NCERT Class 12 Maths Solutions

Exercise 6.2

1. Show that the function given by $f(x) = 3x + 17$ is strictly increasing on R.

Sol. Given: $f(x) = 3x + 17$

$\therefore \quad f'(x) = 3(1) + 0 = 3 > 0$ *i.e.,* + ve for all $x \in$ R.

$\therefore$ $f(x)$ is strictly increasing on R.

2. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

Sol. Given: $f(x) = e^{2x}$

$\therefore \quad f'(x) = e^{2x} \dfrac{d}{dx} 2x = e^{2x}(2) = 2e^{2x} > 0$ *i.e.,* + ve for all $x \in$ R.

[$\because$ We know that e is approximately equal to 2.718 and is always positive]

$\therefore$ $f(x)$ is strictly increasing on R.

Remark. $e^{-2} = \dfrac{1}{(e^2)} > 0$ and $e^0 = 1 > 0$.

3. Show that the function given by $f(x) = \sin x$ is (*a*) strictly increasing in $\left(0, \dfrac{\pi}{2}\right)$ (*b*) strictly decreasing in $\left(\dfrac{\pi}{2}, \pi\right)$ (*c*) neither increasing nor decreasing in $(0, \pi)$.

Sol. Given: $f(x) = \sin x$

$\therefore \quad f'(x) = \cos x$

(*a*) We know that $f'(x) = \cos x > 0$ *i.e.,* + ve in first quadrant *i.e.,* in $\left(0, \dfrac{\pi}{2}\right)$.

$\therefore$ $f(x)$ is strictly increasing in $\left(0, \dfrac{\pi}{2}\right)$.

(b) We know that $f'(x) = \cos x < 0$ *i.e.,* – ve in second quadrant *i.e.,* in $\left(\frac{\pi}{2}, \pi\right)$.

$\therefore$ $f(x)$ is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) Because $f'(x) = \cos x > 0$ *i.e.,* + ve in $\left(0, \frac{\pi}{2}\right)$ and $f'(x) = \cos x < 0$ *i.e.,* – ve in $\left(\frac{\pi}{2}, \pi\right)$ and $f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$\therefore$ $f'(x)$ does not keep the same sign in the interval $(0, \pi)$.

Hence $f(x)$ is neither increasing nor decreasing in $(0, \pi)$.

4. Find the intervals in which the function f given by

$$f(x) = 2x^2 - 3x \text{ is}$$

(a) strictly increasing (b) strictly decreasing.

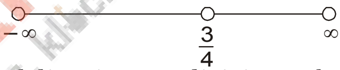
Sol. Given: $f(x) = 2x^2 - 3x$

$\therefore$ $f'(x) = 4x - 3$ ...(i)

Step I. Let us put $f'(x) = 0$ to find turning points *i.e.,* points on the given curve where tangent is parallel to x-axis.

$\therefore$ From (i), $4x - 3 = 0$ *i.e.,* $4x = 3$

or $x = \frac{3}{4}$ (= 0.75).



This turning point divides the real line in two disjoint sub-intervals $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.

Step II.

Interval	**sign of $f'(x) = 4x - 3$ ...(i)**	**Nature of function f**
$\left(-\infty, \frac{3}{4}\right)$	Take $x = 0.5$ (say) then from (i) $f'(x) < 0$	$\therefore$ f is strictly decreasing ↓
$\left(\frac{3}{4}, \infty\right)$	Take $x = 1$ (say) then from (i), $f'(x) > 0$	$\therefore$ f is strictly increasing ↑

Thus, (a) f is strictly increasing in $\left(\frac{3}{4}, \infty\right)$.

(b) f is strictly decreasing in $\left(-\infty, \frac{3}{4}\right)$.

5. Find the intervals in which the function f given by

$$f(x) = 2x^3 - 3x^2 - 36x + 7 \text{ is}$$

(a) strictly increasing (b) strictly decreasing.

Sol. **Given:** $f(x) = 2x^3 - 3x^2 - 36x + 7$

$\therefore \quad f'(x) = 6x^2 - 6x - 36$

Step I. Form factors of $f'(x)$

$f'(x) = 6(x^2 - x - 6)$

(**Caution:** Don't omit 6. It can't be cancelled only from R.H.S.)

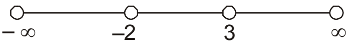
or $\quad f'(x) = 6(x^2 - 3x + 2x - 6) = 6[x(x - 3) + 2(x - 3)]$

$= 6(x + 2)(x - 3) \quad \quad ...(i)$

Step II. Put $f'(x) = 0 \Rightarrow 6(x + 2)(x - 3) = 0$

But $6 \neq 0 \quad \therefore$ Either $x + 2 = 0$ or $x - 3 = 0$

i.e., $x = -2, x = 3$.



These turning points $x = -2$ and $x = 3$ divide the real line into three disjoint sub-intervals $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$.

Step III.

Interval	**sign of $f'(x)$ $= 6(x + 2)(x - 3)$ ...(i)**	**Nature of function f**
$(-\infty, -2)$	Take $x = -3$ (say). Then from (*i*), $f'(x) = (+)(-)(-)$ $= (+)$ *i.e.,* > 0	$\therefore$ f is strictly increasing $\uparrow$ in $(-\infty, -2)$
$(-2, 3)$	Take $x = 2$ (say). Then from (*i*), $f'(x) = (+)(+)(-)$ $= (-)$ *i.e.,* < 0	$\therefore$ f is strictly decreasing $\downarrow$ in $(-2, 3)$
$(3, \infty)$	Take $x = 4$ (say). Then from (*i*), $f'(x) = (+)(+)(+)$ $= (+)$ *i.e.,* > 0	$\therefore$ f is strictly increasing $\uparrow$ in $(3, \infty)$

Thus, (*a*) f is strictly increasing in $(-\infty, -2)$ and $(3, \infty)$.

(*b*) f is strictly decreasing in $(-2, 3)$.

6. Find the intervals in which the following functions are strictly increasing or decreasing.

(*a*) $x^2 + 2x - 5$ **(*b*) $10 - 6x - 2x^2$**

(*c*) $-2x^3 - 9x^2 - 12x + 1$ **(*d*) $6 - 9x - x^2$**

(*e*) $(x + 1)^3(x - 3)^3$.

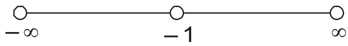
Sol. (*a*) **Given:** $f(x) = x^2 + 2x - 5$

$\therefore \quad f'(x) = 2x + 2 = 2(x + 1) \quad \quad ...(i)$

Step I. Put $f'(x) = 0 \Rightarrow 2(x + 1) = 0$

But $2 \neq 0$. Therefore, $x + 1 = 0$ *i.e.,* $x = -1$.

This turning point $x = -1$ divides the real line into two disjoint sub-intervals $(-\infty, -1)$ and $(-1, \infty)$.



Step II.

Interval	sign of $f'(x) = 2(x+1)$ ...(*i*)	Nature of function f
$(-\infty, -1)$	Take $x = -2$ (say). Then from (*i*), $f'(x) = (-)$ *i.e.,* < 0	$\therefore$ f is strictly decreasing $\downarrow$
$(-1, \infty)$	Take $x = 0$ (say). Then from (*i*), $f'(x) = (+)$ *i.e.,* > 0	$\therefore$ f is strictly increasing $\uparrow$

Thus, f is strictly increasing in $(-1, \infty)$ (*i.e.,* $x > -1$) and strictly decreasing in $(-\infty, -1)$ (*i.e.,* $x < -1$).

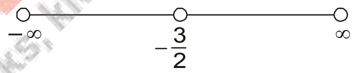
(*b*) **Given:** $f(x) = 10 - 6x - 2x^2$

$\therefore \quad f'(x) = -6 - 4x = -2(3 + 2x) \quad \text{...}(i)$

Step I. Put $f'(x) = 0 \Rightarrow -2(3 + 2x) = 0$

But $-2 \neq 0$. Therefore, $3 + 2x = 0$ *i.e.,* $2x = -3$

i.e., $x = -\dfrac{3}{2}$.



This turning point $x = -\dfrac{3}{2}$ divides the real line into two disjoint sub-intervals $\left(-\infty, -\dfrac{3}{2}\right)$ and $\left(-\dfrac{3}{2}, \infty\right)$.

Step III.

Interval	sign of $f'(x) = -2(3+2x)$ ...(*i*)	Nature of function f
$\left(-\infty, -\dfrac{3}{2}\right)$	Take $x = -2$ (say). Then from (*i*), $f'(x) = (-)(-) = (+)$ *i.e.,* > 0	$\therefore$ f is strictly increasing $\uparrow$
$\left(-\dfrac{3}{2}, \infty\right)$	Take $x = -1$ (say). Then from (*i*), $f'(x) = (-)(+) = (-)$ *i.e.,* < 0	$\therefore$ f is strictly decreasing $\downarrow$

Thus, f is strictly increasing in $\left(-\infty, -\dfrac{3}{2}\right)$ (*i.e.,* for $x < -\dfrac{3}{2}$) and strictly decreasing in $\left(-\frac{3}{2}, \infty\right)$ (*i.e.,* for $x > -\frac{3}{2}$).

(*c*) Let $f(x) = -2x^3 - 9x^2 - 12x + 1$

$\therefore \quad f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2)$

Step I. Forming factors of $f'(x)$

$$= -6(x^2 + x + 2x + 2) = -6[x(x + 1) + 2(x + 1)]$$

or $\quad f'(x) = -6(x + 1)(x + 2) \qquad ...(i)$

Step II. $f'(x) = 0$ gives $x = -1$ or $x = -2$

The points $x = -2$ and $x = -1$ (arranged in ascending order) divide the real line into 3 disjoint intervals, namely, $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$.

Step III. Nature of $f(x)$

Interval	**sign of $f'(x)$ $= -6(x + 1)(x+2)$ (*i*)**	**Nature of function *f***
$(-\infty, -2)$	Take $x = -3$ (say), Then from (*i*), $f'(x) = (-)\ (-)\ (-)$ $= (-)$ *i.e.,* < 0	$\therefore$ *f* is strictly decreasing in $(-\infty, -2)$ ↓
$(-2, -1)$	Take $x = -1.5$ (say), Then from (*i*), $f'(x) = (-)\ (-)\ (+) = +$ *i.e.,* > 0	$\therefore$ *f* is strictly increasing in $(-2, -1)$ ↑
$(-1, \infty)$	Take $x = 0$ (say), then from (*i*) $f'(x) = (-)(+)(+) = (-)$ *i.e.,* < 0	$\therefore$ *f* is strictly decreasing in $(-1, \infty)$ ↓

$\therefore$ *f* is strictly increasing in $(-2, -1)$ and strictly decreasing in $(-\infty, -2)$ and $(-1, \infty)$

(*d*) Let $f(x) = 6 - 9x - x^2 \quad \therefore \quad f'(x) = -9 - 2x.$

$f(x)$ is strictly **increasing** if $f'(x) > 0$, *i.e.,*

if $-9 - 2x > 0$

or $\quad -2x > 9 \quad$ or $\quad x < -\frac{9}{2}$

$\therefore$ *f* is strictly increasing ↑, in the interval $\left(-\infty, -\frac{9}{2}\right)$.

$f(x)$ is strictly **decreasing** if $f'(x) < 0$, *i.e.,* if $-9 - 2x < 0$

or $\quad -2x < 9 \quad$ or $\quad x > -\frac{9}{2}$

$\therefore$ *f* is strictly decreasing ↓ in the interval $\left(-\frac{9}{2}, \infty\right)$.

(e) Let $f(x) = (x + 1)^3 (x - 3)^3$

then $f'(x) = (x + 1)^3 . 3(x - 3)^2 + (x - 3)^3 . 3(x + 1)^2$

$= 3(x + 1)^2 (x - 3)^2 (x + 1 + x - 3)$

$= 3(x + 1)^2 (x - 3)^2 (2x - 2)$

$= 6(x + 1)^2 (x - 3)^2 (x - 1)$

The factors $(x + 1)^2$ and $(x - 3)^2$ are non-negative for all x.

$\therefore$ $f(x)$ is strictly **increasing** if

$f'(x) > 0$, *i.e.,* if $x - 1 > 0$ or $x > 1$

$f(x)$ is strictly **decreasing** if $f'(x) < 0$, *i.e.,* if $x - 1 < 0$

or $x < 1$.

Thus, f is strictly increasing $\uparrow$ in $(1, \infty)$ and strictly decreasing $\downarrow$ in $(-\infty, 1)$.

7. Show that $y = \log (1 + x) - \dfrac{2x}{2+x}$, $x > -1$ is an increasing function of x throughout its domain.

Sol. Given: $y = \log (1 + x) - \dfrac{2x}{2+x}$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{1+x} \frac{d}{dx}(1 + x) - \left[\frac{(2+x)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(2+x)}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \left[\frac{(2+x)2 - 2x}{(2+x)^2}\right] = \frac{1}{1+x} - \frac{(4+2x-2x)}{(2+x)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+x} - \frac{4}{(2+x)^2} = \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}$$

$$= \frac{4 + x^2 + 4x - 4 - 4x}{(1+x)(2+x)^2} = \frac{x^2}{(1+x)(2+x)^2} \quad ...(i)$$

Domain of the given function is given to be $x > -1$

$\Rightarrow$ $x + 1 > 0$. Also $(2 + x)^2 > 0$ and $x^2 \geq 0$

$\therefore$ From (i), $\dfrac{dy}{dx} \geq 0$ for all x in the domain $(x > -1)$.

$\therefore$ The given function is an increasing function of x (in its domain namely $x > -1$).

Note 1. For an increasing function $\dfrac{dy}{dx} = f'(x) \geq 0$ and for a strictly increasing function $\dfrac{dy}{dx} = f'(x) > 0$.

Note 2. For a decreasing function $\dfrac{dy}{dx} = f'(x) \leq 0$ and for a strictly decreasing function $\dfrac{dy}{dx} = f'(x) < 0$.

8. Find the value of x for which $y = (x(x - 2))^2$ is an increasing function.

Sol. Given: $y\ (= f(x)) = (x(x - 2))^2$.

Step I. Find $\frac{dy}{dx}$ and form factors of R.H.S. of value of $\frac{dy}{dx}$.

$$\therefore \quad \frac{dy}{dx} = 2x(x-2)\frac{d}{dx}[x(x-2)]$$

$$\left[\because \frac{d}{dx}(f(x))^n = n(f(x))^{n-1}\frac{d}{dx}f(x)\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = 2x(x-2)\left[x\frac{d}{dx}(x-2) + (x-2)\frac{d}{dx}x\right] \quad \text{(Product Rule)}$$

$$= 2x(x-2)[x + x - 2] = 2x(x-2)(2x-2)$$

$$\text{or} \quad \frac{dy}{dx} = 4x(x-2)(x-1) \qquad ...(i)$$

Step II. Put $\frac{dy}{dx} = 0$.

$\therefore$ From (i) $4x(x - 2)(x - 1) = 0$

But $4 \neq 0$ $\therefore$ Either $x = 0$ or $x - 2 = 0$ or $x - 1 = 0$

$\Rightarrow$ $x = 0$, $x = 2$, $x = 1$

These three turning points $x = 0$, $x = 1$, $x = 2$ (arranged in their ascending order divide the real line into three sub-intervals $(-\infty, 0]$, $[0, 1]$, $[1, 2]$, $[2, \infty)$.

Step III

Interval	**sign of $\frac{dy}{dx}$ $= 4x(x - 2)(x - 1)$ ...(i)**	**Nature of $y = f(x)$**
$(-\infty, 0]$	Take $x = -1$ (say). Then from (i), $\frac{dy}{dx} = (-)\,(-)\,(-)$ $= (-)$ (or $= 0$ at $x = 0$) *i.e.*, ≤ 0	$\therefore$ $f(x)$ is decreasing ↓
$[0, 1]$	Take $x = \frac{1}{2}$ (say). Then from (i), $\frac{dy}{dx} = (+)\,(-)\,(-)$ $= (+)$ (or $= 0$ at $x = 0$, $x = 1$) *i.e.*, ≥ 0	$\therefore$ $f(x)$ is increasing ↑
$[1, 2]$	Take $x = 1.5$ (say). Then from (i), $\frac{dy}{dx} = (+)\,(-)\,(+)$	$\therefore$ $f(x)$ is decreasing ↓

	$= (-)$ (or $= 0$ at $x = 1, x = 2$) *i.e.,* ≤ 0	
$[2, \infty)$	Take $x = 3$ (say). Then from (*i*), $\frac{dy}{dx} = (+)\,(+)\,(+)$ $= (+)$ (or $= 0$ at $x = 2$) *i.e.,* ≥ 0	$\therefore$ $f(x)$ is increasing $\uparrow$

Therefore, $f(x)$ is an increasing function in the intervals $[0, 1]$ (*i.e.,* $0 \le x \le 1$) and $[2, \infty)$ (*i.e.,* $x \ge 2$).

Remark. (We have included the turning points in the sub-intervals because we are to discuss for increasing function and not for strictly increasing function. See Notes 1 and 2 at the end of solution of Q. No. 7).

9. Prove that $y = \frac{4 \sin \theta}{(2 + \cos \theta)} - \theta$ **is an increasing function of θ in** $\left[0, \frac{\pi}{2}\right]$.

Sol. Here $y = \frac{4 \sin \theta}{(2 + \cos \theta)} - \theta$

$$\Rightarrow \frac{dy}{d\theta} = \frac{(2 + \cos \theta) \,.\, 4 \cos \theta - 4 \sin \theta\,(-\sin \theta)}{(2 + \cos \theta)^2} - 1$$

$$= \frac{8 \cos \theta + 4 \cos^2 \theta + 4 \sin^2 \theta}{(2 + \cos \theta)^2} - 1$$

$$= \frac{8 \cos \theta + 4\,(\cos^2 \theta + \sin^2 \theta) - (2 + \cos \theta)^2}{(2 + \cos \theta)^2} \qquad \text{(Taking L.C.M.)}$$

$$= \frac{8 \cos \theta + 4 - (2 + \cos \theta)^2}{(2 + \cos \theta)^2} = \frac{(8 \cos \theta + 4) - (4 + 4 \cos \theta + \cos^2 \theta)}{(2 + \cos \theta)^2}$$

$$\text{or} \quad \frac{dy}{d\theta} = \frac{4 \cos \theta - \cos^2 \theta}{(2 + \cos \theta)^2} = \frac{\cos \theta\,(4 - \cos \theta)}{(2 + \cos \theta)^2} \qquad ...(i)$$

Since $0 \le \theta \le \frac{\pi}{2}$,

we have $0 \le \cos \theta \le 1$ and, therefore, $4 - \cos \theta > 0$. Also $(2 + \cos \theta)^2 > 0$

$\therefore$ From (*i*), $\frac{dy}{d\theta} \ge 0$ for $0 \le \theta \le \frac{\pi}{2}$.

Hence, y is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

10. Prove that the logarithmic function is strictly increasing on (0, ∞).

Sol. Given: $f(x) = \log x$

$\therefore \quad f'(x) = \frac{1}{x} > 0$ for all x in $(0, \infty)$ $\quad [\because \; x \in (0, \infty) \Rightarrow x > 0]$

$\therefore \quad f(x)$ is strictly increasing on $(0, \infty)$.

11. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (– 1, 1).

Sol. Given: $\quad f(x) = x^2 - x + 1$

$\therefore \quad f'(x) = 2x - 1$

$f(x)$ is strictly increasing if $f'(x) > 0$ *i.e.*, if $2x - 1 > 0$

i.e., if $2x > 1$ or $x > \frac{1}{2}$

$f(x)$ is strictly decreasing if

$$f'(x) < 0 \text{ i.e., if } 2x - 1 < 0 \text{ i.e., } x < \frac{1}{2}$$

$\therefore f(x)$ is strictly increasing for $x > \frac{1}{2}$ *i.e.*, on the interval $\left(\frac{1}{2}, 1\right)$

$[\because$ The given interval is $(-1, 1)]$

and $f(x)$ is strictly decreasing for $x < \frac{1}{2}$ *i.e.*, on the interval

$\left(-1, \frac{1}{2}\right)$. $\quad [\because$ The given interval is $(-1, 1)]$

$\therefore f(x)$ is neither strictly increasing nor strictly decreasing on the interval $(-1, 1)$.

12. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x$ (B) $\cos 2x$ (C) $\cos 3x$ (D) $\tan x$.

Sol. (A) Let $f(x) = \cos x$ then $f'(x) = -\sin x$

$\because \quad 0 < x < \frac{\pi}{2}$ in $\left(0, \frac{\pi}{2}\right)$, therefore $\sin x > 0$

[Because $\sin x$ is positive in both first and second quadrants]

$\Rightarrow \; -\sin x < 0 \quad \therefore \; f'(x) = -\sin x < 0$ on $\left(0, \frac{\pi}{2}\right)$

$\Rightarrow \; f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

(B) Let $f(x) = \cos 2x$ then $f'(x) = -2 \sin 2x$

$\because \quad 0 < x < \frac{\pi}{2}, \quad \therefore \quad 0 < 2x < \pi$

$\Rightarrow \quad \sin 2x > 0 \quad \Rightarrow \quad -2 \sin 2x < 0$

$\therefore f'(x) = -2 \sin 2x < 0$ on $\left(0, \frac{\pi}{2}\right)$

$\Rightarrow f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

(C) Let $f(x) = \cos 3x$ then $f'(x) = -3 \sin 3x$

$\because \quad 0 < x < \frac{\pi}{2}, \quad \therefore \quad 0 < 3x < \frac{3\pi}{2} = 270°$

Now for $0 < 3x < \pi$, $\left(i.e., 0 < x < \frac{\pi}{3}\right)$ $\sin 3x > 0$

($\because$ sin θ is positive in first two quadrants)

$\Rightarrow f'(x) = -3 \sin 3x < 0 \Rightarrow f'(x) < 0$

$\Rightarrow f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{3}\right)$

and for $\pi < 3x < \frac{3\pi}{2}$, $\sin 3x < 0$

[Because sin θ is negative in third quadrant]

$\therefore f'(x) = -3 \sin 3x > 0 \Rightarrow f'(x) > 0$

$\Rightarrow f(x)$ is strictly increasing on $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\therefore$ $f(x)$ is neither strictly increasing nor strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

(D) Let $f(x) = \tan x$ then $f'(x) = \sec^2 x > 0$

$\Rightarrow f(x)$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

Hence, only the functions in options **(A)** and **(B)** are strictly decreasing.

13. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ is strictly decreasing?

(A) (0, 1) **(B) $\left(\frac{\pi}{2}, \pi\right)$** **(C) $\left(0, \frac{\pi}{2}\right)$** **(D) None of these.**

Sol. Given: $f(x) = x^{100} + \sin x - 1$

$\therefore \quad f'(x) = 100\, x^{99} + \cos x$ ...(*i*)

Let us test option (A) (0, 1)

On (0, 1); $x > 0$ and hence $100\, x^{99} > 0$

For $\cos x$; interval (0, 1) $\Rightarrow$ (0, 1 radian)

$\Rightarrow$ (0, 57° nearly) ($\because$ π radians = 180°

$\Rightarrow \quad 1 \text{ radian} = \frac{180°}{\pi}$

$= \frac{180°}{\left(\frac{22}{7}\right)} = 180° \times \frac{7}{22} = \frac{90° \times 7}{11} = \frac{630°}{11} = 57°$ nearly)

$\Rightarrow$ x is in first quadrant and hence cos x is positive.

$\therefore$ From (i), $f'(x) = 100x^{99} + \cos x > 0$ and hence $f(x)$ is strictly increasing on (0, 1).

$\therefore$ Option (A) is not the correct option.

Let us test option (B) $\left(\frac{\pi}{2}, \pi\right)$

For $100x^{99}$, $x \in \left(\frac{\pi}{2}, \pi\right)$

$$\Rightarrow \quad x \in \left(\frac{\left(\frac{22}{7}\right)}{2}, \frac{22}{7}\right) = \left(\frac{11}{7}, \frac{22}{7}\right) = (1.5, 3.1)$$

$\Rightarrow$ $x > 1$ $\Rightarrow$ $x^{99} > 1$ and hence $100x^{99} > 100$.

For cos x, $\left(\frac{\pi}{2}, \pi\right)$ $\Rightarrow$ Second quadrant and hence cos x is negative and has value between – 1 and 0.

($\because$ $-1 \le \cos \theta \le 1$)

$\therefore$ From (i), $f'(x) = 100x^{99} + \cos x > 100 - 1 = 99 > 0$

$\therefore$ $f(x)$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

$\therefore$ Option (B) is not the correct option.

Let us test option (C) $\left(0, \frac{\pi}{2}\right)$

On $\left(0, \frac{\pi}{2}\right)$ *i.e.,* (0, 1.5) both terms $100x^{99}$ and cos x are positive and hence from (i), $f'(x) = 100x^{99} + \cos x$ is positive.

$\therefore$ $f(x)$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ also.

$\therefore$ Option (C) is also not the correct option.

$\therefore$ Option (D) is the correct answer.

14. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$ strictly increasing on (1, 2).

Sol. Here $f(x) = x^2 + ax + 1$ ...(i)

Differentiating (i) w.r.t. x, $f'(x) = 2x + a$ ...(ii)

Because $f(x)$ is strictly increasing on (1, 2) (given),

$\therefore$ $f'(x) = 2x + a > 0$ for all x in (1, 2) ...(iii)

Now on (1, 2), $1 < x < 2$

Multiplying by 2, $2 < 2x < 4$ for all x in (1, 2).

Adding a to all sides

$$2 + a < 2x + a < 4 + a \text{ for all } x \text{ in } (1, 2)$$

or $\quad 2 + a < f'(x) < 4 + a$ for all x in (1, 2) [By (ii)]

$\therefore$ Minimum value of $f'(x)$ is $2 + a$ and maximum value of

$f'(x)$ is $4 + a$. ...(*iv*)

But from (*iii*), $f'(x) > 0$ for all x in $(1, 2)$

$\therefore$ $2 + a > 0$ and $4 + a > 0$ [By (*iv*)]

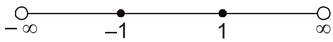
$\therefore$ $a > -2$ and $a > -4$

$\therefore$ $a > -2$ [$\because$ $a > -2 \Rightarrow a > -4$ automatically]

$\therefore$ Least value of a is -2.

15. Let I be any interval disjoint from [– 1, 1]. Prove that the function f given by $f(x) = x + \frac{1}{x}$ is strictly increasing on I.

Sol. Given: $f(x) = x + \frac{1}{x} = x + x^{-1}$



$$\therefore \quad f'(x) = 1 + (-1)x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$

Forming factors, $f'(x) = \frac{(x - 1(x + 1)}{x^2}$ ...(*i*)

Given: I is an interval disjoint from $[-1, 1]$.

i.e., $\text{I} = (-\infty, \infty) - [-1, 1] = (-\infty, -1) \cup (1, \infty)$

$\therefore$ For every $x \in \text{I}$, either $x < -1$ or $x > 1$

For $x < -1$ (For example, $x = -2$ (say)),

from (*i*), $f'(x) = \frac{(-)(-)}{(+)} = (+)$ *i.e.,* > 0

For $x > 1$ (For example, $x = 2$ (say)),

from (*i*), $f'(x) = \frac{(+)(+)}{(+)} = (+)$ *i.e.,* > 0

$\therefore$ $f'(x) > 0$ for all $x \in \text{I}$ $\therefore$ $f(x)$ is strictly increasing on I.

16. Prove that the function f given by $f(x) = \log \sin x$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

Sol. Given: $f(x) = \log \sin x$

$$\therefore \quad f'(x) = \frac{1}{\sin x} \frac{d}{dx} \sin x = \frac{1}{\sin x} (\cos x) = \cot x \quad \text{...}(i)$$

On the interval $\left(0, \frac{\pi}{2}\right)$ *i.e.,* in first quadrant, from (*i*), $f'(x) = \cot x > 0$

$\therefore$ $f(x)$ is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

On the interval $\left(\frac{\pi}{2}, \pi\right)$ *i.e.,* in second quadrant, from (*i*), $f'(x) = \cot x < 0$.

$\therefore$ $f(x)$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

17. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

Sol. Given: $f(x) = \log \cos x$

$$\therefore\ f'(x) = \frac{1}{\cos x}\frac{d}{dx}(\cos x) = \frac{1}{\cos x}(-\sin x) = -\tan x \qquad ...(i)$$

We know that on the interval $\left(0, \frac{\pi}{2}\right)$ *i.e.,* in first quadrant, $\tan x$ is positive and hence from (i), $f'(x) = -\tan x$ is negative *i.e.,* < 0.

$\therefore\ f(x)$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

We know that on the interval $\left(\frac{\pi}{2}, \pi\right)$ *i.e.,* in second quadrant; $\tan x$ is negative and hence from (i),

$f'(x) = -\tan x$ is positive *i.e.,* > 0.

$\therefore\ f(x)$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

18. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

Sol. Given: $f(x) = x^3 - 3x^2 + 3x - 100$.

Then $f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1)$

$= 3(x - 1)^2 \geq 0$ for all x in R

$\therefore\ f(x)$ is increasing on R.

19. The interval in which $y = x^2 e^{-x}$ is increasing is

(A) $(-\infty, \infty)$ (B) $(-2, 0)$ (C) $(2, \infty)$ (D) $(0, 2)$.

Sol. Given: $y\ (= f(x)) = x^2 e^{-x}$

$$\therefore\ \frac{dy}{dx} = x^2 \frac{d}{dx}e^{-x} + e^{-x}\frac{d}{dx}x^2 = x^2 e^{-x}(-1) + e^{-x}(2x)$$

$$= -x^2 e^{-x} + 2x e^{-x} = x e^{-x}(-x + 2)$$

or $\dfrac{dy}{dx} = \dfrac{x(2 - x)}{e^x}$

Out of the intervals mentioned in the options (A), (B), (C) and (D), $\frac{dy}{dx} > 0$ for all x in interval $(0, 2)$ of option (D).

$\therefore\ y\ (= f(x))$ is strictly increasing and hence increasing in interval $(0, 2)$ of option D.

Note. For a subjective solution of this question, proceed as in solution of Q. No. 6 (a), (b), (c).

Remark. Increasing (decreasing) function or monotonically increasing (or monotonically decreasing) function have the same meaning.