z_2 - z_1$

i.e., $-3, -5, -3$ ...(*i*)

$= a_1, b_1, c_1$ (say)

Again direction ratios of the line joining B and C are

$5-(-1), 8-(-2), 7-1 = 6, 10, 6$ ...(*ii*)

$= a_2, b_2, c_2$ (say)

From (*i*) and (*ii*) direction ratios of AB and BC are proportional

i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\left[\because \frac{-3}{6} = \frac{-5}{10}, \frac{-3}{6} \left(\text{each} = \frac{-1}{2}\right)\right]$

Therefore, AB is parallel to BC. But point B is common to both AB and BC. Hence, points A, B, C are collinear.

5. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, – 4), (– 1, 1, 2) and (– 5, – 5, – 2).

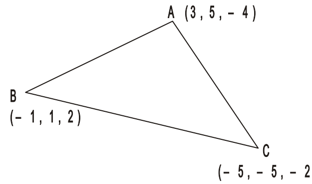
Sol. Direction ratios of line AB are – 1 – 3, 1 – 5, 2 – (– 4)

i.e., – 4, – 4, 6 | $x_2 - x_1, y_2 - y_1, z_2 - z_1$

Dividing each by $\sqrt{a^2 + b^2 + c^2} = \sqrt{(-4)^2 + (-4)^2 + 6^2}$

$$= \sqrt{16 + 16 + 36} = \sqrt{68} = \sqrt{4 \times 17} = 2\sqrt{17}.$$

direction cosines of line AB are



$$\frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}.$$

i.e., $\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$

Direction ratios of line BC are

$$-5 - (-1), -5 - 1, -2 - 2 = -4, -6, -4$$

Dividing each by $\sqrt{(-4)^2 + (-6)^2 + (-4)^2} = \sqrt{16 + 36 + 16}$

$$= \sqrt{68} = \sqrt{4 \times 17} = 2\sqrt{17}$$

Direction cosines of line BC are $\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$

i.e., $\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

Direction ratios of line CA are

$$3 - (-5), 5 - (-5), -4 - (-2) = 8, 10, -2$$

Dividing each by $\sqrt{(8)^2 + (10)^2 + (-2)^2} = \sqrt{64 + 100 + 4}$

$$= \sqrt{168} = \sqrt{4 \times 42} = 2\sqrt{42}.$$

Direction ratios of line CA are

$$\frac{8}{2\sqrt{42}}, \frac{10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}} = \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$$

Note. If l, m, n are direction cosines of a line, then $-l$, $-m$, $-n$ are also direction cosines of the same line.