NCERT Class 12 Maths Solutions

Chapter - 4

Exercise 4.6

Examine the consistency of the system of equations in Exercises 1 to 3.

1. $x + 2y = 2$
$2x + 3y = 3.$

Sol. Given linear equations are

$$x + 2y = 2$$
$$2x + 3y = 3$$

Their matrix form is $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $(\Rightarrow \quad AX = B)$

Comparing $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = 3 - 4 = -1 \neq 0$$

$\therefore$ (Unique) solution and hence equations are consistent.

2. $\mathbf{2x - y = 5}$

$\mathbf{x + y = 4.}$

Sol. Given linear equations are

$$2x - y = 5$$
$$x + y = 4$$

Their matrix form is $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$ $(\Rightarrow \quad AX = B)$

Comparing $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = 2 - (-1) = 3 \neq 0$$

$\therefore$ (Unique) solution and hence equations are consistent.

3. $\mathbf{x + 3y = 5}$

$\mathbf{2x + 6y = 8.}$

Sol. Given linear equations are

$$x + 3y = 5$$
$$2x + 6y = 8$$

Their matrix form is $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ $(\Rightarrow \quad AX = B)$

Comparing $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ $|A| = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} = 6 - 6 = 0$

So we are to find (adj. A) B

$$\text{adj. } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \quad \left(\because \text{ adj.} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \right)$$

$$\therefore \text{ (adj. A) } B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq O$$

$$\left(\because \text{ The matrix } \begin{bmatrix} 6 \\ -2 \end{bmatrix} \text{ has non-zero entries} \right)$$

$\therefore$ Given Equations are Inconsistent *i.e.,* have no common solution.

Examine the consistency of the system of equations in Exercises 4 to 6.

4. $\mathbf{x + y + z = 1}$

$\mathbf{2x + 3y + 2z = 2}$

$\mathbf{ax + ay + 2az = 4}$

Sol. The given equations are

$$x + y + z = 1 \quad \text{...(i)}$$
$$2x + 3y + 2z = 2 \quad \text{...(ii)}$$
$$ax + ay + 2az = 4 \quad \text{...(iii)}$$

Their matrix form is $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ $(\Rightarrow AX = B)$

$\therefore$ Matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}$ $\therefore$ $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix}$

Expanding along first row,

$$|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)$$
$$= 4a - 2a - a = a$$

Case I. $a \neq 0$ $\therefore$ $|A| = a \neq 0$

$\therefore$ (Unique) solution and hence equations are consistent.

Case II. $a = 0$ $\therefore$ $|A| = a = 0$.

Putting $a = 0$ in given equation (iii), we have $0 = 4$ which is impossible.

$\therefore$ Given equations are inconsistent if $a = 0$.

5. $\mathbf{3x - y - 2z = 2}$
$\mathbf{2y - z = -1}$
$\mathbf{3x - 5y = 3}$

Sol. The given equations are

$$3x - y - 2z = 2$$
$$2y - z = -1 \quad i.e., \quad 0x + 2y - z = -1$$

and $3x - 5y = 3$ *i.e.,* $3x - 5y + 0z = 3$

Their matrix form is $\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ $(\Rightarrow AX = B)$

Comparing $A = \begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$\therefore$ $|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding along first row,

$$|A| = 3(0 - 5) - (-1)(0 + 3) + (-2)(0 - 6)$$
$$= 3(-5) + 3 + 12 = -15 + 15 = 0$$

So now we are to find (adj. A) B

To find adj. A for $|A| = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

$$A_{11} = + \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} = (0 - 5) = -5,$$

$$A_{12} = - \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} = -(0 + 3) = -3,$$

$$A_{13} = + \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = (0 - 6) = -6,$$

$$A_{21} = - \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} = -(0 - 10) = 10,$$

$$A_{22} = + \begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} = (0 + 6) = 6,$$

$$A_{23} = - \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = -(-15 + 3) = 12,$$

$$A_{31} = + \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} = (1 + 4) = 5,$$

$$A_{32} = - \begin{vmatrix} 3 & -2 \\ 0 & -1 \end{vmatrix} = -(-3 - 0) = 3,$$

$$A_{33} = + \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = +(6 - 0) = 6.$$

$$\therefore \quad \text{adj. } A = \begin{bmatrix} -5 & -3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}' = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$\therefore \quad (\text{adj. } A)\, B = \begin{bmatrix} -5 & 10 & 5 \\ -3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ -6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \neq O \qquad \left[\because \text{ The matrix } \begin{bmatrix} -5 \\ -3 \\ -6 \end{bmatrix} \text{ has non-zero entries} \right]$$

$\therefore$ Given equations are inconsistent.

6. $\mathbf{5x - y + 4z = 5}$
$\mathbf{2x + 3y + 5z = 2}$
$\mathbf{5x - 2y + 6z = -1}$

Sol. The given equations are

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

Their matrix form is $\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ $(\Rightarrow AX = B)$

$$\therefore A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} \qquad |A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

Expanding along first row

$$= 5(18 + 10) - (-1)(12 - 25) + 4(-4 - 15)$$
$$= 5(28) + (-13) + 4(-19)$$
$$= 140 - 13 - 76 = 140 - 89 = 51 \neq 0$$

$\therefore$ Given system of equations has a (unique) solution and hence equations are consistent.

Solve the system of linear equations, using matrix method, in Exercises 7 to 10.

7. $\mathbf{5x + 2y = 4}$

$\mathbf{7x + 3y = 5.}$

Sol. The given equations are

$5x + 2y = 4$

$7x + 3y = 5$

Their matrix form is $\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ $(\Rightarrow \quad AX = B)$

Comparing $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 15 - 14 = 1 \neq 0$$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B}$

$$\Rightarrow \quad X = \frac{1}{|A|} (\text{adj. } A) \,.\, B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix} \left(\because \text{ adj.} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \right)$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Equating corresponding entries, we have $x = 2$ and $y = -3$.

8. $\mathbf{2x - y = -2}$

$\mathbf{3x + 4y = 3.}$

Sol. The given equations are

$2x - y = -2$

$3x + 4y = 3$

Their matrix form is $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ $(\Rightarrow \quad AX = B)$

Comparing $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix} = 8 - (-3) = 8 + 3 = 11 \neq 0$$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B}$

$$\Rightarrow X = \frac{1}{|A|} (\text{adj. } A) \,.\, B \quad \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad = \frac{1}{11} \begin{bmatrix} -8+3 \\ 6+6 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$$

Equating corresponding entries, we have $x = -\frac{5}{11}$ and $y = \frac{12}{11}$.

9. $\mathbf{4x - 3y = 3}$
$\mathbf{3x - 5y = 7.}$

Sol. The given equations are
$4x - 3y = 3$
$3x - 5y = 7$

Their matrix form is $\begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$ ($\Rightarrow$ AX = B)

Comparing $A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = -20 - (-9) = -20 + 9 = -11 \neq 0$$

∴ Solution is unique and $\mathbf{X = A^{-1}B}$

$$\Rightarrow \quad X = \frac{1}{|A|} (\text{adj. } A)\, B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11} \begin{bmatrix} 6 \\ 19 \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix}$$

Equating corresponding entries, we have $x = -\frac{6}{11}$ and $y = -\frac{19}{11}$.

10. $\mathbf{5x + 2y = 3}$
$\mathbf{3x + 2y = 5.}$

Sol. The given equations are
$5x + 2y = 3$
$3x + 2y = 5$

Their matrix form is $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ ($\Rightarrow$ AX = B)

Comparing $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$$| A | = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 10 - 6 = 4 \neq 0$$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B}$

$$\Rightarrow \quad X = \frac{1}{|A|} (\text{adj. } A) B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \left(\because \text{adj.} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \right)$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Equating corresponding entries, we have $x = -1$ and $y = 4$.

Solve the system of linear equations, using matrix method, in Exercises 11 to 14.

11. $\mathbf{2x + y + z = 1}$

$$\mathbf{x - 2y - z = \frac{3}{2}}$$

$$\mathbf{3y - 5z = 9.}$$

Sol. The given equations are

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$ or $0.x + 3y - 5z = 9$

Their matrix form is $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

$(\Rightarrow AX = B)$

Comparing $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

$$| A | = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

Expanding along first row, $= 2(10 + 3) - 1(-5 - 0) + 1(3 - 0)$

or $| A | = 2(13) + 5 + 3 = 26 + 5 + 3 = 34 \neq 0$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B} = \frac{1}{|A|}$ (adj. A) B ...(*i*)

Let us find adj. A

$$A_{11} = + \begin{vmatrix} -2 & -1 \\ 3 & -5 \end{vmatrix} = 10 + 3 = 13,$$

$$A_{12} = -\begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix} = -(-5-0) = 5,$$

$$A_{13} = +\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix} = (3-0) = 3,$$

$$A_{21} = -\begin{vmatrix} 1 & 1 \\ 3 & -5 \end{vmatrix} = -(-5-3) = 8,$$

$$A_{22} = +\begin{vmatrix} 2 & 1 \\ 0 & -5 \end{vmatrix} = (-10-0) = -10,$$

$$A_{23} = -\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = -(6-0) = -6,$$

$$A_{31} = +\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} = (-1+2) = 1,$$

$$A_{32} = -\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -(-2-1) = 3,$$

$$A_{33} = +\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -4-1 = -5.$$

$$\therefore \quad \text{Adj. A} = \begin{bmatrix} 13 & 5 & 3 \\ 8 & -10 & -6 \\ 1 & 3 & -5 \end{bmatrix}' = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

Putting values in eqn. (*i*), $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \dfrac{3}{2} \\ 9 \end{bmatrix}$

$$= \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{1}{2} \\ \dfrac{-3}{2} \end{bmatrix}$$

Equating corresponding entries, we have $x = 1$,

$$y = \frac{1}{2},\ z = -\frac{3}{2}.$$

12. $\boldsymbol{x - y + z = 4}$
$\boldsymbol{2x + y - 3z = 0}$
$\boldsymbol{x + y + z = 2.}$

Sol. The given equations are

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

Their matrix form is $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ $(\Rightarrow AX = B)$

Comparing $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding along first row,

$$= 1(1 + 3) - (-1)(2 + 3) + 1(2 - 1)$$

or $\quad |A| = 4 + 5 + 1 = 10 \neq 0$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B} = \dfrac{1}{|A|}$ (adj. A) B ...(*i*)

To find adj. A

$$A_{11} = + \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = (1 + 3) = 4,$$

$$A_{12} = - \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2 + 3) = -5,$$

$$A_{13} = + \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = (2 - 1) = 1,$$

$$A_{21} = - \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1 - 1) = 2,$$

$$A_{22} = + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = (1 - 1) = 0,$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -(1 + 1) = -2,$$

$$A_{31} = + \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = (3 - 1) = 2,$$

$$A_{32} = - \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -(-3 - 2) = 5,$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3.$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Putting these values in eqn. (*i*), we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4-0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Equating corresponding entries, we have

$x = 2, \; y = -1, z = 1.$

13. $\mathbf{2x + 3y + 3z = 5}$

$\mathbf{x - 2y + z = -4}$

$\mathbf{3x - y - 2z = 3.}$

Sol. The given equations are

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3y - y - 2z = 3$

Their matrix form is $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$ ($\Rightarrow$ AX = B)

Comparing $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

Expanding along first row, $|A| = 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6)$

$= 2(5) - 3(-5) + 3(5) = 10 + 15 + 15 = 40 \neq 0$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B} = \frac{1}{|A|}$ (adj. A) B ...(*i*)

Let us find adj. A

$A_{11} = + \begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} = 4 + 1 = 5,$

$A_{12} = - \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2 - 3) = 5,$

$A_{13} = + \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = -1 + 6 = 5,$

$A_{21} = - \begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} = -(-6 + 3) = 3,$

$$A_{22} = + \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -4 - 9 = -13,$$

$$A_{23} = - \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = -(-2 - 9) = 11,$$

$$A_{31} = + \begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 3 + 6 = 9,$$

$$A_{32} = - \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2 - 3) = 1,$$

$$A_{33} = + \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -4 - 3 = -7.$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}' = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Putting these values in eqn. (*i*), $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$= \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Equating corresponding entries, we have $x = 1$, $y = 2$, $z = -1$.

14. $\boldsymbol{x - y + 2z = 7}$

$\boldsymbol{3x + 4y - 5z = -5}$

$\boldsymbol{2x - y + 3z = 12.}$

Sol. The given equations are

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

Their matrix form is $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$ ($\Rightarrow$ AX = B)

Comparing, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$$| A | = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

Expanding along first row,

$$| A | = 1(12 - 5) - (-1)(9 + 10) + 2(-3 - 8)$$
$$= 7 + 19 - 22 = 4 \neq 0$$

$\therefore$ Solution is unique and $\mathbf{X = A^{-1}B} = \dfrac{1}{|A|}$ (adj. A) B ...(*i*)

Let us find adj. A

$$A_{11} = + \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 12 - 5 = 7,$$

$$A_{12} = - \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -(9 + 10) = -19,$$

$$A_{13} = + \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3 - 8 = -11,$$

$$A_{21} = - \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -(-3 + 2) = 1,$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1,$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -(-1 + 2) = -1,$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 5 - 8 = -3,$$

$$A_{32} = - \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -(-5 - 6) = 11,$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 4 + 3 = 7.$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}' = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Putting values in eqn. (*i*),

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Equating corresponding entries, we have $x = 2$, $y = 1$, $z = 3$.

15. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3.$$

Sol. Given: Matrix A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

To find A^{-1} $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

Expanding along first row,

$$|A| = 2(-4 + 4) - (-3)(-6 + 4) + 5(3 - 2)$$
$$= 0 + 3(-2) + 5 = -6 + 5 = -1 \neq 0$$

$\therefore$ A^{-1} exists and $A^{-1} = \frac{1}{|A|}$ (adj. A) ...(*i*)

To find adj. A from $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$$A_{11} = + \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} = (-4 + 4) = 0,$$

$$A_{12} = - \begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} = -(-6 + 4) = 2,$$

$$A_{13} = + \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1,$$

$$A_{21} = - \begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} = -(6 - 5) = -1,$$

$$A_{22} = + \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} = -4 - 5 = -9,$$

$$A_{23} = - \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2 + 3) = -5,$$

$$A_{31} = + \begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} = (12 - 10) = 2,$$

$$A_{32} = - \begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} = -(-8 - 15) = 23,$$

$$A_{33} = + \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = (4 + 9) = 13.$$

$$\therefore \quad \text{adj. } A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}' = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Putting this value of adj. A in (*i*),

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \ldots(ii) \left(\because \frac{1}{-1} = -1 \right)$$

Now using (this) A^{-1}, we are to solve the equations

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Their matrix form is $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ $(\Rightarrow AX = B)$

Comparing $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

Solution is unique and $X = A^{-1}B$ $\quad (\because \; A^{-1}$ exists by (*ii*))

Putting values, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Equating corresponding entries, we have $x = 1, y = 2, z = 3$.

16. The cost of 4 kg onion, 3 kg wheat and 2 kg rice is ₹ 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is ₹ 90. The cost of 6 kg onion, 2 kg wheat and 3 kg rice is ₹ 70. Find cost of each item per kg by matrix method.

Sol. Let ₹ x, ₹ y, ₹ z per kg be the prices of onion, wheat and rice respectively.

$\therefore$ According to the given data, we have the following three equations

$$4x + 3y + 2z = 60,$$
$$2x + 4y + 6z = 90,$$
and
$$6x + 2y + 3z = 70.$$

We know that these equations can be expressed in the matrix form as

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

or $\qquad AX = B,$

where $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$ $|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix}$

Expanding along first row,

$$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24)$$
$$= 0 - 3(-30) + 2(-20) = 90 - 40 = 50 \neq 0$$

Hence A is non-singular

$\therefore$ A^{-1} exists.

$\therefore$ Unique solution is $X = A^{-1} B$ ...(*i*)

$A_{11} = +(12 - 12) = 0,$ $\quad A_{12} = -(6 - 36) = 30,$

$A_{13} = +(4 - 24) = -20$

$A_{21} = -(9 - 4) = -5,$ $\quad A_{22} = +(12 - 12) = 0,$

$A_{23} = -(8 - 18) = 10$ $\quad A_{31} = +(18 - 8) = 10,$

$A_{32} = -(24 - 4) = -20,$ $\quad A_{33} = +(16 - 6) = 10$

$$\therefore \text{ adj. } A = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}' = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Putting values of X, A^{-1} and B in (*i*), we have

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$= \frac{1}{50} \begin{bmatrix} -450 + 700 \\ 1800 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} = \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

or $\qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$

$\Rightarrow \quad x = 5, y = 8, z = 8.$

$\therefore$ The cost of onion, wheat and rice are respectively ₹ 5, ₹ 8 and ₹ 8 per kg.