Exercise 3.1

1. In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$, **write**

(*i*) The order of the matrix (*ii*) The number of elements

(*iii*) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

Sol. (*i*) There are 3 horizontal lines (rows) and 4 vertical lines (columns) in the given matrix A.

$\therefore$ Order of the matrix A is **3 × 4**.

(*ii*) The number of elements in this matrix A is 3 × 4 = 12.

($\because$ The number of elements in a $m \times n$ matrix is $m \cdot n$)

(*iii*) $a_{13} \Rightarrow$ Element in first row and third column = 19

$a_{21} \Rightarrow$ Element in second row and first column = 35

$a_{33} \Rightarrow$ Element in third row and third column = – 5

$a_{24} \Rightarrow$ Element in second row and fourth column = 12

$a_{23} \Rightarrow$ Element in second row and third column $= \frac{5}{2}$.

2. If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Sol. We know that a matrix having mn elements is of order $m \times n$.

(*i*) Now 24 = 1 × 24, 2 × 12, 3 × 8, 4 × 6 and hence

= 24 × 1, 12 × 2, 8 × 3, 6 × 4 also.

$\therefore$ There are 8 possible matrices having 24 elements of orders 1 × 24, 2 × 12, 3 × 8, 4 × 6, 24 × 1, 12 × 2, 8 × 3, 6 × 4.

(*ii*) Again (prime number) 13 = 1 × 13 and 13 × 1 only.

$\therefore$ There are 2 possible matrices of order 1 × 13 (Row matrix) and 13 × 1 (Column matrix)

3. If a matrix has 18 elements, what are the possible orders it can have? What if has 5 elements?

Sol. We know that a matrix having mn elements is of order $m \times n$.

(*i*) Now 18 = 1 × 18, 2 × 9, 3 × 6 and hence 18 × 1, 9 × 2, 6 × 3 also.

$\therefore$ There are 6 possible matrices having 18 elements of orders 1×18, 2×9, 3×6, 18×1, 9×2 and 6×3.

(*ii*) Again (Prime number) $5 = 1 \times 5$ and 5×1 only.

$\therefore$ There are 2 possible matrices of order 1×5 and 5×1.

4. Construct a 2×2 matrix A = $[a_{ij}]$ whose elements are given by:

(*i*) $a_{ij} = \dfrac{(i+j)^2}{2}$ **(*ii*)** $a_{ij} = \dfrac{i}{j}$ **(*iii*)** $a_{ij} = \dfrac{(i+2j)^2}{2}$

Sol. To construct a 2×2 matrix $A = [a_{ij}]$

(*i*) **Given:** $a_{ij} = \dfrac{(i+j)^2}{2}$ ...(*i*)

In (*i*),

Put $i = 1, j = 1$, $\therefore$ $a_{11} = \dfrac{(1+1)^2}{2} = \dfrac{2^2}{2} = \dfrac{4}{2} = 2$

Put $i = 1, j = 2$, $\therefore$ $a_{12} = \dfrac{(1+2)^2}{2} = \dfrac{3^2}{2} = \dfrac{9}{2}$

Put $i = 2, j = 1$; $\therefore$ $a_{21} = \dfrac{(2+1)^2}{2} = \dfrac{9}{2}$

Put $i = 2, j = 2$; $\therefore$ $a_{22} = \dfrac{(2+2)^2}{2} = \dfrac{4^2}{2} = \dfrac{16}{2} = 8$

$$\therefore \quad A_{2 \times 2} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}.$$

(*ii*) **Given:** $a_{ij} = \dfrac{i}{j}$ ...(*i*)

In (*i*),

Put $i = 1, j = 1$, $\therefore$ $a_{11} = \dfrac{1}{1} = 1$

Put $i = 1, j = 2$, $\therefore$ $a_{12} = \dfrac{1}{2}$

Put $i = 2, j = 1$; $\therefore$ $a_{21} = \dfrac{2}{1} = 2$

Put $i = 2, j = 2$; $\therefore$ $a_{22} = \dfrac{2}{2} = 1$

$$\therefore \quad A_{2 \times 2} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}.$$

(*iii*) **Given:** $a_{ij} = \dfrac{(i+2j)^2}{2}$ ...(*i*)

In (*i*),

Put $i = 1, j = 1$; $\quad \therefore \quad a_{11} = \dfrac{(1+2)^2}{2} = \dfrac{3^2}{2} = \dfrac{9}{2}$

Put $i = 1, j = 2$; $\quad \therefore \quad a_{12} = \dfrac{(1+4)^2}{2} = \dfrac{5^2}{2} = \dfrac{25}{2}$

Put $i = 2, j = 1$; $\quad \therefore \quad a_{21} = \dfrac{(2+2)^2}{2} = \dfrac{16}{2} = 8$

Put $i = 2, j = 2$; $\quad \therefore \quad a_{22} = \dfrac{(2+4)^2}{2} = \dfrac{6^2}{2} = \dfrac{36}{2} = 18$

$$\therefore \quad A_{2 \times 2} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{9}{2} & \dfrac{25}{2} \\ 8 & 18 \end{bmatrix}.$$

5. Construct a 3 × 4 matrix, whose elements are given by:

(*i*) $a_{ij} = \dfrac{1}{2} \mid -3i + j \mid$ (*ii*) $a_{ij} = 2i - j$.

Sol. (*i*) To construct a 3 × 4 matrix say A.

Given: $a_{ij} = \dfrac{1}{2} \mid -3i + j \mid$ ...(*i*)

In (*i*),

Put $i = 1, j = 1$,

$\therefore \quad a_{11} = \dfrac{1}{2} \mid -3+1 \mid = \dfrac{1}{2} \mid -2 \mid = \dfrac{1}{2}(2) = 1$

Put $i = 1, j = 2$,

$\therefore \quad a_{12} = \dfrac{1}{2} \mid -3+2 \mid = \dfrac{1}{2} \mid -1 \mid = \dfrac{1}{2}(1) = \dfrac{1}{2}$

$i = 1, j = 3$,

$\therefore \quad a_{13} = \dfrac{1}{2} \mid -3+3 \mid = \dfrac{1}{2} \mid 0 \mid = \dfrac{1}{2}(0) = 0$

$i = 1, j = 4$,

$\therefore \quad a_{14} = \dfrac{1}{2} \mid -3+4 \mid = \dfrac{1}{2} \mid 1 \mid = \dfrac{1}{2}(1) = \dfrac{1}{2}$

$i = 2, j = 1$,

$\therefore \quad a_{21} = \dfrac{1}{2} \mid -6+1 \mid = \dfrac{1}{2} \mid -5 \mid = \dfrac{5}{2}$

$i = 2, j = 2$,

$\therefore \quad a_{22} = \dfrac{1}{2} \mid -6+2 \mid = \dfrac{1}{2} \mid -4 \mid = \dfrac{4}{2} = 2$

$i = 2, j = 3$,

$\therefore \quad a_{23} = \dfrac{1}{2} \mid -6+3 \mid = \dfrac{1}{2} \mid -3 \mid = \dfrac{3}{2}$

$i = 2, j = 4$,

$\therefore \quad a_{24} = \dfrac{1}{2} \mid -6+4 \mid = \dfrac{1}{2} \mid -2 \mid = \dfrac{2}{2} = 1$

$i = 3, j = 1,$

$\therefore \quad a_{31} = \frac{1}{2} | -9 + 1 | = \frac{1}{2} | -8 | = \frac{8}{2} = 4$

$i = 3, j = 2,$

$\therefore \quad a_{32} = \frac{1}{2} | -9 + 2 | = \frac{1}{2} | -7 | = \frac{7}{2}$

$i = 3, j = 3,$

$\therefore \quad a_{33} = \frac{1}{2} | -9 + 3 | = \frac{1}{2} | -6 | = \frac{6}{2} = 3$

$i = 3, j = 4,$

$\therefore \quad a_{34} = \frac{1}{2} | -9 + 4 | = \frac{1}{2} | -5 | = \frac{5}{2}$

$$\therefore \quad A_{3 \times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}.$$

(*ii*) **Given:** $a_{ij} = 2i - j$

$\therefore \quad a_{11} = 2 - 1 = 1,$ $\quad a_{12} = 2 - 2 = 0$

$a_{13} = 2 - 3 = -1,$ $\quad a_{14} = 2 - 4 = -2$

$a_{21} = 4 - 1 = 3,$ $\quad a_{22} = 4 - 2 = 2$

$a_{23} = 4 - 3 = 1,$ $\quad a_{24} = 4 - 4 = 0$

$a_{31} = 6 - 1 = 5,$ $\quad a_{32} = 6 - 2 = 4$

$a_{33} = 6 - 3 = 3,$ $\quad a_{34} = 6 - 4 = 2$

$$\therefore \quad A_{3 \times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}.$$

6. Find the values of x, y and z from the following equations:

(*i*) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$ **(*ii*)** $\begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(*iii*) $\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Sol. (*i*) **Given:** $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

By definition of Equal matrices, equating corresponding entries, we have $4 = y$, $3 = z$, $x = 1$, $5 = 5$

$\therefore \quad x = 1, y = 4, z = 3.$

(*ii*) **Given:** $\begin{bmatrix} x + y & 2 \\ 5 + z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Equating corresponding entries, we have

$$x + y = 6 \quad ...(i)$$

$$5 + z = 5 \quad i.e., \quad z = 5 - 5 = 0$$

and $$xy = 8 \quad ...(ii)$$

Let us solve (i) and (ii) for x and y.

From (i), $y = 6 - x$

Putting this value of y in (ii), we have

$$x(6 - x) = 8 \quad \text{or} \quad 6x - x^2 = 8$$

or $\quad -x^2 + 6x - 8 = 0$ or $\quad x^2 - 6x + 8 = 0$

or $\quad x^2 - 4x - 2x + 8 = 0$ or $\quad x(x - 4) - 2(x - 4) = 0$

or $\quad (x - 4)(x - 2) = 0$

$\therefore$ Either $\quad x - 4 = 0$ or $x - 2 = 0$

i.e., $x = 4$ or $x = 2$.

When $x = 4$, then $\quad y = 6 - x = 6 - 4 = 2$

$\therefore \quad x = 4, \ y = 2, z = 0.$

When $x = 2$, then $\quad y = 6 - x = 6 - 2 = 4$

$\therefore \quad x = 2, \ y = 4, \quad z = 0.$

(iii) **Given:** $\begin{bmatrix} x + y + z \\ x + z \\ y + z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Equating corresponding entries, we have

$$x + y + z = 9 \quad ...(i)$$

$$x + z = 5 \quad ...(ii)$$

$$y + z = 7 \quad ...(iii)$$

Eqn. (i) – eqn. (ii) gives $y = 9 - 5 = 4$

Eqn. (i) – eqn. (iii) gives $x = 9 - 7 = 2$

Putting $x = 2$ and $y = 4$ in (i), $2 + 4 + z = 9$

or $\quad 6 + z = 9$

$\therefore \quad z = 3$

Hence $\quad x = 2, y = 4, z = 3.$

7. Find the values of *a*, *b*, *c* and *d* from the equation

$$\begin{bmatrix} a - b & 2a + c \\ 2a - b & 3c + d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}.$$

Sol. Equating corresponding entries of given equal matrices, we have

$$a - b = -1 \quad ...(i)$$

$$2a - b = 0 \quad ...(ii)$$

$$2a + c = 5 \quad ...(iii)$$

and $$3c + d = 13 \quad ...(iv)$$

Eqn. (i) – eqn. (ii) gives $-a = -1$ or $a = 1$

Putting $a = 1$ in (i), $\quad 1 - b = -1$ or $-b = -2$ or $b = 2$

Putting $a = 1$ in (iii), $2 + c = 5 \quad \Rightarrow \quad c = 5 - 2 = 3$

Putting $c = 3$ in (iv), $9 + d = 13$ or $\quad d = 13 - 9 = 4$

$\therefore \quad a = 1, b = 2, c = 3, d = 4.$

8. A = $[a_{ij}]_{m \times n}$ is a square matrix, if

(A) $m < n$ (B) $m > n$ (C) $m = n$ (D) None of these.

Sol. (C) is the correct option.

($\because$ By definition of square matrix $m = n$)

9. Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \frac{-1}{3}, y = 7$ (B) Not possible to find

(C) $y = 7, x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}, y = \frac{-2}{3}$.

Sol. According to given, matrix $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ = matrix $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating corresponding entries, we have

$3x + 7 = 0 \Rightarrow 3x = -7 \Rightarrow x = -\frac{7}{3}$ ...(i)

$5 = y - 2 \Rightarrow 5 + 2 = y \Rightarrow y = 7$

$y + 1 = 8 \Rightarrow y = 8 - 1 = 7$

and $2 - 3x = 4 \Rightarrow -3x = 2 \Rightarrow x = -\frac{2}{3}$ ...(ii)

The two values of $x = -\frac{7}{3}$ given by (i) and $x = -\frac{2}{3}$ given by (ii) are not equal.

$\therefore$ No values of x and y exist to make the two matrices equal.

$\therefore$ Option (B) is the correct answer.

10. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:

(A) 27 (B) 18 (C) 81 (D) 512.

Sol. We know that general matrix of order 3 × 3 is

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

This matrix has 3 × 3 = 9 elements.

The number of choices for a_{11} is 2 (as 0 or 1 can be used)

Similarly, the number of choices for each other element is 2.

Hence, total possible arrangements (matrices)

$$= \underbrace{2 \times 2 \times \ldots \times 2}_{9 \text{ times}}$$ (By fundamental principle of counting)

$= 2^9 = 512$

$\therefore$ Option (D) is the correct answer.