NCERT Class 12 Maths Solutions

Chapter - 10

Exercise 10.2

1. Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k},$$

$$\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}.$$

Sol. Given: $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Therefore, $|\vec{a}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{1+1+1} = \sqrt{3}$.

$$\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}.$$

Therefore, $|\vec{b}| = \sqrt{4 + 49 + 9} = \sqrt{62}$.

$$\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}.$$

Therefore, $|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2}$

$$= \sqrt{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = \sqrt{\frac{3}{3}} = \sqrt{1} = 1.$$

2. Write two different vectors having same magnitude.

Sol. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$.

Clearly, $\vec{a} \neq \vec{b}$. ($\because$ Coefficients of $\hat{i}$ and $\hat{j}$ are same in vectors $\vec{a}$ and $\vec{b}$ but coefficients of $\hat{k}$ in $\vec{a}$ and $\vec{b}$ are unequal as $1 \neq -1$).

But $|\vec{a}| = \sqrt{x^2+y^2+z^2} = \sqrt{1+1+1} = \sqrt{3}$

and $|\vec{b}| = \sqrt{1+1+1} = \sqrt{3}$ $\therefore$ $|\vec{a}| = |\vec{b}|$.

Remark. In this way, we can construct an infinite number of possible answers.

3. Write two different vectors having same direction.

Sol. Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ ...(*i*)

and $\vec{b} = 2(\hat{i} + 2\hat{j} + 3\hat{k})$ ...(*ii*)

$= 2\vec{a}$ [By (*i*)]

$\therefore$ $\vec{b} = m\vec{a}$ where $m = 2 > 0$.

$\therefore$ Vectors $\vec{a}$ and $\vec{b}$ have the same direction.

But $\vec{b} \neq \vec{a}$ [$\because \vec{b} = 2\vec{a} \Rightarrow |\vec{b}| = |2||\vec{a}| = 2|\vec{a}| \neq |\vec{a}|$]

Remark. In this way, we can construct an infinite number of possible answers.

4. Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

Sol. Given: $2\hat{i} + 3\hat{j} = x\hat{i} + y\hat{j}$.

Comparing coefficients of $\hat{i}$ and $\hat{j}$ on both sides, we have

$$x = 2 \text{ and } y = 3.$$

5. Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (– 5, 7).

Sol. Let $\overrightarrow{AB}$ be the vector with initial point A(2, 1) and terminal point B(– 5, 7).

$\Rightarrow$ P.V. (Position Vector) of point A is (2, 1) $= 2\hat{i} + \hat{j}$ and P.V. of point B is (– 5, 7) $= -5\hat{i} + 7\hat{j}$.

$\therefore \quad \vec{AB}$ = P.V. of point B – P.V. of point A

$= (-5\hat{i} + 7\hat{j}) - (2\hat{i} + \hat{j}) = -5\hat{i} + 7\hat{j} - 2\hat{i} - \hat{j}$

$\Rightarrow \quad \vec{AB} = -7\hat{i} + 6\hat{j}$.

$\therefore$ By definition, scalar components of the vectors $\vec{AB}$ are coefficients of $\hat{i}$ and $\hat{j}$ in $\vec{AB}$ *i.e.*, – 7 and 6 and vector components of the vector $\vec{AB}$ are $-7\hat{i}$ and $6\hat{j}$.

6. Find the sum of the vectors:

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k},\ \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$$

and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Sol. **Given:** $\vec{a} = \hat{i} - 2\hat{j} + \hat{k},\ \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$

and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Adding $\vec{a} + \vec{b} + \vec{c} = 0\hat{i} - 4\hat{j} - \hat{k} = -4\hat{j} - \hat{k}$.

7. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Sol. We know that a unit vector in the direction of the vector

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k} \text{ is } \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{1+1+4}}$$

$$\Rightarrow \quad \hat{a} = \frac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}.$$

8. Find the unit vector in the direction of the vector $\vec{PQ}$ where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively.

Sol. Because points P and Q are P(1, 2, 3) and Q(4, 5, 6) (given),

therefore, position vector of point P = $\vec{OP} = 1\hat{i} + 2\hat{j} + 3\hat{k}$

and position vector of point Q = $\vec{OQ} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

where O is the origin.

$\therefore \quad \vec{PQ}$ = Position vector of point Q – Position vector of point P

$= \vec{OQ} - \vec{OP} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore, a unit vector in the direction of vector $\vec{PQ}$

$$= \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{9+9+9=27=9\times 3}}$$

$$= \frac{3(\hat{i} + \hat{j} + \hat{k})}{3\sqrt{3}} = \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} = \frac{1}{\sqrt{3}}\,\hat{i} + \frac{1}{\sqrt{3}}\,\hat{j} + \frac{1}{\sqrt{3}}\,\hat{k}.$$

9. For given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$; find the unit vector in the direction of $\vec{a} + \vec{b}$.

Sol. Given: Vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{j}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$

$\therefore \quad \vec{a} + \vec{b} = 2\hat{i} - \hat{j} + 2\hat{k} - \hat{i} + \hat{j} - \hat{k} = \hat{i} + 0\hat{j} + \hat{k}$

$\therefore \quad |\vec{a} + \vec{b}| = \sqrt{(1)^2 + (0)^2 + (1)^2} = \sqrt{2}$

$\therefore$ A unit vector in the direction of $\vec{a} + \vec{b}$ is

$$\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + 0\hat{j} + \hat{k}}{\sqrt{2}} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\,\hat{i} + \frac{1}{\sqrt{2}}\,\hat{k}.$$

10. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Sol. Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$.

$\therefore$ A vector in the direction of vector $\vec{a}$ which has magnitude 8 units

$$= 8\hat{a} = 8\,\frac{\vec{a}}{|\vec{a}|} = \frac{8(5\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{25 + 1 + 4}}$$

$$= \frac{8}{\sqrt{30}}\,(5\hat{i} - \hat{j} + 2\hat{k}) = \frac{40}{\sqrt{30}}\,\hat{i} - \frac{8}{\sqrt{30}}\,\hat{j} + \frac{16}{\sqrt{30}}\,\hat{k}.$$

11. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Sol. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ ...(*i*)

and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$= -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$ [By (*i*)]

$\Rightarrow \quad \vec{b} = -2\vec{a} = m\vec{a}$ where $m = -2 < 0$

$\therefore$ Vectors $\vec{a}$ and $\vec{b}$ are collinear (unlike because $m = -2 < 0$).

12. Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

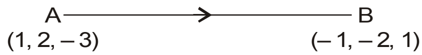
Sol. The given vector is $(\vec{a}) = \hat{i} + 2\hat{j} + 3\hat{k}$

$$= \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

We know that direction cosines of a vector $\vec{a}$ are coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\hat{a}$ *i.e.*, $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

13. Find the direction cosines of the vector joining the points A(1, 2, – 3) and B(– 1, – 2, 1) directed from A to B.

Sol. Given: Points A(1, 2, – 3) and B(– 1, – 2, 1).



$\Rightarrow$ P.V. (Position Vector, $\vec{OA}$) of point A is A(1, 2, – 3) $= \hat{i} + 2\hat{j} - 3\hat{k}$

and P.V. of point B is B(– 1, – 2, 1) $= -\hat{i} - 2\hat{j} + \hat{k}$.

$\therefore$ Vector $\vec{AB}$ (directed from A to B)

= P.V. of point B – P.V. of point A

$$= -\hat{i} - 2\hat{j} + \hat{k} - (\hat{i} + 2\hat{j} - 3\hat{k})$$

$$= -\hat{i} - 2\hat{j} + \hat{k} - \hat{i} - 2\hat{j} + 3\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

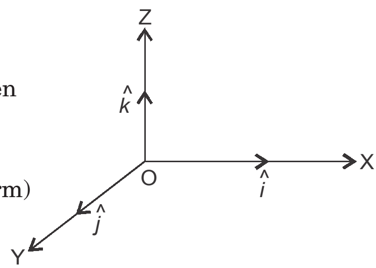
$$\therefore \quad AB = |\vec{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = 6$$

$$\therefore \quad \text{A unit vector along } \vec{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

$$= \frac{-2\hat{i} - 4\hat{j} + 4\hat{k}}{6} = -\frac{2}{6}\hat{i} - \frac{4}{6}\hat{j} + \frac{4}{6}\hat{k} = \frac{-1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}.$$

We know that Direction Cosines of the vector $\vec{AB}$ are the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in a unit vector along $\vec{AB}$ *i.e.*, $\frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$.

14. Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY and OZ.



Sol. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Let us find angle θ_1 (say) between vector $\vec{a}$ and OX ($\Rightarrow \hat{i}$)

($\because \hat{i}$ represents OX in vector form)

$$\therefore \quad \cos\theta_1 = \frac{\vec{a} \cdot \hat{i}}{|\vec{a}|\,|\hat{i}|}$$

$$\Rightarrow \quad \cos\theta_1 = \frac{(\hat{i}+\hat{j}+\hat{k}).(\hat{i}+0\hat{j}+0\hat{k})}{|\hat{i}+\hat{j}+\hat{k}||\hat{i}+0\hat{j}+0\hat{k}|}$$

$$\Rightarrow \quad \cos\theta_1 = \frac{1(1)+1(0)+1(0)}{\sqrt{1+1+1}\sqrt{1+0+0}} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta_1 = \cos^{-1}\frac{1}{\sqrt{3}}$$

Similarly, angle θ_2 between vectors $\vec{a}$ and $\hat{j}$ (OY) is $\cos^{-1}\frac{1}{\sqrt{3}}$

and angle θ_3 between vectors $\vec{a}$ and $\hat{k}$ (OZ) is also $\cos^{-1}\frac{1}{\sqrt{3}}$.

$\therefore \quad \theta_1 = \theta_2 = \theta_3$.

$\therefore$Vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is equally inclined to OX, OY and OZ

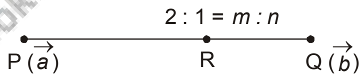
15. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2 : 1
(*i*) internally (*ii*) externally.

Sol. P.V. of point P is $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$

and P.V. of point Q is $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$

(*i*) Therefore P.V. of point R dividing PQ internally (*i.e.*, R lies within the segment PQ) in the ratio 2 : 1 (= $m : n$) (= PR : QR)

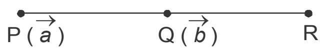
is $\dfrac{m\vec{b} + n\vec{a}}{m + n}$



$$= \frac{2(-\hat{i}+\hat{j}+\hat{k})+\hat{i}+2\hat{j}-\hat{k}}{2+1} = \frac{-2\hat{i}+2\hat{j}+2\hat{k}+\hat{i}+2\hat{j}-\hat{k}}{3}$$

$$= \frac{-\hat{i}+4\hat{j}+\hat{k}}{3} = \frac{-1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}.$$

(*ii*) P.V. of point R dividing PQ externally (*i.e.*, R lies outside PQ and to the right of point Q because ratio 2 : 1 = $\frac{2}{1}$ > 1 as PR is 2 times PQ *i.e.*, $\left.\frac{PR}{QR} = \frac{2}{1}\right)$ is $\dfrac{m\vec{b} - n\vec{a}}{m - n}$



$$= \frac{2(-\hat{i}+\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-\hat{k})}{2-1}$$

$$= -2\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} - 2\hat{j} + \hat{k} = -3\hat{i} + \hat{k}.$$

Remark. In the above question 15(*ii*), had R been dividing PQ externally in the ratio 1 : 2; then R will lie to the left of point P and $\frac{PR}{QR} = \frac{1}{2}$.

16. Find the position vector of the mid-point of the vector joining the points P(2, 3, 4) and Q(4, 1, – 2).

Sol. Given: Point P is (2, 3, 4) and Q is (4, 1, – 2).

$\Rightarrow$ P.V. of point P(2, 3, 4) is $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

and P.V. of point Q(4, 1, – 2) is $\vec{b} = 4\hat{i} + \hat{j} - 2\hat{k}$.

$\therefore$ P.V. of mid-point R of PQ is $\dfrac{\vec{a}+\vec{b}}{2}$.

[By Formula of Internal division]

$$= \frac{2\hat{i}+3\hat{j}+4\hat{k}+4\hat{i}+\hat{j}-2\hat{k}}{2} = \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}.$$

17. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right-angled triangle.

Sol. Given: P.V. of points A, B, C respectively are $\vec{a}\ (= \overrightarrow{OA}) = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b}\ (= \overrightarrow{OB}) = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}\ (= \overrightarrow{OC}) = \hat{i} - 3\hat{j} - 5\hat{k}$, where O is the origin.

Step I. $\therefore$ $\overrightarrow{AB}$ = P.V. of point B – P.V. of point A

$= 2\hat{i} - \hat{j} + \hat{k} - (3\hat{i} - 4\hat{j} - 4\hat{k}) = 2\hat{i} - \hat{j} + \hat{k} - 3\hat{i} + 4\hat{j} + 4\hat{k}$

or $\overrightarrow{AB} = -\hat{i} + 3\hat{j} + 5\hat{k}$ ...(*i*)

$\overrightarrow{BC}$ = P.V. of point C – P.V. of point B

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k}) = \hat{i} - 3\hat{j} - 5\hat{k} - 2\hat{i} + \hat{j} - \hat{k}$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$ ...(*ii*)

$\overrightarrow{AC}$ = P.V. of point C – P.V. of point A

$= \hat{i} - 3\hat{j} - 5\hat{k} - (3\hat{i} - 4\hat{j} - 4\hat{k}) = \hat{i} - 3\hat{j} - 5\hat{k} - 3\hat{i} + 4\hat{j} + 4\hat{k}$

$= -2\hat{i} + \hat{j} - \hat{k}$ ...(*iii*)

Adding (*i*) and (*ii*),

$\overrightarrow{AB} + \overrightarrow{BC} = -\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 6\hat{k}$

$-2\hat{i} + \hat{j} - \hat{k} = \overrightarrow{AC}$ [By (*iii*)]

$\therefore$ By Triangle Law of addition of Vectors, Points A, B, C are the Vertices of a triangle or points A, B, C are collinear.

Step II.

From (*i*) AB = $|\overrightarrow{AB}| = \sqrt{1+9+25} = \sqrt{35}$

From (*ii*), BC = $|\overrightarrow{BC}| = \sqrt{1+4+36} = \sqrt{41}$

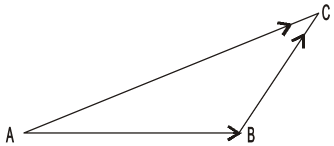
From (*iii*), AC = $|\overrightarrow{AC}| = \sqrt{4+1+1} = \sqrt{6}$

We can observe that (Longest side BC)^2 = $(\sqrt{41})^2 = 41 = 35 + 6$

$= AB^2 + AC^2$

$\therefore$ Points A, B, C are the vertices of a right-angled triangle.

18. In triangle ABC (Fig. below), which of the following is not true:



(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

(B) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$

(C) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$

(D) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

Sol. Option (C) is not true.

Because we know by Triangle Law of Addition of vectors that

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, *i.e.*, $\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$ $\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

But for option (C), $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{AC} = 2\overrightarrow{AC} \neq \vec{0}$.

Option (D) is same as option (A).

19. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:

(A) $\vec{b} = \lambda\vec{a}$**, for some scalar** λ**.** **(B)** $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

Sol. Option (D) is not true because two collinear vectors can have **different** directions and also different magnitudes.

The options (A) and (C) are true by definition of collinear vectors. Option (B) is a particular case of option (A) (taking $\lambda = \pm 1$).