NCERT Class 12 Maths Solutions

Chapter - 6

1. Find the rate of change of the area of a circle with respect to its radius r when

(a) $r = 3$ cm **(b) $r = 4$ cm.**

Sol. Let z denote the area of a circle of variable radius r.

We know that z (area of circle) $= \pi r^2$

$\therefore$ By Note 1 above, rate of change of area z w.r.t. radius r

$$= \frac{dz}{dr} = \pi(2r) = 2\pi r \qquad ...(i)$$

(a) **When $r = 3$ cm** (given), $\therefore$ From (i), $\frac{dz}{dr} = 2\pi(3) = 6\pi$ sq. cm

(b) **When $r = 4$ cm** (given), $\therefore$ From (i), $\frac{dz}{dr} = 2\pi(4) = 8\pi$ sq. cm.

2. The volume of a cube is increasing at the rate of 8 cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm?

Sol. Let x cm be the edge of a cube (for example, a room whose length, breadth and height are equal) at time t.

Given: Rate of **Increase** of volume of cube = 8 cm^3/sec.

$$\Rightarrow \quad \frac{d}{dt}(x.x.x) \quad i.e., \quad \frac{d}{dt}\boldsymbol{x^3} \text{ is positive and } = 8$$

(8 cm^3/sec $\Rightarrow$ rate of Increase w.r.t. time)

$$\Rightarrow \quad 3x^2 \frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{x} = 8 \quad \Rightarrow \quad \frac{dx}{dt} = \frac{8}{3x^2} \qquad ...(i)$$

Let z denote the surface area of the cube. $\therefore$ z = $\boldsymbol{6x^2}$

(Area of four walls + Area of floor + Area of ceiling)

$\therefore$ Rate of change of surface area of cube

$$= \frac{dz}{dt} = 6\frac{d}{dt}x^2 = 6\left(2x\frac{dx}{dt}\right) = 12x\left(\frac{8}{3x^2}\right) \qquad [\text{By } (i)]$$

$$= 4\left(\frac{8}{x}\right) = \frac{32}{x} \text{ cm}^2\text{/sec.}$$

Putting x = 12 cm (given), $\frac{dz}{dt} = \frac{32}{12} = \frac{8}{3}$ cm^2/sec

Since $\frac{dz}{dt}$ is positive, therefore, surface area is increasing at the rate of $\frac{8}{3}$ cm^2/sec.

3. The radius of a circle is increasing uniformely at the rate of 3 cm per second. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

Sol. Let x cm denote the radius of a circle at time t.

Given: Rate of increase of radius of circle = 3 cm/sec.

$$\Rightarrow \quad \frac{dx}{dt} \text{ is positive and } = 3 \text{ cm/sec} \qquad ...(i)$$

Let z denote the area of the circle.

$\therefore$ z = $\boldsymbol{\pi x^2}$.

$\therefore$ Rate of change of area of circle = $\frac{dz}{dt} = \pi\frac{d}{dt}x^2$

$$= \pi.2x\frac{\boldsymbol{dx}}{\boldsymbol{dt}} = 2\pi x(3) \qquad [\text{By } (i)]$$

$$= 6\pi x.$$

Putting x = 10 cm (given), $\frac{dz}{dt}$ = $6\pi(10)$ = 60π cm^2/sec

Since $\frac{dz}{dt}$ is positive, therefore area of circle is **increasing** at the rate of 60π cm^2/sec.

4. An edge of a variable cube is increasing at the rate of 3 cm per second. How fast is the volume of the cube increasing when the edge is 10 cm long?

Sol. Let x cm be the edge of variable cube at time t.

Given: Rate of increase of edge x is 3 cm/sec.

$\therefore \quad \frac{dx}{dt}$ is positive and = 3 cm/sec ...(*i*)

Let z denote the volume of the cube.

$\therefore \quad z = x^3$

$\therefore$ Rate of change of volume of cube

$$= \frac{dz}{dt} = \frac{d}{dt}x^3 = 3x^2\frac{dx}{dt} = 3x^2(3) \quad \text{[By (i)]}$$

or $\quad \frac{dz}{dt} = 9x^2$ cm^3/sec.

Putting x = 10 cm (given), $\frac{dz}{dt} = 9(10)^2 = 9(100) = 900$ cm^3/sec.

Since $\frac{dz}{dt}$ is positive, therefore volume of the cube is **increasing** at the rate of 900 cm^3/sec.

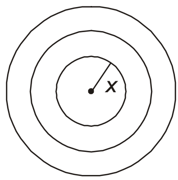
5. A stone is dropped into a quite lake and waves move in circles at the rate of 5 cm/sec. At the instant when radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

Sol. Let x cm be radius of circular wave at time t.

Given: Waves move in circles at the rate of 5 cm/sec.

$\Rightarrow$ Radius x of circular wave increases at the rate of 5 cm/sec.

$\Rightarrow \quad \frac{dx}{dt}$ is positive and = 5 cm/sec. ...(*i*)



Let z denote the enclosed area of the circular wave at time t.

$\therefore \quad z = \pi x^2$.

$\therefore$ Rate of change of area $= \frac{dz}{dt} = \pi\frac{d}{dt}x^2 = \pi.2x\frac{dx}{dt}$

$= 2\pi x(5)$ [By (*i*)] $= 10\pi x$

Putting x = 8 cm (given), $\frac{dz}{dt} = 10\pi(8) = 80\pi$ cm^2/sec.

Since $\frac{dz}{dt}$ is positive, therefore area of circular wave is increasing at the rate of 80π cm^2/sec.

6. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

Sol. Let x be the radius of the circle at time t.

Given: Rate of increase of radius of circle = 0.7 cm/sec.

$\Rightarrow \quad \frac{dx}{dt}$ is positive and = 0.7 cm/sec. ...(*i*)

Let z denote the circumference of the circle at time t.

$\therefore$ $\quad z = 2\pi x$ **(Formula)**

$\therefore$ Rate of change of circumference of circle

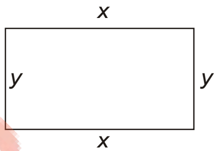
$$= \frac{dz}{dt} = \frac{d}{dt}(2\pi x) = 2\pi \frac{dx}{dt} = 2\pi(0.7) \qquad \text{(By } (i))$$

$$= 1.4\pi \text{ cm/sec.}$$

7. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle.

Sol. Given: Rate of decrease of length x of rectangle is 5 cm/minute.

$\Rightarrow \frac{dx}{dt}$ is negative and $= -5$ cm/minute ...(i)



Given: Rate of increase of width y of rectangle is 4 cm/minute.

$\Rightarrow \frac{dy}{dt}$ is positive and $= 4$ cm/minute ...(ii)

(a) Let z denote the perimeter of rectangle.

$\therefore$ $\quad z = x + y + x + y = 2x + 2y$

$$\therefore \quad \frac{dz}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$$

Putting values from (i) and (ii),

$$\frac{dz}{dt} = 2(-5) + 2(4) = -10 + 8 = -2 \text{ is negative.}$$

$\therefore$ Perimeter z of the rectangle is **decreasing** at the rate of 2 cm/sec.

(Even when x = 8 cm and y = 6 cm).

(b) Let z denote the area of rectangle.

$\therefore$ $\quad z = xy$

$$\therefore \quad \frac{dz}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} \qquad | \text{ By Product Rule}$$

Putting x = 8 cm and y = 6 cm (given) and putting values of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ from (i) and (ii),

$$\frac{dz}{dt} = 8(4) + 6(-5) = 32 - 30 = 2 \text{ is positive.}$$

$\therefore$ Area z of the rectangle is **increasing** at the rate of 2 sq cm/minute even when x = 8 cm and y = 6 cm.

8. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

Sol. Let x cm be the radius of the spherical balloon at time t.

Given: Rate at which the balloon is being inflated *i.e.,* rate at which the volume of the balloon is increasing = 900 cu. cm sec.

$$\Rightarrow \quad \frac{d}{dt}\left(\frac{4\pi}{3}x^3\right) = 900$$

$$\Rightarrow \quad \frac{4\pi}{3}\frac{d}{dt}x^3 = 900 \quad \Rightarrow \quad \frac{4\pi}{3} \cdot 3x^2\frac{dx}{dt} = 900$$

$$\Rightarrow \quad 4\pi x^2\frac{dx}{dt} = 900 \quad \Rightarrow \quad \frac{dx}{dt} = \frac{900}{4\pi x^2}$$

Putting $x = 15$ cm (given), $\frac{dx}{dt} = \frac{900}{4\pi(15)^2} = \frac{900}{4\pi(225)}$

$$= \frac{900}{900\pi} = \frac{1}{\pi} \text{ is positive.}$$

$\therefore$ Radius of balloon is **increasing** at the rate of $\frac{1}{\pi}$ cm sec.

9. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm.

Sol. We know that the volume V of a balloon with radius x is $V = \frac{4}{3}\pi x^3$

$\therefore$ Rate of change of volume with respect to radius x is given by

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right) = \frac{4}{3}\pi \cdot 3x^2 = 4\pi x^2$$

$\therefore$ When $x = 10$ cm, $\frac{dV}{dx} = 4\pi(10)^2 = 400\pi$

i.e., the volume is increasing at the rate of $4\pi(10)^2 = 400\pi$ cm^3/cm.

10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall? (*Important*)

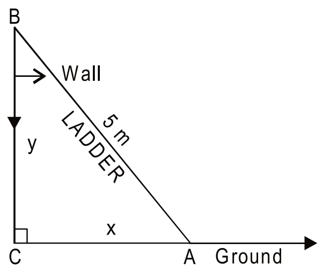
Sol. Let AB be the ladder and C, the junction of wall and ground, AB = 5 m

Let CA = x metres, CB = y metres.

We know that as the end A moves away from C, the end B moves towards C.

[$\because$ Length of ladder can't change]

i.e., as x increases, y decreases.



$$\text{Now } \frac{dx}{dt} = 2 \text{ cm/s}$$

...(i) (given)

In ΔABC, by Pythagoras Theorem $AC^2 + BC^2 = AB^2$

or $$x^2 + y^2 = 5^2 = 25 \quad ...(ii)$$

Differentiating both sides w.r.t. t, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

or $$2x(2) + 2y = 0 \quad \text{or} \quad 2y\frac{dy}{dt} = -4x$$

$$\therefore \quad \frac{dy}{dt} = -\frac{2x}{y} \quad ...(iii)$$

When $x = 4$ (given), from (*ii*), $16 + y^2 = 25$ or $y^2 = 9$ or $y = 3$

$\therefore$ From (*iii*), $\frac{dy}{dt} = -\frac{2 \times 4}{3} = -\frac{8}{3}$ cm/s.

Note. The negative sign indicates that y decreases as t increases.

11. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

Sol. Given: Equation of the curve is $6y = x^3 + 2$ ...(*i*)

Let (x, y) be the required point on curve (*i*).

Given: y-coordinate is changing 8 times as fast as the x coordinate.

$\Rightarrow$ Rate of change of y w.r.t. x is 8

$$\Rightarrow \quad \frac{dy}{dx} = 8 \quad ...(ii)$$

Differentiating both sides of (*i*) w.r.t. x, we have $6\frac{dy}{dx} = 3x^2$

Putting $\frac{dy}{dx} = 8$ from (*ii*), $48 = 3x^2 \Rightarrow x^2 = \frac{48}{3} = 16 \therefore x = \pm 4$

When $x = 4$, from (*i*), $6y = 64 + 2 = 66 \quad \therefore \quad y = \frac{66}{6} = 11$

$\therefore$ One required point is (4, 11).

When $x = -4$, from (*i*) $6y = -64 + 2 = -62$,

$$\therefore \quad y = \frac{-62}{6} = \frac{-31}{3}$$

$\therefore$ Second required point is $\left(-4, \frac{-31}{3}\right)$.

$\therefore$ Required points on curve (*i*) are (4, 11) and $\left(-4, \frac{-31}{3}\right)$.

12. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing when the radius is 1 cm?

Sol. Let x cm be the radius of the air bubble at time t.

Given: Rate of increase of radius of air bubble (spherical as we all know) $= \frac{1}{2}$ cm/sec.

$\Rightarrow \quad \dfrac{dx}{dt}$ is positive and $= \dfrac{1}{2}$ cm/sec. ...(*i*)

Let z denote the volume of the air bubble.

$\therefore \qquad \boldsymbol{z = \dfrac{4\pi}{3} x^3}$

$\therefore \quad \dfrac{dz}{dt}$ = Rate of change of volume of air bubble

$$= \frac{4\pi}{3} \frac{d}{dt} x^3 = \frac{4\pi}{3} \, . \, 3x^2 \frac{dx}{dt} = 4\pi x^2 \left(\frac{1}{2}\right) \text{ [By } (i)] = 2\pi x^2$$

Putting $x = 1$ cm (given), $\dfrac{dz}{dt} = 2\pi(1)^2 = 2\pi$ which is positive.

$\therefore$ Required rate of increase of volume of air bubble is 2π cm^3/sec.

13. A balloon, which always remains spherical, has a variable diameter $\dfrac{3}{2}(2x + 1)$. Find the rate of change of its volume with respect to x.

Sol. Diameter of the balloon $= \dfrac{3}{2}(2x + 1)$ (given)

$\therefore$ Radius of balloon $= \dfrac{1}{2}$(diameter) $= \dfrac{1}{2} \, . \, \dfrac{3}{2}(2x + 1) = \dfrac{3}{4}(2x + 1)$

$\therefore$ Volume of balloon (V) $= \boldsymbol{\dfrac{4}{3} \pi \text{ (radius)}^3}$

$$= \frac{4\pi}{3} \left(\frac{3}{4}(2x + 1)\right)^3 = \frac{4}{3}\pi \, . \, \frac{27}{64}(2x + 1)^3$$

$$= \frac{9\pi}{16}(2x + 1)^3 \text{ cu. units}$$

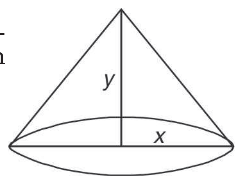
$\therefore$ Rate of change of volume w.r.t. x,

$$= \frac{dV}{dx} = \frac{9\pi}{16} \, . \, 3(2x + 1)^2 \, . \, \frac{d}{dx}(2x + 1)$$

$$= \frac{27\pi}{16}(2x + 1)^2 \, . \, 2 = \frac{27\pi}{8}(2x + 1)^2.$$

14. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Sol. Let the height and base radius of the sand-cone formed at time t sec be y cm and x cm respectively. Then $y = \dfrac{1}{6}x$ (given) or $x = 6y$.



Volume of cone (V) $= \dfrac{1}{3}\pi x^2 \, y$

Putting $x = 6y$, $\qquad V = \dfrac{1}{3}\pi(6y)^2 \, y = 12 \, \pi y^3$

$\therefore \quad \dfrac{dV}{dy} = 36 \, \pi y^2$ ...(*i*)

It is given that sand is pouring from a pipe to form a sand-cone at the rate of 12 cm^3/sec.

$\therefore \quad \frac{dV}{dt} = 12 \quad \Rightarrow \quad \frac{dV}{dy} \times \frac{dy}{dt} = 12$

$\Rightarrow \quad 36\pi y^2 \times \frac{dy}{dt} = 12$ (By (*i*)) $\Rightarrow \frac{dy}{dt} = \frac{1}{3\pi y^2}$

When $y = 4$ cm, (given); $\frac{dy}{dt} = \frac{1}{3\pi \times 4^2} = \frac{1}{48\pi}$ cm/sec.

15. The total cost C(*x*) in rupees associated with the production of *x* units of an item is given by

$$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000.$$

Find the marginal cost when 17 units are produced.

Sol. Marginal cost *is defined as the rate of change of total cost with respect to the number of units produced.*

$\therefore$ Marginal cost (MC) $= \frac{dC}{dx}$

$= \frac{d}{dx}(0.007x^3 - 0.003x^2 + 15x + 4000)$

$= 0.021x^2 - 0.006x + 15$

$\therefore$ When $x = 17$, MC $= 0.021 \times (17)^2 - 0.006 \times (17) + 15$

$= 0.021(289) - 0.102 + 15$

$= 6.069 - 0.102 + 15 = 20.967$

Hence, the required marginal cost = ₹ 20.97.

16. The total revenue in rupees received from the sale of *x* units of a product is given by

$$R(x) = 13x^2 + 26x + 15.$$

Find the marginal revenue when *x* = 7.

Sol. Marginal Revenue *is defined as the rate of change of total revenue with respect to the number of units sold.*

$\therefore$ Marginal revenue (MR) $= \frac{dR}{dx}$

$= \frac{d}{dx}(13x^2 + 26x + 15) = 26x + 26$

When $x = 7$, MR $= 26 \times 7 + 26 = 208$

Hence, the required marginal revenue = ₹ 208.

Choose the correct answer in Exercises 17 and 18.

17. The rate of change of the area of a circle with respect to its radius *r* at *r* = 6 cm is

(A) 10π (B) 12π (C) 8π (D) 11π.

Sol. Let z denote the area of a circle of radius r.

$\therefore \quad z = \pi r^2$

$\therefore$ Rate of change of area z w.r.t. radius $r = \frac{dz}{dr} = 2\pi r$

Putting $r = 6$ cm (given), $\dfrac{dz}{dr} = 2\pi(6) = 12\pi$

$\therefore$ Option (B) is the correct answer.

18. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. The marginal revenue, when $x = 15$ is

(A) 116 **(B) 96** **(C) 90** **(D) 126**

Sol. Given: Total revenue $R(x) = 3x^2 + 36x + 5$

$\therefore$ Marginal revenue $= \dfrac{d}{dx}R(x) = 6x + 36$

Putting $x = 15$ (given), $\dfrac{d(R(x))}{dx} = 6(15) + 36$

$\therefore$ $90 + 36 = 126$

$\therefore$ Option (D) is the correct answer.