NCERT Class 12 Maths Solutions

Chapter - 7

Integrals

Exercise 7.1

Find an antiderivative (or integral) of the following functions by the method of inspection in Exercises 1 to 5.

1. **sin** $2x$

Sol. To find an anti derivative of sin $2x$ by Inspection Method.

We know that $\frac{d}{dx}$ (cos 2x) = – 2 sin 2x

Dividing by – 2, $\frac{-1}{2} \frac{d}{dx}$ (cos 2x) = sin 2x

or $\frac{d}{dx}\left(\frac{-1}{2}\cos 2x\right)$ = sin 2x

∴ By definition; **an** integral or **an** antiderivative of sin 2x is $\frac{-1}{2}$ cos 2x.

Note. In fact anti derivative or integral of sin 2x is $\frac{-1}{2}$ cos 2x + c. For different values of c, we get different antiderivatives. So we omitted c for writing **an** anti derivative.

2. cos 3*x*

Sol. To find an anti derivative of cos 3x by Inspection Method.

We know that $\frac{d}{dx}$ (sin 3x) = 3 cos 3x

Dividing by 3, $\frac{1}{3} \frac{d}{dx}$ (sin 3x) = cos 3x or $\frac{d}{dx}\left(\frac{1}{3}\sin 3x\right) = \cos 3x$

∴ By definition, **an** integral or **an** antiderivative of cos 3x is $\frac{1}{3}$ sin 3x.

(See note after solution of Q.No.1 for not adding c to the answer.)

3. e^{2x}.

Sol. To find an antiderivative of e^{2x} by Inspection Method.

We know that $\frac{d}{dx} e^{2x} = e^{2x} \frac{d}{dx} (2x) = 2e^{2x}$

Dividing by 2, $\frac{1}{2} \frac{d}{dx} e^{2x} = e^{2x}$ or $\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right) = e^{2x}$

∴ An antiderivative of e^{2x} is $\frac{1}{2} e^{2x}$.

4. $(ax + b)^2$.

Sol. To find an anti derivative of $(ax + b)^2$.

We know that $\frac{d}{dx} (ax + b)^3 = 3(ax + b)^2 \frac{d}{dx} (ax + b) = 3(ax + b)^2 a$.

Dividing by 3a, $\frac{1}{3a} \frac{d}{dx} (ax + b)^3 = (ax + b)^2$

or $\frac{d}{dx}\left[\frac{1}{3a}(ax + b)^3\right] = (ax + b)^2$

∴ An anti derivative of $(ax + b)^2$ is $\frac{1}{3a} (ax + b)^3$.

5. sin 2x – 4e^{3x}.

Sol. To find an anti derivative of sin 2x – 4e^{3x} by Inspection Method.

We know that $\frac{d}{dx}(\cos 2x) = -2 \sin 2x$

Dividing by -2, $\frac{d}{dx}\left(\frac{-1}{2}\cos 2x\right) = \sin 2x$ ...(i)

Again $\frac{d}{dx} e^{3x} = 3e^{3x}$ $\therefore$ $\frac{d}{dx}\left(\frac{1}{3}e^{3x}\right) = e^{3x}$

Multiplying by -4, $\frac{d}{dx}\left(\frac{-4}{3}e^{3x}\right) = -4e^{3x}$ ...(ii)

Adding eqns. (i) and (ii)

$$\frac{d}{dx}\left(\frac{-1}{2}\cos 2x\right) + \frac{d}{dx}\left(\frac{-4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

or $$\frac{d}{dx}\left(\frac{-1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

$\therefore$ An anti derivative of $\sin 2x - 4e^{3x}$ is $\frac{-1}{2}\cos 2x - \frac{4}{3}e^{3x}$.

Evaluate the following integrals in Exercises 6 to 11.

6. $\int (4e^{3x} + 1)\, dx.$

Sol. $\int (4e^{3x} + 1)\, dx = \int 4e^{3x}\, dx + \int 1\, dx$

$= 4\int e^{3x}\, dx + x = 4\frac{e^{3x}}{3} + x + c.$ $\left[\because \int e^{ax}\, dx = \frac{e^{ax}}{a} \text{ and } \int 1\, dx = x\right]$

7. $\int x^2\left(1 - \frac{1}{x^2}\right) dx.$

Sol. $\int x^2\left(1 - \frac{1}{x^2}\right) dx = \int\left(x^2 - \frac{x^2}{x^2}\right) dx = \int (x^2 - 1)\, dx$

$= \int x^2\, dx - \int 1\, dx = \frac{x^3}{3} - x + c.$ $\left[\because \int x^n\, dx = \frac{x^{n+1}}{n+1} \text{ if } n \neq -1\right]$

8. $\int (ax^2 + bx + c)\, dx$.

Sol. $\int (ax^2 + bx + c)\, dx = \int ax^2\, dx + \int bx\, dx + \int c\, dx$

$= a\int x^2\, dx + b\int x^1\, dx + c\int 1\, dx = a\frac{x^3}{3} + b\frac{x^2}{2} + cx + c_1$

where c_1 is the constant of integration.

9. $\int (2x^2 + e^x)\, dx.$

Sol. $\int (2x^2 + e^x)\, dx = \int 2x^2\, dx + \int e^x\, dx$

$= 2\int x^2\, dx + \int e^x\, dx = 2\frac{x^{2+1}}{2+1} + e^x + c = 2\frac{x^3}{3} + e^x + c.$

10. $\int\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx.$

Sol. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \, dx$

Opening the square = $\int \left((\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x}\,\frac{1}{\sqrt{x}}\right) dx$

$= \int \left(x + \frac{1}{x} - 2\right) dx \;=\; \int x \; dx + \int \frac{1}{x} \; dx - \int 2 \; dx$

$= \frac{x^2}{2} + \log |x| - 2x + c.$ $\qquad\left[\because \int 2\,dx = 2\int 1\,dx = 2x\right]$

11. $\int \frac{\boldsymbol{x^3 + 5x^2 - 4}}{\boldsymbol{x^2}} \; \boldsymbol{dx.}$

Sol. $\int \frac{x^3 + 5x^2 - 4}{x^2} \; dx = \int \left(\frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2}\right) dx$

$\left[\text{Using } \frac{a+b-c}{d} = \frac{a}{d} + \frac{b}{d} - \frac{c}{d}\right]$

$= \int (x + 5 - 4x^{-2}) \; dx = \int x^1 \; dx + \int 5 \; dx - \int 4x^{-2} \; dx$

$= \frac{x^2}{2} + 5\int 1 \; dx - 4\int x^{-2} \; dx = \frac{x^2}{2} + 5x - 4\,\frac{x^{-2+1}}{-2+1} + c$

$= \frac{x^2}{2} + 5x + \frac{4}{x} + c.$

Evaluate the following integrals in Exercises 12 to 16.

12. $\int \frac{\boldsymbol{x^3 + 3x + 4}}{\boldsymbol{\sqrt{x}}} \; \boldsymbol{dx.}$

Sol. $\int \frac{x^3 + 3x + 4}{\sqrt{x}} \; dx = \int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right) dx$

$= \int (x^{3-1/2} + 3x^{1-1/2} + 4x^{-1/2})\,dx = \int (x^{5/2} + 3x^{1/2} + 4x^{-1/2}) \; dx$

$= \int x^{5/2} \; dx + 3\int x^{1/2} \; dx + 4\int x^{-1/2} \; dx$

$= \frac{x^{5/2+1}}{\frac{5}{2}+1} + 3\,\frac{x^{1/2+1}}{\frac{1}{2}+1} + 4\,\frac{x^{-1/2+1}}{\frac{-1}{2}+1} + c = \frac{x^{7/2}}{\frac{7}{2}} + 3\,\frac{x^{3/2}}{\frac{3}{2}} + 4\,\frac{x^{1/2}}{\frac{1}{2}} + c$

$= \frac{2}{7}\,x^{7/2} + 2x^{3/2} + 8x^{1/2} + c.$

13. $\int \frac{\boldsymbol{x^3 - x^2 + x - 1}}{\boldsymbol{x - 1}} \; \boldsymbol{dx.}$

Sol. $\int \frac{x^3 - x^2 + x - 1}{x - 1} \; dx = \int \frac{x^2(x-1) + (x-1)}{x - 1} \; dx$

$= \int \frac{(x-1)(x^2+1)}{(x-1)} \; dx = \int (x^2 + 1) \; dx$

$$=\int x^2\ dx + \int 1\ dx = \frac{x^{2+1}}{2+1} + x + c\ = \frac{x^3}{3} + x + c.$$

14. $\int (1-x)\sqrt{x}\ dx.$

Sol. $\int (1-x)\sqrt{x}\, dx = \int (\sqrt{x} - x\sqrt{x})\ dx$

$$= \int (x^{1/2} - x^1 x^{1/2})\ dx = \int (x^{1/2} - x^{1+1/2})\ dx$$

$$= \int (x^{1/2} - x^{3/2})\ dx = \frac{x^{1/2+1}}{\frac{1}{2}+1} - \frac{x^{3/2+1}}{\frac{3}{2}+1} + c$$

$$= \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{5/2}}{\frac{5}{2}} + c = \frac{2}{3}\ x^{3/2} - \frac{2}{5}\ x^{5/2} + c.$$

15. $\int \sqrt{x}\,(3x^2 + 2x + 3)\ dx.$

Sol. $\int \sqrt{x}\,(3x^2 + 2x + 3)\ dx = \int x^{1/2}\,(3x^2 + 2x + 3)\ dx$

$$= \int (3x^2\, x^{1/2} + 2x\, x^{1/2} + 3x^{1/2})\ dx\ = \int (3x^{5/2} + 2x^{3/2} + 3x^{1/2})\ dx$$

$$\left(\because\ \ 2 + \frac{1}{2} = \frac{4+1}{2} = \frac{5}{2}, 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}\right)$$

$$= 3\int x^{5/2}\ dx + 2\int x^{3/2}\ dx + 3\int x^{1/2}\ dx$$

$$= 3\,\frac{x^{5/2+1}}{\frac{5}{2}+1} + 2\,\frac{x^{3/2+1}}{\frac{3}{2}+1} + 3\,\frac{x^{1/2+1}}{\frac{1}{2}+1} + c = 3\,\frac{x^{7/2}}{\frac{7}{2}} + 2\,\frac{x^{5/2}}{\frac{5}{2}} + 3\,\frac{x^{3/2}}{\frac{3}{2}} + c$$

$$= \frac{6}{7}\ x^{7/2} + \frac{4}{5}\ x^{5/2} + 2x^{3/2} + c.$$

16. $\int (2x - 3\cos x + e^x)\ dx.$

Sol. $\int (2x - 3\cos x + e^x)\ dx = \int 2x\ dx - \int 3\cos x\ dx + \int e^x\, dx$

$$= 2\int x^1\ dx - 3\int \cos x\ dx + \int e^x\, dx\ = 2\,\frac{x^2}{2} - 3\ \sin x + e^x + c$$

$$= x^2 - 3\ \sin x + e^x + c.$$

Evaluate the following integrals in Exercises 17 to 20.

17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\ dx.$

Sol. $\int (2x^2 - 3\sin x + 5\sqrt{x})\ dx$

$$= 2\int x^2\ dx - 3\int \sin x\ dx + 5\int x^{1/2}\ dx$$

$$= 2\,\frac{x^{2+1}}{2+1} - 3(-\cos x) + 5\ \frac{x^{1/2+1}}{\frac{1}{2}+1} + c = 2\ \frac{x^3}{3} + 3\ \cos x + 5\,\frac{x^{3/2}}{\frac{3}{2}} + c$$

$$= 2\ \frac{x^3}{3} + 3\ \cos x + \frac{10}{3}\ x^{3/2} + c.$$

18. $\int$ **sec** $\boldsymbol{x}$ **(sec** $\boldsymbol{x}$ **+ tan** $\boldsymbol{x}$**)** $\boldsymbol{dx}$**.**

Sol. $\int \sec x\,(\sec x + \tan x)\ dx = \int (\sec^2 x + \sec x \tan x)\,dx$

$= \int \sec^2 x\ dx + \int \sec x \tan x\ dx \qquad = \tan x + \sec x + c.$

19. $\int \dfrac{\sec^2 x}{\text{cosec}^2 x}\ dx.$

Sol. $\int \dfrac{\sec^2 x}{\text{cosec}^2 x}\ dx = \int \dfrac{\dfrac{1}{\cos^2 x}}{\dfrac{1}{\sin^2 x}}\ dx = \int \dfrac{\sin^2 x}{\cos^2 x}\ dx$

$= \int \tan^2 x\ dx = \int (\sec^2 x - 1)\ dx$

$(\because\ \sec^2 x - \tan^2 x = 1 \ \Rightarrow\ \sec^2 x - 1 = \tan^2 x)$

$= \int \sec^2 x\ dx - \int 1\ dx = \tan x - x + c.$

Note. Similarly $\int \cot^2 x\ dx = \int (\text{cosec}^2 x - 1)\ dx$

$= \int \text{cosec}^2 x\,dx - \int 1\ dx = -\cot x - x + c.$

20. $\int \dfrac{2 - 3\sin x}{\cos^2 x}\ dx.$

Sol. $\int \dfrac{2 - 3\sin x}{\cos^2 x}\ dx = \int \left(\dfrac{2}{\cos^2 x} - \dfrac{3\sin x}{\cos^2 x}\right) dx$

$= \int \left(2\sec^2 x - \dfrac{3\sin x}{\cos x \cos x}\right) dx = \int (2\sec^2 x - 3\tan x \sec x)\ dx$

$= 2\int \sec^2 x\ dx - 3\int \sec x \tan x\ dx \ = 2\tan x - 3\sec x + c.$

21. Choose the correct answer:

The anti derivative of $\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$ **equals**

(A) $\dfrac{1}{3}x^{1/3} + 2x^{1/2} + C$ **(B)** $\dfrac{2}{3}\ x^{2/3} + \dfrac{1}{2}\ x^2 + C$

(C) $\dfrac{2}{3}\ x^{3/2} + 2x^{1/2} + C$ **(D)** $\dfrac{3}{2}\ x^{3/2} + \dfrac{1}{2}\ x^{1/2} + C.$

Sol. The anti derivative of the $\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)$

$= \int \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right) dx = \int (x^{1/2} + x^{-1/2})\ dx$

$= \int x^{1/2}\ dx + \int x^{-1/2}\ dx = \dfrac{x^{1/2+1}}{\dfrac{1}{2}+1} + \dfrac{x^{1/2+1}}{\dfrac{1}{2}+1} + C$

$= \dfrac{x^{3/2}}{\dfrac{3}{2}} + \dfrac{x^{1/2}}{\dfrac{1}{2}} + C = \dfrac{2}{3}\ x^{3/2} + 2x^{1/2} + C$

$\therefore$ Option (C) is the correct answer.

22. Choose the correct answer:

If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ **(B)** $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ **(D)** $x^3 + \frac{1}{x^4} - \frac{129}{8}$.

Sol. Given: $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ and $f(2) = 0$

$\therefore$ By definition of anti derivative (*i.e.*, Integral),

$$f(x) = \int \left(4x^3 - \frac{3}{x^4}\right) dx = 4\int x^3 \, dx - 3\int \frac{1}{x^4} \, dx$$

$$= 4 \cdot \frac{x^4}{4} - 3\int x^{-4} \, dx = x^4 - 3 \frac{x^{-3}}{-3} + c$$

or $f(x) = x^4 + \frac{1}{(x^3)} + c$ ...(*i*)

To find *c*. Let us make use of $f(2) = 0$ (given)

Putting $x = 2$ on both sides of (*i*),

$f(2) = 16 + \frac{1}{8} + c$ or $0 = \frac{128+1}{8} + c$

($\because$ $f(2) = 0$ (given))

or $c + \frac{129}{8} = 0$ or $c = \frac{-129}{8}$

Putting $c = \frac{-129}{8}$ in (*i*), $f(x) = x^4 + \frac{1}{(x^3)} - \frac{129}{8}$

$\therefore$ Option (A) is the correct answer.