NCERT Class 12 Maths

Solutions

Chapter - 5

Exercise 5.2

Differentiate the functions w.r.t. x in Exercises 1 to 8.

1. $\sin(x^2 + 5)$.

Sol. Let $y = \sin(x^2 + 5)$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\sin(x^2+5) = \cos(x^2+5)\,\frac{d}{dx}(x^2+5)$$

$$\left[\because \frac{d}{dx}\sin f(x) = \cos f(x)\,\frac{d}{dx}f(x)\right]$$

$$= \cos(x^2+5)\,.\,(2x+0)$$

$$\because \left[\frac{d}{dx}x^n = n\,x^{n-1} \text{ and } \frac{d}{dx}(c) = 0\right]$$

$$= 2x\cos(x^2+5).$$

Caution. $\sin(x^2 + 5)$ is not the product of two functions. It is composite function: sine of $(x^2 + 5)$.

2. $\cos(\sin x)$.

Sol. Let $y = \cos(\sin x)$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\cos(\sin x) = -\sin(\sin x)\,\frac{d}{dx}\sin x$$

$$\left[\because \frac{d}{dx}\cos f(x) = -\sin f(x)\,\frac{d}{dx}f(x)\right]$$

$$= -\sin(\sin x)\,.\,\cos x = -\cos x\sin(\sin x).$$

3. sin (*ax* + *b*).

Sol. Let $y = \sin(ax + b)$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx} \sin(ax + b) = \cos(ax + b) \frac{d}{dx}(ax + b)$$

$$= \cos(ax + b) \left[a \frac{d}{dx}(x) + \frac{d}{dx}(b)\right]$$

$$= \cos(ax + b)\,[a(1) + 0]$$

$$= a \cos(ax + b).$$

Note. It may be noted that letters *a* to *q* of English Alphabet are treated as constants (similar to 3, 5 etc.) as per convention.

4. sec (tan $\sqrt{x}$).

Sol. Let $y = \sec(\tan \sqrt{x})$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx} \sec(\tan \sqrt{x})$$

$$= \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \frac{d}{dx}(\tan \sqrt{x})$$

$$\left[\because \frac{d}{dx} \sec f(x) = \sec f(x) \tan f(x) \frac{d}{dx} f(x)\right]$$

$$= \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \sec^2(\sqrt{x}) \frac{d}{dx} \sqrt{x}$$

$$\left[\because \frac{d}{dx} f(x) = \sec^2 f(x) \frac{d}{dx} f(x)\right]$$

$$= \sec(\tan \sqrt{x}) \tan(\tan \sqrt{x}) \sec^2 \sqrt{x} \; \frac{1}{2\sqrt{x}}$$

$$\left[\because \frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}\right]$$

5. $\dfrac{\sin(ax + b)}{\cos(cx + d)}$.

Sol. Let $y = \dfrac{\sin(ax + b)}{\cos(cx + d)}$

$$\therefore \quad \frac{dy}{dx} = \frac{\cos(cx + d) \frac{d}{dx} \sin(ax + b) - \sin(ax + b) \frac{d}{dx} \cos(cx + d)}{\cos^2(cx + d)}$$

$$\left[\because \text{By Quotient Rule } \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{(\text{DEN.}) \frac{d}{dx}(\text{NUM}) - \text{NUM} \frac{d}{dx}(\text{DEN})}{(\text{DEN})^2}\right]$$

$$= \frac{\cos(cx + d) \cos(ax + b) \frac{d}{dx}(ax + b) - \sin(ax + b)(-\sin(cx + d)) \frac{d}{dx}(cx + d)}{\cos^2(cx + d)}$$

$$= \frac{a \cos(cx+d)\cos(ax+b) + c\sin(ax+b)\sin(cx+d)}{\cos^2(cx+d)}$$

$$\left[\because \frac{d}{dx}(ax+b) = \frac{d}{dx}(ax) + \frac{d}{dx}(b) = a\frac{d}{dx}(x) + 0 = a \cdot 1 = a \right.$$

$$\left. \text{Similarly } \frac{d}{dx}(cx+d) = c \right]$$

6. $\cos x^3 \sin^2 (x^5)$.

Sol. Let $y = \cos x^3 \sin^2 (x^5) = \cos x^3 (\sin x^5)^2$

$$\therefore \quad \frac{dy}{dx} = \cos x^3 \frac{d}{dx} (\sin x^5)^2 + (\sin x^5)^2 \frac{d}{dx} \cos x^3$$

$$\left[\because \text{By Product Rule } \frac{d}{dx}(uv) = \text{I} \frac{d}{dx}(\text{II}) + \text{II} \frac{d}{dx}(\text{I})\right]$$

$$= \cos x^3 \cdot 2 (\sin x^5) \frac{d}{dx} \sin x^5 + (\sin x^5)^2 (-\sin x^3) \frac{d}{dx} x^3$$

$$= \cos x^3 \cdot 2 (\sin x^5) \cos x^5 (5x^4) + \sin^2 x^5 (-\sin x^3)\, 3x^2$$

$$\left[\because \frac{d}{dx}\sin x^5 = \cos x^5 \frac{d}{dx} x^5 = \cos x^5 (5x^4)\right]$$

$$= 10x^4 \cos x^3 \sin x^5 \cos x^5 - 3x^2 \sin^2 x^5 \sin x^3$$

$$= x^2 \sin x^5 [10x^2 \cos x^3 \cos x^5 - 3 \sin x^5 \sin x^3].$$

7. $2\sqrt{\cot (x^2)}$.

Sol. Let $y = 2\sqrt{\cot (x^2)} = 2 (\cot (x^2))^{1/2}$

$$\therefore \quad \frac{dy}{dx} = 2 \cdot \frac{1}{2} (\cot x^2)^{1/2 - 1} \frac{d}{dx} (\cot (x^2))$$

$$\left| \because \frac{d}{dx}(f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x) \right.$$

$$= (\cot x^2)^{-1/2} \left(-\operatorname{cosec}^2 (x^2) \frac{d}{dx} x^2\right)$$

$$\left| \because \frac{d}{dx} \cot f(x) = -\operatorname{cosec}^2 (f(x)) \frac{d}{dx} f(x) \right.$$

$$= \frac{-\operatorname{cosec}^2 (x^2)}{\sqrt{\cot x^2}} (2x) = \frac{-2x \operatorname{cosec}^2 (x^2)}{\sqrt{\cot (x^2)}}.$$

8. $\cos (\sqrt{x})$.

Sol. Let $y = \cos (\sqrt{x})$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx} \cos (\sqrt{x}) = -\sin \sqrt{x} \frac{d}{dx} \sqrt{x}$$

$$\left[\because \frac{d}{dx} \cos f(x) = -\sin f(x) \frac{d}{dx} f(x)\right]$$

$$= -\sin \sqrt{x} \frac{1}{2\sqrt{x}} \left[\because \frac{d}{dx}\sqrt{x} = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{1/2 - 1} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}\right]$$

9. Prove that the function f given by $f(x) = |x - 1|$, $x \in \mathrm{R}$ is not differentiable at $x = 1$.

Sol. Definition. A function $f(x)$ is said to be differentiable at a point $x = c$ if $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists

(and then this limit is called $f'(c)$ *i.e.,* value of $f'(x)$ or $\frac{dy}{dx}$ at $x = c$)

Here $f(x) = |x - 1|, x \in \mathrm{R}$ ...(*i*)

To prove: $f(x)$ is not differentiable at $x = 1$.

Putting $x = 1$ on (*i*), $f(1) = |1 - 1| = |0| = 0$

Left Hand Derivative $= \mathrm{L}f'(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$

$$= \lim_{x \to 1^-} \frac{|x - 1| - 0}{x - 1} = \lim_{x \to 1^-} \frac{-(x - 1)}{x - 1}$$

$[\because x \to 1^- \Rightarrow x < 1 \Rightarrow x - 1 < 0 \Rightarrow |x - 1| = -(x - 1)]$

$$= \lim_{x \to 1^-} (-1) = -1 \qquad ...(ii)$$

Right Hand derivative $= \mathrm{R}f'(1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$

$$= \lim_{x \to 1^+} \frac{|x - 1| - 0}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)}{x - 1}$$

$(\because x \to 1^+ \Rightarrow x > 1 \Rightarrow x - 1 > 0 \Rightarrow |x - 1| = x - 1)$

$$= \lim_{x \to 1^+} 1 = 1 \qquad ...(iii)$$

From (*ii*) and (*iii*), $\mathrm{L}f'(1) \neq \mathrm{R}f'(1)$

$\therefore$ $f(x)$ is not differentiable at $x = 1$.

Note. In problems on limits of Modulus function, and bracket function (*i.e.,* greatest Integer Function), we have to find both left hand limit and right hand limit (we have used this concept quite few times in Exercise 5.1).

10. Prove that the greatest integer function defined by

$$f(x) = [x],\ 0 < x < 3$$

is not differentiable at $x = 1$ and $x = 2$.

Sol. Given: $f(x) = [x], 0 < x < 3$ ...(*i*)

Differentiability at $x = 1$

Putting $x = 1$ in (*i*), $f(1) = [1] = 1$

Left Hand derivative $= \mathrm{L}f'(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{[x] - 1}{x - 1}$

Put $x = 1 - h$, $h \to 0^+$

$$= \lim_{h \to 0^+} \frac{[1-h]-1}{1-h-1} \qquad = \lim_{h \to 0^+} \frac{0-1}{-h} = \lim_{h \to 0^+} \frac{1}{h}$$

[We know that as $h \to 0^+$, $[c - h] = c - 1$ if c is an integer. Therefore $[1 - h] = 1 - 1 = 0$]

Put $h = 0$, $= \frac{1}{0} = \infty$ does not exist.

$\therefore$ $f(x)$ is not differentiable at $x = 1$.

(We need not find R$f'(1)$ as L$f'(1)$ does not exist).

Differentiability at x = 2

Putting $x = 2$ in (i), $f(2) = [2] = 2$

Left Hand derivative = L$f'(2)$ = $\lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2} = \lim_{x \to 2^-} \frac{[x]-2}{x-2}$

Put $x = 2 - h$ as $h \to 0^+$

$$= \lim_{h \to 0^+} \frac{[2-h]-2}{2-h-2} = \lim_{h \to 0^+} \frac{1-2}{-h} = \lim_{h \to 0^+} \frac{-1}{-h}$$

(For $h \to 0^+$, $[2 - h] = 2 - 1 = 1$)

$$= \lim_{h \to 0^+} \frac{1}{h} = \frac{1}{0} = \infty$$ does not exist.

$\therefore$ $f(x)$ is not differentiable at $x = 2$.

Note. For $h \to 0^+$, $[c + h] = c$ if c is an integer.