NCERT Class 12 Maths Solutions

Chapter - 7

Exercise 7.2

Integrate the functions in Exercises 1 to 8:

1. $\dfrac{2x}{1+x^2}$

Sol. To evaluate $\int \frac{2x}{1+x^2}\, dx$

Put $1 + x^2 = t$. Therefore $2x = \frac{dt}{dx}$ or $2x\, dx = dt$

$\therefore \int \frac{2x}{1+x^2}\, dx = \int \frac{dt}{t} = \int \frac{1}{t}\, dt = \log |t| + c$

Putting $t = 1 + x^2$, $= \log |1 + x^2| + c = \log (1 + x^2) + c.$

($\because$ $1 + x^2 > 0$. Therefore $|1 + x^2| = 1 + x^2$)

2. $\dfrac{(\log x)^2}{x}$.

Sol. To evaluate $\int \frac{(\log x)^2}{x}\, dx$

Put $\log x = t$. Therefore $\frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{dx}{x} = dt$

$$\therefore \int \frac{(\log x)^2}{x}\, dx = \int t^2\, dt = \frac{t^3}{3} + c$$

Putting $t = \log x$, $= \frac{1}{3}(\log x)^3 + c$.

3. $\mathbf{\dfrac{1}{x + x\log x}}$

Sol. To evaluate $\int \frac{1}{x + x\log x}\, dx = \int \frac{1}{x(1 + \log x)}\, dx$

Put $1 + \log x = t$. Therefore $\frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{dx}{x} = dt$

$$\therefore \int \frac{1}{x + x\log x}\, dx = \int \frac{1}{1 + \log x}\frac{dx}{x} = \int \frac{1}{t}\, dt = \log|t| + c$$

Putting $t = 1 + \log x$, $\log|1 + \log x| + c$.

4. sin *x* sin (cos *x*)

Sol. To evaluate $\int \sin x \sin(\cos x)\, dx = -\int \sin(\cos x)(-\sin x)\, dx$

Put $\cos x = t$. Therefore $-\sin x = \frac{dt}{dx}$

$$\therefore \quad -\sin x\, dx = dt$$

$$\therefore \int \sin x \sin(\cos x)\, dx = -\int \sin(\cos x)(-\sin x\, dx)$$

$$= -\int \sin t\, dt = -(-\cos t) + c$$

$$= \cos t + c$$

Putting $t = \cos x$, $= \cos(\cos x) + c$.

5. sin (*ax* + *b*) cos (*ax* + *b*)

Sol. To evaluate $\int \sin(ax + b)\cos(ax + b)\, dx$

$$= \frac{1}{2}\int 2\sin(ax + b)\cos(ax + b)\, dx = \frac{1}{2}\int \sin 2(ax + b)\, dx$$

$(\because\ 2\sin\theta\cos\theta = \sin 2\theta)$

$$= \frac{1}{2}\int \sin(2ax + 2b)\, dx = \frac{1}{2}\frac{[-\cos(2ax + 2b)]}{2a \to \text{Coeff. of } x} + c$$

$$= \frac{-1}{4a}\cos 2(ax + b) + c.$$

6. $\sqrt{\boldsymbol{ax + b}}$

Sol. To evaluate $\int \sqrt{ax + b}\, dx = \int (ax + b)^{1/2}\, dx$

$$= \frac{(ax + b)^{\frac{1}{2} + 1}}{\left(\frac{1}{2} + 1\right) a \to \text{Coeff. of } x} + c = \frac{(ax + b)^{\frac{3}{2}}}{\frac{3}{2}a} + c$$

$$\left[\because \int (ax + b)^n\, dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + c \text{ if } n \neq -1\right]$$

$= \frac{2}{3a} (ax + b)^{3/2} + c.$

7. $\boldsymbol{x\sqrt{x+2}}$

Sol. To evaluate $\int x\sqrt{x+2}\ dx$

$= \int x\sqrt{x+2}\ dx = \int ((x+2)-2)\sqrt{x+2}\ dx$

$= \int \left((x+2)(x+2)^{\frac{1}{2}} - 2(x+2)^{\frac{1}{2}}\right) dx = \int \left((x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right) dx$

$= \int (x+2)^{\frac{3}{2}}\ dx - 2\int (x+2)^{\frac{1}{2}}\ dx$

$= \frac{(x+2)^{\frac{3}{2}+1}}{\left(\frac{3}{2}+1\right) 1 \to \text{Coeff. of } x} - 2\frac{(x+2)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right).1} + c = \frac{(x+2)^{\frac{5}{2}}}{\frac{5}{2}} - 2\frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c.$

OR

To evaluate $\int x\sqrt{x+2}\ dx$

Put $\sqrt{\textbf{Linear}}$ **= *t*,** *i.e.,* $\sqrt{x+2} = t.$

Squaring $x + 2 = t^2$ $(\Rightarrow x = t^2 - 2)$

$\therefore \frac{dx}{dt} = 2t,$ *i.e.,* $\frac{dx}{dt} = 2t$ or $dx = 2t\ dt$

$\therefore \int x\sqrt{x+2}\ dx = \int (t^2 - 2)\,t\,.\,2t\ dt = \int 2t^2(t^2-2)\ dt$

$= \int 2t^2(t^2-2)\ dt = 2\int t^4\ dt - 4\int t^2\ dt = 2\frac{t^5}{5} - 4\frac{t^3}{3} + c$

Putting $t = \sqrt{x+2}$, $= \frac{2}{5}(\sqrt{x+2})^5 - \frac{4}{3}(\sqrt{x+2})^3 + c$

$= \frac{2}{5}((x+2)^{1/2})^5 - \frac{4}{3}((x+2)^{1/2})^3 + c = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c.$

8. $\boldsymbol{x\sqrt{1+2x^2}}$

Sol. To evaluate $\int x\sqrt{1+2x^2}\ dx$

Let $I = \int x\sqrt{1+2x^2}\ dx = \frac{1}{4}\int \sqrt{1+2x^2}\,(4x\ dx)$ ...(*i*)

$\left[\because \frac{d}{dx}(1+2x^2) = 0 + 2\,.\,2x = 4x\right]$

Put $1 + 2x^2 = t$. Therefore $4x = \frac{dt}{dx}$ or $4x\ dx = dt$

$\therefore$ From (*i*), $I = \frac{1}{4}\int \sqrt{t}\ dt = \frac{1}{4}\int t^{1/2}\ dt$

$$= \frac{1}{4} \frac{t^{3/2}}{\frac{3}{2}} + c = \frac{1}{4} \cdot \frac{2}{3} t^{3/2} + c$$

Putting $t = 1 + 2x^2$, $= \frac{1}{6} (1 + 2x^2)^{3/2} + c$.

Integrate the functions in Exercises 9 to 17:

9. $(4x + 2) \sqrt{x^2 + x + 1}$.

Sol. Let $I = \int (4x + 2) \sqrt{x^2 + x + 1} \, dx = \int 2(2x + 1) \sqrt{x^2 + x + 1} \, dx$

$$= \int 2\sqrt{x^2 + x + 1} \, (2x + 1) \, dx \qquad ...(i)$$

Put $x^2 + x + 1 = t$. Therefore $(2x + 1) = \frac{dt}{dx}$

$\therefore \quad (2x + 1) \, dx = dt$

$\therefore \quad$ From (i), $I = \int 2\sqrt{t} \, dt = 2\int t^{1/2} \, dt$

$$= 2 \frac{t^{3/2}}{\frac{3}{2}} + c = \frac{4}{3} t^{3/2} + c$$

Putting $t = x^2 + x + 1$, $I = \frac{4}{3} (x^2 + x + 1)^{3/2} + c$.

10. $\frac{1}{x - \sqrt{x}}$

Sol. Let $I = \int \frac{1}{x - \sqrt{x}} \, dx \qquad ...(i)$

Put $\sqrt{\textbf{Linear}} = t$, *i.e.,* $\sqrt{x} = t$

Squaring $x = t^2$. Therefore $\frac{dx}{dt} = 2t$ or $dx = 2t \, dt$

$\therefore \quad$ From (i), $I = \int \frac{1}{t^2 - t} 2t \, dt = 2 \int \frac{t}{t(t-1)} \, dt$

$$= 2\int \frac{1}{t-1} \, dt = 2 \log | t - 1 | + c \left(\because \int \frac{1}{ax+b} dx = \frac{1}{a} \log | ax + b | \right)$$

Putting $t = \sqrt{x}$, $I = 2 \log | \sqrt{x} - 1 | + c$.

11. $\frac{x}{\sqrt{x+4}}$, $x > 0$

Sol. Let $I = \int \frac{x}{\sqrt{x+4}} \, dx \qquad ...(i)$

$$= \int \frac{x + 4 - 4}{\sqrt{x+4}} \, dx = \int \left(\frac{x+4}{\sqrt{x+4}} - \frac{4}{\sqrt{x+4}} \right) dx$$

$$= \int \sqrt{x+4} \, dx - 4 \int \frac{1}{\sqrt{x+4}} \, dx \left[\because \frac{t}{\sqrt{t}} = \frac{t\sqrt{t}}{\sqrt{t}\sqrt{t}} = \frac{t\sqrt{t}}{t} = \sqrt{t} \right]$$

$$= \int (x+4)^{1/2}\, dx - 4\int (x+4)^{-1/2}\, dx$$

$$= \frac{(x+4)^{3/2}}{\frac{3}{2}(1)} - \frac{4(x+4)^{1/2}}{\frac{1}{2}(1)} + c = \frac{2}{3}(x+4)^{3/2} - 8(x+4)^{1/2} + c$$

$$= \frac{2}{3}(x+4)\sqrt{x+4} - 8\sqrt{x+4} + c$$

$$\left[\because\ t^{3/2} = t^{\frac{2}{2}+\frac{1}{2}} = t^{1+\frac{1}{2}} = t^1 \,.\, t^{1/2} = t\sqrt{t}\right]$$

$$= 2\sqrt{x+4}\left(\frac{x+4}{3} - 4\right) + c = 2\sqrt{x+4}\left(\frac{x+4-12}{3}\right) + c$$

$$= \frac{2}{3}\sqrt{x+4}\,(x-8) + c.$$

OR

Put $\sqrt{\textbf{Linear}}$ **= *t*,** *i.e.,* $\sqrt{x+4} = t.$

Squaring $x + 4 = t^2 \Rightarrow x = t^2 - 4.$

Therefore $\frac{dx}{dt} = 2t$ or $dx = 2t\,dt$

$$\therefore\quad I = \int \frac{x}{\sqrt{x+4}}\, dx = \int \frac{t^2-4}{t}\,.\,2t\,dt$$

$$= 2\int (t^2-4)\, dt = 2\left[\int t^2\, dt - 4\int 1\, dt\right]$$

$$= 2\left[\frac{t^3}{3} - 4t\right] + c = \frac{2t}{3}(t^2 - 12) + c.$$

Putting $t = \sqrt{x+4}$, $= \frac{2}{3}\sqrt{x+4}\,(x+4-12) + c$

$$= \frac{2}{3}\sqrt{x+4}\,(x-8) + c.$$

12. $(x^3 - 1)^{1/3}\, x^5$

Sol. Let $I = \int (x^3-1)^{1/3}\, x^5\, dx = \int (x^3-1)^{1/3}\, x^3\, x^2\, dx$

$$= \frac{1}{3}\int (x^3-1)^{1/3}\, x^3\, (3x^2\, dx) \quad ...(i) \quad \left[\because\ \frac{d}{dx}(x^3-1) = 3x^2\right]$$

Put $x^3 - 1 = t \Rightarrow x^3 = t + 1$

$$\therefore\quad 3x^2 = \frac{dt}{dx} \Rightarrow 3x^2\, dx = dt$$

$\therefore$ From (i), $I = \frac{1}{3}\int t^{1/3}(t+1)\, dt$

$$= \frac{1}{3}\int (t^{4/3} + t^{1/3})\, dt \qquad \left[\because\ \frac{1}{3} + 1 = \frac{1+3}{3} = \frac{4}{3}\right]$$

$$= \frac{1}{3}\left(\int t^{4/3}\, dt + \int t^{1/3}\, dt\right)$$

$$= \frac{1}{3}\left(\frac{t^{7/3}}{\frac{7}{3}} + \frac{t^{4/3}}{\frac{4}{3}}\right) + c = \frac{1}{3}\left(\frac{3}{7}t^{7/3} + \frac{3}{4}t^{4/3}\right) + c = \frac{1}{7}t^{7/3} + \frac{1}{4}t^{4/3} + c$$

Putting $t = x^3 - 1$, $= \frac{1}{7}(x^3 - 1)^{7/3} + \frac{1}{4}(x^3 - 1)^{4/3} + c.$

13. $\frac{x^2}{(2+3x^3)^3}$

Sol. Let $I = \int \frac{x^2}{(2+3x^3)^3}\, dx$

$$= \frac{1}{9}\int \frac{9x^2}{(2+3x^3)^3}\, dx \qquad ...(i) \qquad \left[\because \frac{d}{dx}(2+3x^3) = 9x^2\right]$$

Put $2 + 3x^3 = t$. Therefore $9x^2 = \frac{dt}{dx} \Rightarrow 9x^2\, dx = dt$

$\therefore$ From (i), $I = \frac{1}{9}\int t^{-3}\, dt = \frac{1}{9}\frac{t^{-2}}{-2} + c = \frac{-1}{18t^2} + c$

Putting $t = 2 + 3x^3$; $= \frac{-1}{18(2+3x^3)^2} + c.$

14. $\frac{1}{x(\log x)^m}$, $x > 0$ ***(Important)***

Sol. Let $I = \int \frac{1}{x(\log x)^m}\, dx \ (x > 0) \Rightarrow I = \int \frac{\frac{1}{x}dx}{(\log x)^m}$ ...(i)

Put $\log x = t$. Therefore $\frac{1}{x} = \frac{dt}{dx} \Rightarrow \frac{dx}{x} = dt$

$\therefore$ From (i), $I = \int \frac{dt}{t^m} = \int t^{-m}\, dt = \frac{t^{-m+1}}{-m+1} + c$

(Assuming $m \neq 1$)

Putting $t = \log x$, $= \frac{(\log x)^{1-m}}{1-m} + c.$

15. $\frac{x}{9-4x^2}$

Sol. Let $I = \int \frac{x}{9-4x^2}\, dx = \frac{-1}{8}\int \frac{-8x}{9-4x^2}\, dx$ ...(i)

$$\left[\because \frac{d}{dx}(9-4x^2) = -8x\right]$$

Put $9 - 4x^2 = t$. Therefore $-8x = \frac{dt}{dx} \Rightarrow -8x\, dx = dt$

$\therefore$ From (i), $I = \frac{-1}{8}\int \frac{dt}{t} = \frac{-1}{8}\int \frac{1}{t}\, dt = \frac{-1}{8}\log|t| + c$

Putting $t = 9 - 4x^2$, $= \frac{-1}{8}\log|9 - 4x^2| + c.$

16. e^{2x+3}

Sol. $\int e^{2x+3}\ dx = \dfrac{e^{2x+3}}{2 \rightarrow \text{Coeff. of } x} + c$ $\quad \left[\because \int e^{ax+b}\ dx = \dfrac{e^{ax+b}}{a}\right]$

$= \dfrac{1}{2} e^{2x+3} + c.$

17. $\dfrac{x}{e^{x^2}}$

Sol. Let I $= \int \dfrac{x}{(e^{x^2})}\ dx = \dfrac{1}{2} \int \dfrac{2x}{(e^{x^2})}\ dx$ ...(*i*)

Put $x^2 = t$. Therefore $2x = \dfrac{dt}{dx} \Rightarrow 2x\ dx = dt$.

$\therefore$ From (*i*), $\quad I = \dfrac{1}{2} \int \dfrac{dt}{(e^t)} = \dfrac{1}{2} \int e^{-t}\ dt$

$= \dfrac{1}{2} \dfrac{e^{-t}}{-1 \rightarrow \text{Coeff. of } t} + c = \dfrac{-1}{2(e^t)} + c$

Putting $t = x^2$, $I = \dfrac{-1}{2(e^{x^2})} + c.$

Integrate the functions in Exercises 18 to 26:

18. $\dfrac{e^{\tan^{-1} x}}{1+x^2}$

Sol. Let I $= \int \dfrac{e^{\tan^{-1} x}}{1+x^2}\ dx$ ...(*i*)

Put $\tan^{-1} x = t$.

$\therefore \quad \dfrac{1}{1+x^2} = \dfrac{dt}{dx} \quad \Rightarrow \dfrac{dx}{1+x^2} = dt$

$\therefore$ From (*i*), $\quad I = \int e^t\ dt = e^t + c = e^{\tan^{-1} x} + c.$

19. $\dfrac{e^{2x} - 1}{e^{2x} + 1}$

Sol. Let I $= \int \dfrac{e^{2x} - 1}{e^{2x} + 1}\ dx$

Multiplying every term in integrand by e^{-x},

$I = \int \dfrac{e^x - e^{-x}}{e^x + e^{-x}}\ dx$ ...(*i*) $\quad [\because\ e^{2x} \cdot e^{-x} = e^{2x-x} = e^x]$

Put denominator $e^x + e^{-x} = t$

$\therefore \quad e^x + e^{-x} \dfrac{d}{dx}(-x) = \dfrac{dt}{dx} \quad \Rightarrow \quad (e^x - e^{-x})\ dx = dt$

$\therefore$ From (*i*), $I = \int \dfrac{dt}{t} = \int \dfrac{1}{t}\ dt = \log |t| + c$

Putting $t = e^x + e^{-x}$, $I = \log |e^x + e^{-x}| + c$ or $I = \log (e^x + e^{-x}) + c$

$\left[\because\ e^x + e^{-x} = e^x + \dfrac{1}{(e^x)} > 0 \text{ for all real } x \text{ and hence } |e^x + e^{-x}| = e^x + e^{-x}\right]$

20. $\dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$

Sol. Let $I = \int \dfrac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\, dx = \dfrac{1}{2}\int \dfrac{2(e^{2x} - e^{-2x})}{e^{2x} + e^{-2x}}\, dx$ ...(*i*)

Put denominator $e^{2x} + e^{-2x} = t$

$\therefore \quad e^{2x} \dfrac{d}{dx} 2x + e^{-2x} \dfrac{d}{dx} (-2x) = \dfrac{dt}{dx}$

$\Rightarrow \quad e^{2x} . 2 - 2e^{-2x} = \dfrac{dt}{dx} \quad \Rightarrow \quad 2(e^{2x} - e^{-2x})\, dx = dt$

$\therefore$ From (*i*), $I = \dfrac{1}{2} \int \dfrac{dt}{t} = \dfrac{1}{2} \log | t | + c$

Putting $t = e^{2x} + e^{-2x}$, $= \dfrac{1}{2}\log| e^{2x} + e^{-2x} | + c = \dfrac{1}{2}\log(e^{2x} + e^{-2x}) + c$

$[\because \; e^{2x} + e^{-2x} > 0 \quad \Rightarrow \quad | e^{2x} + e^{-2x} | = e^{2x} + e^{-2x}]$

21. $\tan^2 (2x - 3)$

Sol. $\int \tan^2(2x - 3)\, dx = \int (\sec^2(2x - 3) - 1)\, dx$ $(\because \tan^2 \theta = \sec^2 \theta - 1)$

$= \int \sec^2(2x - 3)\, dx - \int 1\, dx$

$= \dfrac{\tan (2x - 3)}{2 \to \text{Coeff. of } x} - x + c = \dfrac{1}{2} \tan (2x - 3) - x + c$

$\left[\because \int \sec^2 (ax + b)\, dx = \dfrac{1}{a} \tan (ax + b) + c\right]$

22. $\sec^2 (7 - 4x)$

Sol. $\int \sec^2 (7 - 4x)\, dx = \dfrac{\tan (7 - 4x)}{-4 \to \text{Coeff. of } x} + c$

$\left[\because \int \sec^2 (ax + b)\, dx = \dfrac{1}{a} \tan (ax + b) + c\right]$

$= \dfrac{-1}{4} \tan (7 - 4x) + c.$

23. $\dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$

Sol. Let $I = \int \dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$ ...(*i*)

Put $\sin^{-1} x = t \quad \therefore \quad \dfrac{1}{\sqrt{1 - x^2}} = \dfrac{dt}{dx} \quad \Rightarrow \quad \dfrac{dx}{\sqrt{1 - x^2}} = dt$

$\therefore$ From (*i*), $I = \int t\, dt = \dfrac{t^2}{2} + c$

Putting $t = \sin^{-1} x$, $I = \dfrac{1}{2} (\sin^{-1} x)^2 + c.$

24. $\dfrac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x}$

Sol. Let $I = \int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\,dx = \int \frac{2\cos x - 3\sin x}{2(2\sin x + 3\cos x)}\,dx$

$$= \frac{1}{2}\int \frac{2\cos x - 3\sin x}{2\sin x + 3\cos x}\,dx \qquad ...(i)$$

Put DENOMINATOR $2\sin x + 3\cos x = t$

$\therefore \quad 2\cos x - 3\sin x = \frac{dt}{dx} \Rightarrow (2\cos x - 3\sin x)\,dx = dt$

$\therefore \quad$ From (*i*), $I = \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\log|t| + c.$

Putting $t = 2\sin x + 3\cos x$, $= \frac{1}{2}\log|2\sin x + 3\cos x| + c.$

25. $\frac{1}{\cos^2 x\,(1 - \tan x)^2}$

Sol. Let $I = \int \frac{1}{\cos^2 x\,(1 - \tan x)^2}\,dx = \int \frac{\sec^2 x}{(1 - \tan x)^2}\,dx$

$$= -\int \frac{-\sec^2 x}{(1 - \tan x)^2}\,dx \qquad ...(i)$$

Put $1 - \tan x = t.$

$\therefore \quad -\sec^2 x = \frac{dt}{dx} \qquad \Rightarrow -\sec^2 x\,dx = dt$

$\therefore \quad$ From (*i*), $I = -\int \frac{dt}{t^2} = -\int t^{-2}\,dt$

$$= -\frac{t^{-1}}{-1} + c = \frac{1}{t} + c = \frac{1}{1 - \tan x} + c.$$

26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Sol. Let $I = \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx \qquad ...(i)$

Put $\sqrt{\textbf{Linear}}$ **= *t*,** *i.e.*, $\sqrt{x} = t$

Squaring, $x = t^2$. Therefore $\frac{dx}{dt} = 2t \qquad \therefore \quad dx = 2t\,dt$

$\therefore \quad$ From (*i*), $I = \int \frac{\cos t}{t}\,2t\,dt = 2\int \cos t\,dt = 2\sin t + c$

Putting $t = \sqrt{x}$, $I = 2\sin\sqrt{x} + c.$

Integrate the functions in Exercises 27 to 37:

27. $\sqrt{\sin 2x}\,\cos 2x$

Sol. Let $I = \int \sqrt{\sin 2x}\,\cos 2x\,dx = \frac{1}{2}\int \sqrt{\sin 2x}\,(2\cos 2x\,dx) \qquad ...(i)$

Put sin 2*x* = *t*

$\therefore \quad \cos 2x\,\frac{d}{dx}(2x) = \frac{dt}{dx} \qquad \Rightarrow 2\cos 2x\,dx = dt$

$\therefore$ From (i), $\mathrm{I} = \frac{1}{2} \int \sqrt{t}\ dt = \frac{1}{2} \int t^{1/2}\ dt$

$$= \frac{1}{2} \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{1}{2} \frac{t^{3/2}}{\frac{3}{2}} + c = \frac{1}{3} (\sin 2x)^{3/2} + c.$$

28. $\dfrac{\cos x}{\sqrt{1 + \sin x}}$

Sol. Let $\mathrm{I} = \int \dfrac{\cos x}{\sqrt{1+\sin x}}\ dx$ ...(i)

Put $1 + \sin x = t$

$\therefore$ $\cos x = \dfrac{dt}{dx}$ or $\cos x\ dx = dt$

$\therefore$ From (i), $\mathrm{I} = \int \dfrac{dt}{\sqrt{t}} = \int t^{-1/2}\ dt = \dfrac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$

$$= \frac{t^{1/2}}{\frac{1}{2}} + c = 2\sqrt{t} + c = 2\sqrt{1+\sin x} + c.$$

29. $\cot x \log \sin x$

Sol. Let $\mathrm{I} = \int \cot x \log \sin x\ dx$ ...(i)

Put $\log \sin x = t$

$\therefore$ $\dfrac{1}{\sin x} \dfrac{d}{dx} (\sin x) = \dfrac{dt}{dx}$ or $\dfrac{1}{\sin x} \cos x = \dfrac{dt}{dx}$

or $\cot x\ dx = dt$

$\therefore$ From (i), $\mathrm{I} = \int t\ dt = \dfrac{t^2}{2} + c$ $= \dfrac{1}{2} (\log \sin x)^2 + c.$

30. $\dfrac{\sin x}{1 + \cos x}$

Sol. Let $\mathrm{I} = \int \dfrac{\sin x}{1 + \cos x}\ dx$ $= - \int \dfrac{-\sin x}{1+\cos x}\ dx$ ...(i)

Put $1 + \cos x = t$. Therefore $-\sin x = \dfrac{dt}{dx}$

$\therefore$ $-\sin x\ dx = dt$

$\therefore$ From (i), $\mathrm{I} = -\int \dfrac{dt}{t}$ $= -\log |\, t\,| + c$

Putting $t = 1 + \cos x$, $= -\log |\, 1 + \cos x\,| + c$.

31. $\dfrac{\sin x}{(1 + \cos x)^2}$

Sol. Let $\mathrm{I} = \int \dfrac{\sin x}{(1+\cos x)^2}\ dx = - \int \dfrac{-\sin x\ dx}{(1 + \cos x)^2}$ ...(i)

Put $1 + \cos x = t$. Therefore $-\sin x = \dfrac{dt}{dx}$

$\Rightarrow \quad -\sin x \, dx = dt$

$\therefore$ From (*i*), $I = -\int \frac{dt}{t^2} = -\int t^{-2} \, dt = \frac{-t^{-1}}{-1} + c$

$$= \frac{1}{t} + c = \frac{1}{1+\cos x} + c.$$

32. $\mathbf{\frac{1}{1+\cot x}}$

Sol. Let $I = \int \frac{1}{1+\cot x} dx = \int \frac{1}{1+\frac{\cos x}{\sin x}} dx = \int \frac{1}{\left(\frac{\sin x + \cos x}{\sin x}\right)} dx$

$= \int \frac{\sin x}{\sin x + \cos x} dx = \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx = \frac{1}{2} \int \frac{\sin x + \sin x}{\sin x + \cos x} dx$

Adding and subtracting cos x in the numerator of integrand,

$$I = \frac{1}{2} \int \frac{\sin x + \cos x - \cos x + \sin x}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \frac{(\sin x + \cos x) - (\cos x - \sin x)}{\sin x + \cos x} dx$$

$$= \frac{1}{2} \int \left(\frac{\sin x + \cos x}{\sin x + \cos x} - \frac{(\cos x - \sin x)}{\sin x + \cos x} \right) dx \quad \left[\because \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c} \right]$$

$$= \frac{1}{2} \int \left(1 - \frac{(\cos x - \sin x)}{\sin x + \cos x} \right) dx$$

$$= \frac{1}{2} \left[\int 1 \, dx - \int \frac{\cos x - \sin x}{\sin x + \cos x} dx \right] = \frac{1}{2} [x - I_1] \quad ...(i)$$

where $I_1 = \int \frac{\cos x - \sin x}{\sin x + \cos x} dx$

Put DENOMINATOR $\sin x + \cos x = t$

$\therefore \quad \cos x - \sin x = \frac{dt}{dx} \quad \Rightarrow \quad (\cos x - \sin x) \, dx = dt$

$\therefore \quad I_1 = \int \frac{dt}{t} = \log |t| = \log |\sin x + \cos x|.$

Note. Alternative solution for finding $\mathbf{I_1}$

$$I_1 = \int \frac{\cos x - \sin x}{\sin x + \cos x} dx = \log |\sin x + \cos x|$$

$$\left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| \right]$$

Putting this value of I_1 in (*i*), required integral

$$= \frac{1}{2} [x - \log |\sin x + \cos x|] + c.$$

33. $\dfrac{1}{1-\tan x}$

Sol. Let $I = \int \dfrac{1}{1-\tan x}\,dx = \int \dfrac{1}{1-\dfrac{\sin x}{\cos x}}\,dx = \int \dfrac{1}{\left(\dfrac{\cos x - \sin x}{\cos x}\right)}\,dx$

$= \int \dfrac{\cos x}{\cos x - \sin x}\,dx = \dfrac{1}{2}\int \dfrac{2\cos x}{\cos x - \sin x}\,dx = \dfrac{1}{2}\int \dfrac{\cos x + \cos x}{\cos x - \sin x}\,dx$

Subtracting and adding $\sin x$ in the Numerator,

$= \dfrac{1}{2}\int \dfrac{\cos x - \sin x + \sin x + \cos x}{\cos x - \sin x}\,dx$

$= \dfrac{1}{2}\int \left(\dfrac{\cos x - \sin x}{\cos x - \sin x} + \dfrac{\sin x + \cos x}{\cos x - \sin x}\right)dx = \dfrac{1}{2}\int \left(1 + \dfrac{\sin x + \cos x}{\cos x - \sin x}\right)dx$

$= \dfrac{1}{2}\left[\int 1\,dx - \int \dfrac{-\sin x - \cos x}{\cos x - \sin x}\,dx\right]$

$= \dfrac{1}{2}\,[x - \log|\cos x - \sin x|] + c \qquad \left[\because \int \dfrac{f'(x)}{f(x)}\,dx = \log|f(x)|\right]$

Note. Alternative solution for evaluating $\int \dfrac{-\sin x - \cos x}{\cos x - \sin x}\,dx$, put denominator $\cos x - \sin x = t$.

34. $\dfrac{\sqrt{\tan x}}{\sin x \cos x}$

Sol. Let $I = \int \dfrac{\sqrt{\tan x}}{\sin x \cos x}\,dx = \int \dfrac{\sqrt{\tan x}}{\dfrac{\sin x}{\cos x}\cos x \cos x}\,dx$

$= \int \dfrac{\sqrt{\tan x}}{\tan x \cos^2 x}\,dx = \int \dfrac{\sec^2 x}{\sqrt{\tan x}}\,dx \qquad ...(i) \qquad \left[\because \dfrac{\sqrt{t}}{t} = \dfrac{1}{\sqrt{t}}\right]$

Put $\tan x = t$.

$\therefore \quad \sec^2 x = \dfrac{dt}{dx} \qquad \Rightarrow \quad \sec^2 x\,dx = dt$

$\therefore$ From (i),

$I = \int \dfrac{dt}{\sqrt{t}} = \int t^{-1/2}\,dt = \dfrac{t^{1/2}}{\frac{1}{2}} + c \qquad = 2\sqrt{t} + c = 2\sqrt{\tan x} + c.$

35. $\dfrac{(1+\log x)^2}{x}$

Sol. Let $I = \int \dfrac{(1+\log x)^2}{x}\,dx \qquad ...(i)$

Put $1 + \log x = t$

$\therefore \quad \dfrac{1}{x} = \dfrac{dt}{dx} \qquad \Rightarrow \quad \dfrac{dx}{x} = dt$

$\therefore$ From (i), $I = \int t^2\,dt = \dfrac{t^3}{3} + c = \dfrac{1}{3}(1+\log x)^3 + c.$

36. $\dfrac{(x+1)(x+\log x)^2}{x}$

Sol. Let I = $\displaystyle\int \frac{(x+1)(x+\log x)^2}{x}\ dx$ ...(*i*)

Put $x + \log x = t$

$\therefore\quad 1 + \dfrac{1}{x} = \dfrac{dt}{dx} \Rightarrow \dfrac{x+1}{x} = \dfrac{dt}{dx} \Rightarrow \left(\dfrac{x+1}{x}\right) dx = dt$

$\therefore$ From (*i*), I = $\displaystyle\int t^2\, dt = \frac{t^3}{3} + c$

Putting $t = x + \log x$, $\dfrac{1}{3}(x + \log x)^3 + c.$

37. $\dfrac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

Sol. Let I = $\displaystyle\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}\ dx = \frac{1}{4}\int \sin(\tan^{-1} x^4) \cdot \frac{4x^3}{1+x^8} dx$ ...(*i*)

Put $(\tan^{-1} x^4) = t$

[Rule for $\int \sin(f(x))\, f'(x)\, dx$; put $f(x) = t$]

$\therefore\quad \dfrac{1}{1+(x^4)^2}\dfrac{d}{dx}x^4 = \dfrac{dt}{dx}\quad \left[\because \dfrac{d}{dx}\tan^{-1} f(x) = \dfrac{1}{1+(f(x))^2}\dfrac{d}{dx}f(x)\right]$

$\Rightarrow\quad \dfrac{4x^3}{1+x^8}\, dx = dt$

$\therefore$ From (*i*),

$\text{I} = \dfrac{1}{4}\displaystyle\int \sin t\ dt = -\frac{1}{4}\cos t + c = \frac{-1}{4}\cos(\tan^{-1} x^4) + c.$

Choose the correct answer in Exercises 38 and 39:

38. $\displaystyle\int \frac{10x^9 + 10^x \log_e 10\, dx}{x^{10} + 10^x}$ **equals**

(A) $10^x - x^{10} + C$ **(B)** $10^x + x^{10} + C$

(C) $(10^x - x^{10})^{-1} + C$ **(D)** $\log(10^x + x^{10}) + C.$

Sol. Let I = $\displaystyle\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x}\ dx$ ...(*i*)

Put $x^{10} + 10^x = t$

$\therefore\quad (10x^9 + 10^x \log_e 10)\, dx = dt \quad \left[\because \dfrac{d}{dx}(a^x) = a^x \log_e a\right]$

$\therefore$ From (*i*), I = $\displaystyle\int \frac{dt}{t} = \log|t| + c$

Putting $t = x^{10} + 10^x$, I = $\log|x^{10} + 10^x| + c$

or I = $\log(10^x + x^{10}) + c$.

$\therefore$ Option (D) is the correct answer.

OR

$\displaystyle\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x}\ dx = \int \frac{f'(x)}{f(x)}\ dx = \log|f(x)| + c$

$$= \log | x^{10} + 10^x | + c$$

$\therefore$ Option (D) is the correct answer.

39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ **equals**

(A) $\tan x + \cot x + C$ **(B)** $\tan x - \cot x + C$

(C) $\tan x \cot x + C$ **(D)** $\tan x - \cot 2x + C.$

Sol. $\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$ $[\because 1 = \sin^2 x + \cos^2 x]$

$= \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$ $\left[\because \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}\right]$

$= \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx = \int (\sec^2 x + \text{cosec}^2 x) \, dx$

$= \int \sec^2 x \, dx + \int \text{cosec}^2 x \, dx = \tan x - \cot x + c$

$\therefore$ Option (B) is the correct answer.