NCERT Class 12 Maths Solutions

Chapter - 10

Vector Algebra

Exercise 10.3

1. **Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2, respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.**

Sol. Given: $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$. We know that

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Putting values, $\cos\theta = \frac{\sqrt{6}}{\sqrt{3}(2)}$

$$= \frac{\sqrt{6}}{\sqrt{3}\sqrt{4}} = \frac{\sqrt{6}}{\sqrt{12}} = \sqrt{\frac{6}{12}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \qquad \therefore \quad \theta = \frac{\pi}{4}.$$

2. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

Sol. Given: Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$.

$\therefore |\vec{a}| = \sqrt{1+4+9} = \sqrt{14} \quad \left| \because |x\hat{i} + y\hat{j} + z\hat{k}| = \sqrt{x^2+y^2+z^2} \right.$

and $\quad |\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$

Also, $\vec{a}\cdot\vec{b}$ = Product of coefficients of $\hat{i}$ + Product of coefficient of $\hat{j}$ + Product of coefficients of $\hat{k}$

$$= 1(3) + (-2)(-2) + 3(1) = 3 + 4 + 3 = 10$$

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

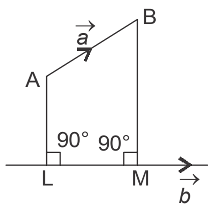
We know that $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{10}{\sqrt{14}\sqrt{14}} = \frac{10}{14} = \frac{5}{7}$

$\therefore \quad \theta = \cos^{-1}\frac{5}{7}$.

3. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

Sol. Let $\vec{a} = \hat{i} - \hat{j} = \hat{i} - \hat{j} + 0\hat{k}$

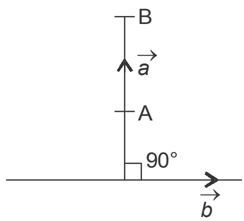
and $\vec{b} = \hat{i} + \hat{j} = \hat{i} + \hat{j} + 0\hat{k}$



Projection of vector $\vec{a}$ and $\vec{b}$

$$= \text{Length LM} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$= \frac{(1)(1) + (-1)(1) + 0(0)}{\sqrt{(1)^2 + (1)^2 + 0^2}} = \frac{1 - 1 + 0}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0.$$



Remark. If projection of vector $\vec{a}$ on $\vec{b}$ is zero, then vector $\vec{a}$ is perpendicular to vector $\vec{b}$.

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

Sol. Let $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ and $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$

We know that projection of vector $\vec{a}$ on vector $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$$= \frac{1(7) + 3(-1) + 7(8)}{\sqrt{(7)^2 + (-1)^2 + (8)^2}} = \frac{7 - 3 + 56}{\sqrt{49 + 1 + 64}} = \frac{60}{\sqrt{114}}.$$

5. Show that each of the given three vectors is a unit vector: $\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$, $\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$, $\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$.

Also show that they are mutually perpendicular to each other.

Sol. Let $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$ ...(*i*)

$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}) = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$ ...(*ii*)

$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}$ ...(*iii*)

$$\therefore \quad |\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2} = \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}}$$

$$= \sqrt{\frac{49}{49}} = \sqrt{1} = 1$$

$$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2} = \sqrt{\frac{9}{49} + \frac{36}{49} + \frac{4}{49}} = \sqrt{\frac{49}{49}}$$

$$= \sqrt{1} = 1$$

$$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2} = \sqrt{\frac{36}{49} + \frac{4}{49} + \frac{9}{49}} = \sqrt{\frac{49}{49}}$$

$$= \sqrt{1} = 1$$

$\therefore$ Each of the three given vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ is a unit vector.

From (*i*) and (*ii*),

$$\vec{a} \,.\, \vec{b} = \left(\frac{2}{7}\right) . \left(\frac{3}{7}\right) + \left(\frac{3}{7}\right)\left(\frac{-6}{7}\right) + \left(\frac{6}{7}\right)\left(\frac{2}{7}\right)$$

[$\vec{a} \,.\, \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$]

$$= \frac{6}{49} - \frac{18}{49} + \frac{12}{49} = \frac{6-18+12}{49} = \frac{0}{49} = 0$$

$\therefore$ $\vec{a}$ and $\vec{b}$ are perpendicular to each other.

From (*ii*) and (*iii*),

$$\vec{b} \cdot \vec{c} = \left(\frac{3}{7}\right)\left(\frac{6}{7}\right) + \left(\frac{-6}{7}\right)\left(\frac{2}{7}\right) + \frac{2}{7}\left(\frac{-3}{7}\right)$$

$$= \frac{18}{49} - \frac{12}{49} - \frac{6}{49} = \frac{18-12-6}{49} = \frac{0}{49} = 0$$

$\therefore$ $\vec{b}$ and $\vec{c}$ are perpendicular to each other.

From (*i*) and (*iii*),

$$\vec{a} \cdot \vec{c} = \frac{2}{7}\left(\frac{6}{7}\right) + \frac{3}{7}\left(\frac{2}{7}\right) + \left(\frac{6}{7}\right)\left(\frac{-3}{7}\right)$$

$$= \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = \frac{12+6-18}{49} = \frac{0}{49} = 0$$

$\therefore$ $\vec{a}$ and $\vec{c}$ are perpendicular to each other.

Hence, $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors.

6. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

Sol. Given: $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$ ...(*i*)

$\Rightarrow$ $\vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b} = 8$

$\Rightarrow$ $|\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - |\vec{b}|^2 = 8$

[$\because$ We know that $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ and $\vec{b} \cdot \vec{b} = |\vec{b}|^2$ and $\vec{b} \cdot \vec{a} = \vec{a} \cdot \vec{b}$]

$\Rightarrow$ $|\vec{a}|^2 - |\vec{b}|^2 = 8$ ...(*ii*)

Putting $|\vec{a}| = 8|\vec{b}|$ from (*i*) in (*ii*), $64|\vec{b}|^2 - |\vec{b}|^2 = 8$

or $(64 - 1)|\vec{b}|^2 = 8$ $\Rightarrow$ $63|\vec{b}|^2 = 8$

$\Rightarrow$ $|\vec{b}|^2 = \frac{8}{63}$ $\Rightarrow$ $|\vec{b}| = \sqrt{\frac{8}{63}} = \sqrt{\frac{4 \times 2}{9 \times 7}}$

($\because$ Length *i.e.,* modulus of a vector is never negative.)

$\Rightarrow$ $|\vec{b}| = \frac{2}{3}\sqrt{\frac{2}{7}}$

Putting this value of $|\vec{b}|$ in (*i*),

$$|\vec{a}| = 8\left(\frac{2}{3}\sqrt{\frac{2}{7}}\right) = \frac{16}{3}\sqrt{\frac{2}{7}}.$$

7. Evaluate the product $(3\vec{a} - 5\vec{b}) . (2\vec{a} + 7\vec{b})$.

Sol. The given expression $= (3\vec{a} - 5\vec{b}) . (2\vec{a} + 7\vec{b})$

$= (3\vec{a}) . (2\vec{a}) + (3\vec{a}) . (7\vec{b}) - (5\vec{b}) . (2\vec{a}) - (5\vec{b}) . (7\vec{b})$

$= 6\vec{a} . \vec{a} + 21\vec{a} . \vec{b} - 10\vec{b} . \vec{a} - 35\vec{b} . \vec{b}$

$= 6|\vec{a}|^2 + 21\vec{a} . \vec{b} - 10\vec{a} . \vec{b} - 35|\vec{b}|^2$

$[\because \vec{a} . \vec{a} = |\vec{a}|^2$ and $\vec{b} . \vec{b} = |\vec{b}|^2$ and $\vec{b} . \vec{a} = \vec{a} . \vec{b}]$

$= 6|\vec{a}|^2 + 11\vec{a} . \vec{b} - 35|\vec{b}|^2.$

8. Find the magnitude of two vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{1}{2}$.

Sol. Given: $|\vec{a}| = |\vec{b}|$ and angle θ (say) between $\vec{a}$ and $\vec{b}$ is 60° and their scalar (*i.e.,* dot) product $= \frac{1}{2}$

i.e., $\vec{a} . \vec{b} = \frac{1}{2}$

$\Rightarrow |\vec{a}|\,|\vec{b}| \cos\theta = \frac{1}{2}$ $[\because \vec{a} . \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta]$

Putting $|\vec{b}| = |\vec{a}|$ (given) and θ = 60° (given), we have

$|\vec{a}|\,|\vec{a}| \cos 60° = \frac{1}{2}$ $\Rightarrow |\vec{a}|^2\left(\frac{1}{2}\right) = \frac{1}{2}$

Multiplying by 2, $|\vec{a}|^2 = 1$ $\Rightarrow |\vec{a}| = 1$ ...(*i*)

($\because$ Length of a vector is never negative)

$\therefore$ $|\vec{b}| = |\vec{a}| = 1$ [By (*i*)]

$\therefore$ $|\vec{a}| = 1$ and $|\vec{b}| = 1.$

9. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}) . (\vec{x} + \vec{a}) = 12$.

Sol. Given: $\vec{a}$ is a unit vector $\Rightarrow |\vec{a}| = 1$ ...(*i*)

Also given $(\vec{x} - \vec{a}) . (\vec{x} + \vec{a}) = 12$

$\Rightarrow \vec{x} . \vec{x} + \vec{x} . \vec{a} - \vec{a} . \vec{x} - \vec{a} . \vec{a} = 12$

$\Rightarrow |\vec{x}|^2 + \vec{a} . \vec{x} - \vec{a} . \vec{x} - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

Putting $|\vec{a}| = 1$ from (*i*), $|\vec{x}|^2 - 1 = 12$

$\Rightarrow \quad |\vec{x}|^2 = 13 \qquad\qquad \Rightarrow \quad |\vec{x}| = \sqrt{13}$.

(∵ Length of a vector is never negative.)

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

Sol. Given : $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

and $\vec{c} = 3\hat{i} + \hat{j}$.

Now, $\vec{a} + \lambda\vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-\hat{i} + 2\hat{j} + \hat{k})$

$= 2\hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + 2\lambda\hat{j} + \lambda\hat{k}$

$\Rightarrow \quad \vec{a} + \lambda\vec{b} = (2 - \lambda)\hat{j} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$

Again given $\vec{c} = 3\hat{i} + \hat{j} = 3\hat{i} + \hat{j} + 0\hat{k}$.

Because vector $\vec{a} + \lambda\vec{a}$ is perpendicular to $\vec{c}$, therefore,

$(\vec{a} + \lambda\vec{b}) \cdot \vec{c} = 0$

i.e., Product of coefficients of $\hat{i}$ + ... = 0

$\Rightarrow \quad (2 - \lambda)3 + (2 + 2\lambda)1 + (3 + \lambda)0 = 0$

$\Rightarrow \quad 6 - 3\lambda + 2 + 2\lambda = 0 \qquad \Rightarrow \quad -\lambda + 8 = 0$

$\Rightarrow \quad -\lambda = -8 \qquad \Rightarrow \quad \lambda = 8.$

11. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non-zero vectors $\vec{a}$ and $\vec{b}$.

Sol. Let $\vec{c} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a} = l\vec{b} + m\vec{a}$

where $l = |\vec{a}|$ and $m = |\vec{b}|$

Let $\vec{d} = |\vec{a}|\vec{b} - |\vec{b}|\vec{a} = l\vec{b} - m\vec{a}$

Now, $\vec{c} \cdot \vec{d} = (l\vec{b} + m\vec{a}) \cdot (l\vec{b} - m\vec{a})$

$= l^2\,\vec{b} \cdot \vec{b} - lm\,\vec{b} \cdot \vec{a} + lm\,\vec{a} \cdot \vec{b} - m^2\vec{a}\;\vec{a}$

$= l^2|\vec{b}|^2 - lm\,\vec{a} \cdot \vec{b} + lm\,\vec{a} \cdot \vec{b} - m^2|\vec{a}|^2 = l^2|\vec{b}|^2 - m^2|\vec{a}|$

Putting $l = |\vec{a}|$ and $m = |\vec{b}|$,

$= |\vec{a}|^2\,|\vec{b}|^2 - |\vec{b}|^2\,|\vec{a}|^2 = 0$

i.e., $\vec{c} \cdot \vec{d} = 0$

∴ Vectors $\vec{c}$ and $\vec{d}$ are perpendicular to each other.

12. If $\vec{a}\cdot\vec{a} = 0$ and $\vec{a}\cdot\vec{b} = 0$, then what can be concluded about the vector $\vec{b}$?

Sol. Given: $\vec{a}\cdot\vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$ ...(*i*)

($\Rightarrow$ $\vec{a}$ is a zero vector by definition of zero vector.)

Again given $\vec{a}\cdot\vec{b} = 0 \Rightarrow |\vec{a}||\vec{b}|\cos\theta = 0$

Putting $|\vec{a}| = 0$ from (*i*), we have $0|\vec{b}|\cos\theta = 0$

i.e., $0 = 0$ for all (any) vectors $\vec{b}$. $\therefore$ $\vec{b}$ can be any vector.

Note. $(\vec{a} + \vec{b} + \vec{c})^2 = (\vec{a} + (\vec{b} + \vec{c}))^2$

$= \vec{a}^{\,2} + (\vec{b} + \vec{c})^2 + 2\vec{a}\cdot(\vec{b} + \vec{c})$

$[\because (\vec{A} + \vec{B})^2 = \vec{A}^2 + \vec{B}^2 + 2\vec{A}\cdot\vec{B}]$

$= \vec{a}^{\,2} + \vec{b}^{\,2} + \vec{c}^{\,2} + 2\vec{b}\cdot\vec{c} + 2\vec{a}\cdot\vec{b} + 2\vec{a}\cdot\vec{c}$

Using $\vec{a}\cdot\vec{c} = \vec{c}\cdot\vec{a}$

or $(\vec{a} + \vec{b} + \vec{c})^2 = \vec{a}^{\,2} + \vec{b}^{\,2} + \vec{c}^{\,2} + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

13. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

Sol. Because $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, therefore,

$|\vec{a}| = 1, |\vec{b}| = 1$ and $|\vec{c}| = 1$. ...(*i*)

Again given $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Squaring both sides $(\vec{a} + \vec{b} + \vec{c})^2 = 0$
Using formula of **Note** above

$\Rightarrow \vec{a}^{\,2} + \vec{b}^{\,2} + \vec{c}^{\,2} + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

or $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

Putting $|\vec{a}| = 1, |\vec{b}| = 1, |\vec{c}| = 1$ from (*i*),

$1 + 1 + 1 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -3$

Dividing both sides by 2, $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-3}{2}$.

14. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a}\cdot\vec{b} = 0$. But the converse need not be true. Justify your answer with an example.

Sol. **Case I.** Vector $\vec{a} = \vec{0}$. Therefore, by definition of zero vector,

$$|\vec{a}| = 0 \qquad ...(i)$$

$$\therefore \quad \vec{a}.\vec{b} = |\vec{a}||\vec{b}| \cos\theta = 0\,(|\vec{b}| \cos\theta) \qquad \text{[By } (i)\text{]}$$
$$= 0$$

Case II. Vector $\vec{b} = \vec{0}$. Proceeding as above we can prove that $\vec{a}.\vec{b} = 0$

But the converse is not true.

Let us justify it with an example.

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. Therefore, $|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \neq 0$.

Therefore $\vec{a} \neq \vec{0}$ (By definition of Zero Vector)

Let $\quad \vec{b} = \hat{i} + \hat{j} - 2\hat{k}$.

Therefore, $|\vec{b}| = \sqrt{(1)^2 + (1)^2 + (-2)^2} = \sqrt{6} \neq 0$.

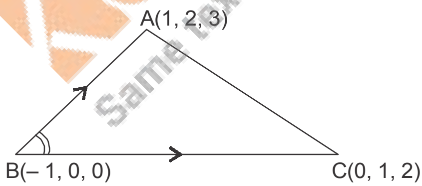
Therefore, $\quad \vec{b} \neq \vec{0}$.

But $\quad \vec{a}.\vec{b} = 1(1) + 1(1) + 1(-2) = 1 + 1 - 2 = 0$

So here $\vec{a}.\vec{b} = 0$ but neither $\vec{a} = \vec{0}$ nor $\vec{b} = \vec{0}$.

15. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (– 1, 0, 0) and (0, 1, 2), respectively, then find ∠ABC.

Sol. **Given:** Vertices A, B, C of a triangle are A(1, 2, 3), B(– 1, 0, 0) and C(0, 1, 2) respectively.



$\therefore$ Position vector (P.V.) of point A (=s $\vec{OA}$) = (1, 2, 3)

$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

Position vector (P.V.) of point B (= $\vec{OB}$) = (– 1, 0, 0)

$$= -\hat{i} + 0\hat{j} + 0\hat{k}$$

and position vector (P.V.) of point C (= $\vec{OC}$) = (0, 1, 2)

$$= 0\hat{i} + \hat{j} + 2\hat{k}$$

We can see from the above figure that ∠ABC is the angle between the vectors $\vec{BA}$ and $\vec{BC}$

Now $\overrightarrow{BA}$ = P.V. of terminal point A – P.V. of initial point B

$= \hat{i} + 2\hat{j} + 3\hat{k} - (-\hat{i} + 0\hat{j} + 0\hat{k})$

$= \hat{i} + 2\hat{j} + 3\hat{k} + \hat{i} - 0\hat{j} - 0\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ...(*i*)

and $\overrightarrow{BC}$ = P.V. of point C – P.V. of point B

$= 0\hat{i} + \hat{j} + 2\hat{k} - (-\hat{i} + 0\hat{j} + 0\hat{k})$

$= 0\hat{i} + \hat{j} + 2\hat{k} + \hat{i} - 0\hat{j} - 0\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$ ...(*ii*)

We know that $\cos \angle ABC = \dfrac{\overrightarrow{BA}\,.\,\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$ | $\cos\theta = \dfrac{\vec{a}\,.\,\vec{b}}{|\vec{a}||\vec{b}|}$

Using (*i*) and (*ii*)

$$= \frac{2(1) + 2(1) + 3(2)}{\sqrt{4+4+9}\,\sqrt{1+1+4}} = \frac{10}{\sqrt{17}\sqrt{6}} = \frac{10}{\sqrt{102}}$$

$\therefore \quad \angle ABC = \cos^{-1} \dfrac{10}{\sqrt{102}}$.

16. Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, – 1) are collinear.

Sol. Given points are A(1, 2, 7), B(2, 6, 3) and C(3, 10, – 1).

$\Rightarrow$ P.V.'s $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ of points A, B, C are

$\overrightarrow{OA} = (1, 2, 7) = \hat{i} + 2\hat{j} + 7\hat{k}$

$\overrightarrow{OB} = (2, 6, 3) = 2\hat{i} + 6\hat{j} + 3\hat{k}$

and $\overrightarrow{OC} = (3, 10, -1) = 3\hat{i} + 10\hat{j} - \hat{k}$

$\therefore$ $\overrightarrow{AB}$ = P.V. of terminal point B – P.V. of initial point A

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - (\hat{i} + 2\hat{j} + 7\hat{k})$

$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$ ...(*i*)

and $\overrightarrow{AC}$ = P.V. of point C – P.V. of point A

$= 3\hat{i} + 10\hat{j} - \hat{k} - (\hat{i} + 2\hat{j} + 7\hat{k})$

$= 3\hat{i} + 10\hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 7\hat{k}$

$= 2\hat{i} + 8\hat{j} - 8\hat{k} = 2(\hat{i} + 4\hat{j} - 4\hat{k})$

$\Rightarrow \quad \overrightarrow{AC} = 2\,\overrightarrow{AB}$ [By (*i*)]

$\Rightarrow$ Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear or parallel. $\left| \because \vec{a} = m\,\vec{b} \right.$

$\Rightarrow$ Points A, B, C are collinear.

($\because$ Vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ have a common point A and hence can't be parallel.)

Remark. When we come to exercise 10.4 and learn that Exercise, we have a second solution for proving points A, B, C to be collinear:

Prove that $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{0}$.

17. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Sol. Let the given (position) vectors be P.V.'s of the points A, B, C respectively.

P.V. of point A is $2\hat{i} - \hat{j} + \hat{k}$ and

P.V. of point B is $\hat{i} - 3\hat{j} - 5\hat{k}$ and

P.V. of point C is $3\hat{i} - 4\hat{j} - 4\hat{k}$.

$\therefore$ $\overrightarrow{AB}$ = P.V. of point B – P.V. of point A

$= \hat{i} - 3\hat{j} - 5\hat{k} - (2\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 3\hat{j} - 5\hat{k} - 2\hat{i} + \hat{j} - \hat{k}$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$ ...(*i*)

and $\overrightarrow{BC}$ = P.V. of point C – P.V. of point B

$= 3\hat{i} - 4\hat{j} - 4\hat{k} - (\hat{i} - 3\hat{j} - 5\hat{k}) = 3\hat{i} - 4\hat{j} - 4\hat{k} - \hat{i} + 3\hat{j} + 5\hat{k}$

$= 2\hat{i} - \hat{j} + \hat{k}$ ...(*ii*)

and $\overrightarrow{AC}$ = P.V. of point C – P.V. of point A

$= 3\hat{i} - 4\hat{j} - 4\hat{k} - (2\hat{i} - \hat{j} + \hat{k}) = 3\hat{i} - 4\hat{j} - 4\hat{k} - 2\hat{i} + \hat{j} - \hat{k}$

$= \hat{i} - 3\hat{j} - 5\hat{k}$ ...(*iii*)

Adding (*i*) and (*ii*), we have

$\overrightarrow{AB} + \overrightarrow{BC} = -\hat{i} - 2\hat{j} - 6\hat{k} + 2\hat{i} - \hat{j} + \hat{k}$

$= \hat{i} - 3\hat{j} - 5\hat{k} = \overrightarrow{AC}$ [By (*iii*)]

$\therefore$ By Triangle Law of addition of vectors, points A, B, C are the vertices of a triangle ABC or points A, B, C are collinear.

Now from (*i*) and (*ii*), $\overrightarrow{AB} . \overrightarrow{BC} = (-1)(2) + (-2)(-1) + (-6)(1)$

$= -2 + 2 - 6 = -6 \neq 0$

From (*ii*) and (*iii*), $\overrightarrow{BC} \cdot \overrightarrow{AC} = 2(1) + (-1)(-3) + 1(-5)$
$= 2 + 3 - 5 = 0$

$\Rightarrow$ $\overrightarrow{BC}$ is perpendicular to $\overrightarrow{AC}$

$\Rightarrow$ Angle C is 90°. $\therefore$ ΔABC is right angled at point C.

$\therefore$ Points A, B, C are the vertices of a right angled triangle.

18. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if

(A) $\lambda = 1$ (B) $\lambda = -1$ (C) $a = |\lambda|$ (D) $a = \frac{1}{|\lambda|}$

Sol. Given: $\vec{a}$ is a non-zero vector of magnitude a

$\Rightarrow$ $|\vec{a}| = 1$ ...(*i*)

Also given: $\lambda \neq 0$ and $\lambda\vec{a}$ is a unit vector.

$\Rightarrow$ $|\lambda\vec{a}| = 1$ $\Rightarrow$ $|\lambda||\vec{a}| = 1$

$\Rightarrow$ $|\lambda|\, a = 1$ $\Rightarrow$ $a = \frac{1}{|\lambda|}$

$\therefore$ Option (D) is the correct answer.