NCERT Class 12 Maths

Solutions

Chapter - 5

Exercise 5.3

Find $\frac{dy}{dx}$ in the following Exercises 1 to 15.

1. $2x + 3y = \sin x$.

Sol. Given: $2x + 3y = \sin x$

Differentiating both sides w.r.t. x, we have

$$\frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}\sin x$$

$$\therefore\ 2 + 3\frac{dy}{dx} = \cos x \ \Rightarrow\ 3\frac{dy}{dx} = \cos x - 2 \quad \therefore\ \frac{dy}{dx} = \frac{\cos x - 2}{3}.$$

2. $2x + 3y = \sin y$.

Sol. Given: $2x + 3y = \sin y$

Differentiating both sides w.r.t. x, we have

$$\frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}\sin y \qquad \therefore\ 2 + 3\frac{dy}{dx} = \cos y\frac{dy}{dx}$$

$$\Rightarrow\ -\cos y\frac{dy}{dx} + 3\frac{dy}{dx} = -2 \ \Rightarrow\ -\frac{dy}{dx}(\cos y - 3) = -2$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{2}{\cos y - 3}.$$

3. $\boldsymbol{ax + by^2 = \cos y.}$

Sol. Given: $ax + by^2 = \cos y$

Differentiating both sides w.r.t. x, we have

$$\frac{d}{dx}(ax) + \frac{d}{dx}(by^2) = \frac{d}{dx}(\cos y) \qquad \therefore\ a + b \,.\, 2y\frac{dy}{dx} = -\sin y\frac{dy}{dx}$$

$$\Rightarrow \qquad 2by\frac{dy}{dx} + \sin y\frac{dy}{dx} = -a$$

$$\Rightarrow \qquad \frac{dy}{dx}(2by + \sin y) = -a \qquad \Rightarrow \qquad \frac{dy}{dx} = \frac{-a}{2by + \sin y}$$

4. $\boldsymbol{xy + y^2 = \tan x + y.}$

Sol. Given: $xy + y^2 = \tan x + y$

Differentiating both sides w.r.t. x, we have

$$\frac{d}{dx}(xy) + \frac{d}{dx}y^2 = \frac{d}{dx}\tan x + \frac{d}{dx}y$$

Applying product rule,

$$x\frac{d}{dx}y + y\frac{d}{dx}x + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow \qquad x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow \qquad x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x - y$$

$$\Rightarrow \qquad (x + 2y - 1)\frac{dy}{dx} = \sec^2 x - y \qquad \therefore \frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}.$$

5. $\boldsymbol{x^2 + xy + y^2 = 100.}$

Sol. Given: $x^2 + xy + y^2 = 100$

Differentiating both sides w.r.t. x,

$$\frac{d}{dx}x^2 + \frac{d}{dx}xy + \frac{d}{dx}y^2 = \frac{d}{dx}(100)$$

$$\therefore \quad 2x + \left(x\frac{d}{dx}y + y\frac{d}{dx}x\right) + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (x + 2y)\frac{dy}{dx} = -2x - y \qquad \Rightarrow \qquad \frac{dy}{dx} = -\frac{(2x + y)}{x + 2y}.$$

6. $\boldsymbol{x^3 + x^2y + xy^2 + y^3 = 81.}$

Sol. Given: $x^3 + x^2y + xy^2 + y^3 = 81$

Differentiating both sides w.r.t. x,

$$\frac{d}{dx}x^3 + \frac{d}{dx}x^2y + \frac{d}{dx}xy^2 + \frac{d}{dx}y^3 = \frac{d}{dx}81$$

$$\therefore \quad 3x^2 + \left(x^2\frac{dy}{dx} + y\,.\,\frac{d}{dx}x^2\right) + x\frac{d}{dx}y^2 + y^2\frac{d}{dx}(x) + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 3x^2 + x^2\frac{dy}{dx} + y \,.\, 2x + x \,.\, 2y\frac{dy}{dx} + y^2.1 + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(x^2 + 2xy + 3y^2) = -3x^2 - 2xy - y^2$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}.$$

7. $\sin^2 y + \cos xy = \pi$.

Sol. Given: $\sin^2 y + \cos xy = \pi$

Differentiating both sides w.r.t. x,

$$\frac{d}{dx}(\sin y)^2 + \frac{d}{dx}\cos xy = \frac{d}{dx}(\pi)$$

$$\therefore \quad 2(\sin y)^1\frac{d}{dx}\sin y - \sin xy\frac{d}{dx}(xy) = 0$$

$$\Rightarrow \quad 2\sin y\cos y\frac{dy}{dx} - \sin xy\left(x\frac{dy}{dx} + y\,.\,1\right) = 0$$

$$\Rightarrow \quad \sin 2y\frac{dy}{dx} - x\sin xy\frac{dy}{dx} - y\sin xy = 0$$

$$\Rightarrow \quad (\sin 2y - x\sin xy)\frac{dy}{dx} = y\sin xy$$

$$\therefore \quad \frac{dy}{dx} = \frac{y\sin xy}{\sin 2y - x\sin xy}.$$

8. $\sin^2 x + \cos^2 y = 1$.

Sol. Given: $\sin^2 x + \cos^2 y = 1$

Differentiating both sides w.r.t. x,

$$\frac{d}{dx}(\sin x)^2 + \frac{d}{dx}(\cos y)^2 = \frac{d}{dx}(1)$$

$$\therefore \quad 2(\sin x)^1\frac{d}{dx}\sin x + 2(\cos y)^1\frac{d}{dx}\cos y = 0$$

$$\Rightarrow \quad 2\sin x\cos x + 2\cos y\left(-\sin y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad 2\sin x\cos x - 2\sin y\cos y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \sin 2x - \sin 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad -\sin 2y\frac{dy}{dx} = -\sin 2x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}.$$

9. $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

Sol. Given: $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

To simplify the given Inverse T-function, put $\boldsymbol{x = \tan\theta}$.

$$\therefore \quad y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \quad y = 2\tan^{-1} x \qquad (\because \; x = \tan\theta \;\Rightarrow\; \theta = \tan^{-1} x)$$

$$\therefore \quad \frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}.$$

10. $\boldsymbol{y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}}$.

Sol. Given: $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

To simplify the given Inverse T-function, put $\boldsymbol{x = \tan\theta}$.

$$\therefore \quad y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta) = 3\theta$$

$$\Rightarrow \quad y = 3\tan^{-1} x \qquad (\because \; x = \tan\theta \;\Rightarrow\; \theta = \tan^{-1} x)$$

$$\therefore \quad \frac{dy}{dx} = 3 \cdot \frac{1}{1+x^2} = \frac{3}{1+x^2}.$$

11. $\boldsymbol{y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1}$.

Sol. Given: $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

To simplify the given Inverse T-function, put $\boldsymbol{x = \tan\theta}$.

$$\therefore \quad y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta)$$

$$= 2\theta = 2\tan^{-1} x \quad (\because \; x = \tan\theta \;\Rightarrow\; \theta = \tan^{-1} x)$$

$$\therefore \quad \frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}.$$

12. $\boldsymbol{y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1}$.

Sol. Given: $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

To simplify the given Inverse T-function, **put $\boldsymbol{x = \tan\theta}$.**

$$\therefore \quad y = \sin^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\cos 2\theta)$$

$$= \sin^{-1}\sin\left(\frac{\pi}{2} - 2\theta\right) = \frac{\pi}{2} - 2\theta$$

$$\Rightarrow \quad y = \frac{\pi}{2} - 2\tan^{-1} x \qquad (\because \; x = \tan\theta \;\Rightarrow\; \theta = \tan^{-1} x)$$

$\therefore \quad \dfrac{dy}{dx} = 0 - 2 \cdot \dfrac{1}{1+x^2} = \dfrac{-2}{1+x^2}$.

13. $\boldsymbol{y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right), -1 < x < 1.}$

Sol. Given: $y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right)$

To simplify the given Inverse T-function **put** $\boldsymbol{x = \tan\theta}$**.**

$\therefore \quad y = \cos^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\sin 2\theta)$

$= \cos^{-1}\cos\left(\dfrac{\pi}{2} - 2\theta\right) = \dfrac{\pi}{2} - 2\theta$

$\Rightarrow \quad y = \dfrac{\pi}{2} - 2\tan^{-1}x \quad (\because x = \tan\theta \Rightarrow \theta = \tan^{-1}x)$

$\therefore \quad \dfrac{dy}{dx} = 0 - 2 \cdot \dfrac{1}{1+x^2} = \dfrac{-2}{1+x^2}$.

14. $\boldsymbol{y = \sin^{-1}(2x\sqrt{1-x^2}), -\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}.}$

Sol. Given: $y = \sin^{-1}(2x\sqrt{1-x^2})$

Put $\boldsymbol{x = \sin\theta}$

To simplify the given Inverse T-function,

put $\boldsymbol{x = \sin\theta}$ (For $\sqrt{a^2 - x^2}$, put $x = a\sin\theta$)

$\therefore \quad y = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\sqrt{\cos^2\theta}) = \sin^{-1}(2\sin\theta\cos\theta)$

$y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$

$[\because x = \sin\theta \Rightarrow \theta = \sin^{-1}x]$

$\therefore \quad \dfrac{dy}{dx} = 2 \cdot \dfrac{1}{\sqrt{1-x^2}}$.

15. $\boldsymbol{y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right) \quad 0 < x < \dfrac{1}{\sqrt{2}}.}$

Sol. Given: $y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$

To simplify the given inverse T-function, **put** $\boldsymbol{x = \cos\theta}$**.**

$\therefore \quad y = \sec^{-1}\left(\dfrac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\dfrac{1}{\cos 2\theta}\right)$

$= \sec^{-1}(\sec 2\theta) = 2\theta = 2\cos^{-1}x \quad (\because x = \cos\theta \Rightarrow \theta = \cos^{-1}x)$

$$\therefore \qquad \frac{dy}{dx} = 2\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-2}{\sqrt{1-x^2}}.$$