1. Determine whether or not each of the definition of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this.

(i) On Z^+, define $*$ by $a * b = a - b$

(ii) On Z^+, define $*$ by $a * b = ab$

(iii) On R, define $*$ by $a * b = ab^2$

(iv) On Z^+, define $*$ by $a * b = |a - b|$

(v) On Z^+, define $*$ by $a * b = a$

Solution:

(i) On Z^+, $*$ is defined by $a * b = a - b$.
Here, the image of $(1, 2)$ under $*$ is $1 * 2 = 1 - 2 = -1 \notin Z^+$.

Hence, the given definition of $*$ is not a binary operation.

(ii) On Z^+, $*$ is defined by $a * b = ab$.

It is clear that for each $a, b \in Z^+$, there is a unique element ab in Z^+.

This means that $*$ takes each pair (a, b) to a unique element $a * b = ab$ in Z^+.

Hence, $*$ is a binary operation.

(iii) On R, $*$ is defined by $a * b = ab^2$.

It is clear that for each $a, b \in R$, there is a unique element ab^2 in R.

This means that $*$ takes each pair (a, b) to a unique element $a * b = ab^2$ in R.

Hence, $*$ is a binary operation.

(iv) On Z^+, $*$ is defined by $a * b = |a - b|$.

Here, the image of $(1, 1)$ under $*$ is $1 * 1 = |1 - 1| = 0 \notin Z^+$

Hence, $*$ is not a binary operation.

(v) On Z^+, $*$ is defined by $a * b = a$.

It is clear that for each $a, b \in Z^+$, there is a unique element a in Z^+.

This means that $*$ takes each pair (a, b) to a unique element $a * b = a$ in Z^+.

Hence, $*$ is a binary operation.

2. For each operation $*$ defined below, determine whether $*$ is binary, commutative or associative.

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z^+, define $a * b = 2^{ab}$

(v) On Z^+, define $a * b = a^b$

(vi) On $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

Solution:

(i) On Z, $*$ is defined by $a * b = a - b$.

If $a, b \in Z$, then $a - b \in Z$.

Hence, the operation $*$ is a binary operation.

It is observed that $1 * 2 = 1 - 2 = -1$ and $2 * 1 = 2 - 1 = 1$.

$\therefore$ $1 * 2 \neq 2 * 1$, where $1, 2 \in Z$

Hence, the operation $*$ is not commutative.

Also, we have

$(1 * 2) * 3 = (1 - 2) * 3 = -1 * 3 = -1 - 3 = -4$

$1 * (2 * 3) = 1 * (2 - 3) = 1 * -1 = 1 - (-1) = 2$

$\therefore$ $(1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in Z$

Hence, the operation $*$ is not associative.

(ii) On Q, $*$ is defined by $a * b = ab + 1$.

If $a, b \in Q$, then $ab + 1 \in Q$.

Hence, the operation $*$ is a binary operation.

We know that: $ab = ba$ for all $a, b \in Q$

$\Rightarrow ab + 1 = ba + 1$ for all $a, b \in Q$

$\Rightarrow a * b = b * a$ for all $a, b \in Q$

Hence, the operation $*$ is commutative.

It can be observed that

$(1 * 2) * 3 = (1 \times 2 + 1) * 3 = 3 * 3 = 3 \times 3 + 1 = 10$

$1 * (2 * 3) = 1 * (2 \times 3 + 1) = 1 * 7 = 1 \times 7 + 1 = 8$

$(1 * 2) * 3 \neq 1 * (2 * 3)$, where 1, 2, 3 $\in Q$

Hence, the operation $*$ is not associative.

Hence, the operation $*$ is commutative and not associative.

(iii) On Q, $*$ is defined by $a * b = \frac{ab}{2}$

If $a, b \in Q$, then $\frac{ab}{2} \in Q$.

Hence, the operation $*$ is a binary operation.

We know that: $ab = ba$ for all $a, b \in Q$

$\Rightarrow \frac{ab}{2} = \frac{ba}{2}$ for all $a, b \in Q$

$\Rightarrow a * b = b * a$ for all $a, b \in Q$

Hence, the operation $*$ is commutative.

Again, for all $a, b, c \in Q$, we have

$(a * b) * c = \left(\frac{ab}{2}\right) * c = \frac{\left(\frac{ab}{2}\right)c}{2} = \frac{abc}{4}$

and

$a * (b * c) = a * \left(\frac{bc}{2}\right) = \frac{a\left(\frac{bc}{2}\right)}{2} = \frac{abc}{4}$

$\therefore (a * b) * c = a * (b * c)$, where $a, b, c \in Q$

Hence, the operation $*$ is associative.

Hence, the operation $*$ is associative and commutative.

(iv) On Z^+, $*$ is defined by $a * b = 2^{ab}$.

If $a, b \in Z^+$, then $2^{ab} \in Z^+$.

Hence, the operation $*$ is a binary operation.

we know that: $ab = ba$ for all $a, b \in Z^+$

$\Rightarrow 2^{ab} = 2^{ba}$ for all $a, b \in Z^+$

$\Rightarrow$ a $* b =$ b $* a$ for all $a, b \in Z^+$

Hence, the operation $*$ is commutative.

It is observed that

$(1 * 2) * 3 = 2^{1\times 2} * 3 = 4 * 3 = 2^{4\times 3} = 2^{12}$ and

$1 * (2 * 3) = 1 * 2^{2\times 3} = 1 * 2^6 = 1 * 64 = 2^{1\times 64} = 2^{64}$

Since, $(1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in Z^+$

Hence, the operation $*$ is not associative.

Hence, the operation $*$ is commutative and not associative.

(v) On Z^+, $*$ is defined by $a * b = a^b$.

If $a, b \in Z^+$, then $a^b \in Z^+$.

Hence, the operation $*$ is a binary operation.

It is observed that

$1 * 2 = 1^2 = 1$ and $2 * 1 = 2^1 = 2$

Since, $1 * 2 \neq 2 * 1$, where $1, 2 \in Z^+$

Hence, the operation $*$ is not commutative.

It is also observed that

$(2 * 3) * 4 = 2^3 * 4 = 8 * 4 = 8^4 = 2^{12}$ and

$2 * (3 * 4) = 2 * 3^4 = 2 * 81 = 2^{81}$

$(2 * 3) * 4 \neq 2 * (3 * 4)$, where $2, 3, 4 \in Z^+$

Hence, the operation $*$ is not associative.

Hence, the operation $*$ is neither associative nor commutative.

(vi) On $R - \{-1\}$, $*$ is defined by $a * b = \frac{a}{b+1}$

If $a, b \in R - \{-1\}$, then $\frac{a}{b+1} \in R - \{-1\}$.

Hence, the operation $*$ is a binary operation.

It is observed that

$1 * 2 = \frac{1}{2+1} = \frac{1}{3}$ and $2 * 1 = \frac{2}{1+1} = \frac{2}{2} = 1$

Since, $1 * 2 \neq 2 * 1$, where $1, 2 \in R - \{-1\}$

Hence, the operation $*$ is not commutative.

It is also observed that

$$(1 * 2) * 3 = \frac{1}{2+1} * 3 = \frac{1}{3} * 3 = \frac{\frac{1}{3}}{3+1} = \frac{1}{12}$$

and

$$1 * (2 * 3) = 1 * \frac{2}{3+1} = 1 * \frac{2}{4} = 1 * \frac{1}{2} = \frac{1}{\frac{1}{2}+1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Since, $(1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in R - \{-1\}$

Hence, the operation $*$ is not associative.

Hence, the operation $*$ is neither associative nor commutative.

3. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a \wedge b$ =min $\{a, b\}$. Write the operation table of the operation $\wedge$.

Solution:

Since, the binary operation on the set $\{1, 2, 3, 4, 5\}$ is defined as $a \wedge b$ = min $\{a, b\}$ for all $a, b \in \{1, 2, 3, 4, 5\}$.

Hence, the operation table for the given operation $\wedge$ can be given as:

$\wedge$	**1**	**2**	**3**	**4**	**5**
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

4. Consider a binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ given by the following multiplication table.

(i) Compute $(2 * 3) * 4$ and $2 * (3 * 4)$

(ii) Is $*$ commutative?

(iii) Compute $(2 * 3) * (4 * 5)$.

(Hint: use the following table)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Solution:

(i) We need to find: $(2 * 3) * 4$ and $2 * (3 * 4)$

Using table,

$(2 * 3) * 4 = 1 * 4 = 1$

$2 * (3 * 4) = 2 * 1 = 1$

(ii) For every $a, b \in \{1, 2, 3, 4, 5\}$, we can observe that $a * b = b * a$. Hence, the operation

$*$ is commutative.

(iii) From table, $(2 * 3) = 1$ and $(4 * 5) = 1$

$\therefore (2 * 3) * (4 * 5) = 1 * 1 = 1$

5. Let $*'$ be the binary operation on the set $\{1 , 2, 3, 4, 5\}$ defined by $a *' b = H.C.F.$ of a and b. Is the operation $*'$ same as the operation $*$ defined in question 4 above? Justify your answer.

Solution:

Given, the binary operation $*'$ on the set {1, 2, 3, 4, 5} is defined as $a *' b = H.C.F$ of a and b.

Hence, the operation table for the operation $*'$ is given as:

$*'$	**1**	**2**	**3**	**4**	**5**
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

We can see that the operation tables for the operations $*$ and $*'$ are the same.

Hence, the operation $*'$ is same as the operation $*$.

6. Let $*$ be the binary operation on N given by $a * b = L.C.M.$ of a and b. Find

(i) $5 * 7, 20 * 16$

(ii) Is $*$ commutative?

(iii) Is $*$ associative?

(iv) Find the identity of $*$ in N

(v) Which elements of N are invertible for the operation $*$?

Solution:

Since, the binary operation $*$ on N is defined as $a^*b = L.C.M.$ of a and b.

(i) Hence, $5 * 7 = L.C.M.$ of 5 and $7 = 35$

$20 * 16 = L.C.M$ of 20 and $16 = 80$

(ii) As we know that

L.C. M of a and $b = L.C.M$ of b and a for all $a, b \in N$.

Hence, $a * b = b * a$

Hence, the operation $*$ is commutative.

(iii) For $a, b, c \in N$, we have

$(a * b) * c = (L.C.M$ of a and $b) * c =$ LCM of a, b, and c

$a * (b * c) = a * (LCM$ of b and $c) = L.C.M$ of a, b, and c

$\therefore (a * b) * c = a * (b * c)$

Hence, the operation $*$ is associative.

(iv) As we know that,

$L.C.M.$ of a and $1 = a = L.C.M.$ 1 and a for all $a \in N$

$\Rightarrow a * 1 = a = 1 * a$ for all $a \in N$

Hence, 1 is the identity of $*$ in N.

(v) An element a in N is invertible with respect to the operation $*$ if and only if there exists

an element b in N, such that $a * b = e = b * a$.

Here, $e = 1$

It means that

$L.C.M$ of a and $b = 1 = L.C.M$ of b and a

This is possible only if a and b are equal to 1.

Hence, 1 is the only invertible element of N with respect to the operation $*$.

7. Is $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a * b = L.C.M.$ of a and b a binary operation? Justify your answer.

Solution:

Given, the operation $*$ on the set $A = \{1, 2, 3, 4, 5\}$ as $a * b = L.C.M.$ of a

and b.

Hence, the operation table for the given operation $*$ is:

$*$	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	6	3	12	15
4	4	4	12	4	20
5	5	10	15	20	5

From the obtained table, we can observe that,

$3 * 2 = 2 * 3 = 6 \notin A,$

$5 * 2 = 2 * 5 = 10 \notin A,$

$3 * 4 = 4 * 3 = 12 \notin A,$

$3 * 5 = 5 * 3 = 15 \notin A,$

$4 * 5 = 5 * 4 = 20 \notin A$

Hence, the given operation $*$ is not a binary operation.

8. Let $*$ be the binary operation on N defined by $a * b = H.C.F.$ of a and b. Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

Solution:

Given, the binary operation $*$ on N as: $a * b = H.C.F.$ of a and b

As we know that,

$H.C.F.$ of a and $b = H.C.F.$ of b and a for all $a, b \in N$.

$\therefore a * b = b * a$

Hence, the operation $*$ is commutative.

For $a, b, c \in N$, we have

$(a * b) * c = (H.C.F.$ of a and $b) * c = H.C.F.$ of a, b and c

$a * (b * c) = a * (H.C.F.$ of b and $c) = H.C.F.$ of a, b, and c

$\therefore (a * b) * c = a * (b * c)$

Hence, the operation $*$ is associative.

Now, an element $e \in N$ will be the identity for the operation $*$ if $a * e = a = e * a$ for all $a \in N$.

But this is not true for any $a \in N$.

Hence, the operation $*$ does not have any identity in N.

9. Let $*$ be a binary operation on the set Q of rational numbers as follows:

(i) $a * b = a - b$

(ii) $a * b = a^2 + b^2$

(iii) $a * b = a + ab$

(iv) $a * b = (a - b)^2$

(v) $a * b = \frac{ab}{4}$

(vi) $a * b = ab^2$

Find which of the binary operations are commutative and which are associative.

Solution:

(i) On Q, the binary operation $*$ is defined as $a * b = a - b$. It is observed that:

$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

and

$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$

$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}$, where $\frac{1}{2}, \frac{1}{3} \in Q$

Hence, the operation $*$ is not commutative.

It is also observed that

$$\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}=\left(\frac{1}{2}-\frac{1}{3}\right)*\frac{1}{4}=\left(\frac{3-2}{6}\right)*\frac{1}{4}=\frac{1}{6}*\frac{1}{4}=\frac{1}{6}-\frac{1}{4}=\frac{2-3}{12}=\frac{-1}{12}$$

and

$$\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{2}*\left(\frac{4-3}{12}\right)=\frac{1}{2}*\frac{1}{12}=\frac{1}{2}-\frac{1}{12}=\frac{6-1}{12}=\frac{5}{12}$$

Since, $\left(\frac{1}{2}*\frac{1}{3}\right)*\frac{1}{4}\neq\frac{1}{2}*\left(\frac{1}{3}*\frac{1}{4}\right)$, where $\frac{1}{2},\frac{1}{3},\frac{1}{4}\in Q$

Hence, the operation $*$ is not associative.

(ii) On Q, the binary operation $*$ is defined as $a*b=a^2+b^2$.

For $a, b\in Q$, we have

$a*b=a^2+b^2=b^2+a^2=b*a$

$\therefore a*b=b*a$

Hence, the operation $*$ is commutative.

It is also found that,

$(1*2)*3=(1^2+2^2)*3=(1+4)*3=5*3=5^2+3^2=34$ and

$1*(2*3)=1*(2^2+3^2)=1*(4+9)=1*13=1^2+13^2=170$

$\therefore (1*2)*3\neq 1*(2*3)$, where 1, 2, 3 $\in Q$

Hence, the operation $*$ is not associative.

(iii) On Q, the binary operation $*$ is defined as $a*b=a+ab$.

It is found that

$1*2=1+1\times 2=1+2=3$

$2*1=2+2\times 1=2+2=4$

Since, $1*2\neq 2*1$, where 1, 2 $\in Q$

Hence, the operation $*$ is not commutative.

It is also observed that

$(1*2)*3=(1+1\times 2)*3=(1+2)*3=3*3=3+3\times 3=3+9=12$ and

$\therefore 1*(2*3)=1*(2+2\times 3)=1*(2+6)=1*8=1+1\times 8=1+8=9$

$\therefore (1*2)*3\neq 1*(2*3)$, where $1, 2, 3\in Q$

Hence, the operation $*$ is not associative.

(iv) On Q, the binary operation $*$ is defined by $a * b = (a-b)^2$.

For $a, b \in Q$, we have

$a * b = (a-b)^2$

$b * a = (b-a)^2 = [-(a-b)]^2 = (a-b)^2$

Since, $a * b = b * a$

Hence, the operation $*$ is commutative.

Now, it is also seen that

$(1 * 2) * 3 = (1-2)^2 * 3 = (-1)^2 * 3 = 1 * 3 = (1-3)^2 = (-2)^2 = 4$

and

$1 * (2 * 3) = 1 * (2-3)^2 = 1 * (-1)^2 = 1 * 1 = (1-1)^2 = 0$

Since, $(1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in Q$

Hence, the operation $*$ is not associative.

(v) On Q, the binary operation $*$ is defined as $a * b = \frac{ab}{4}$.

For $a, b \in Q$, we have

$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$

$a * b = b * a$

Hence, the operation $*$ is commutative.

For $a, b, c \in Q$, we have

$(a * b) * c = \left(\frac{ab}{4}\right) * c = \frac{\left(\frac{ab}{4}\right).c}{4} = \frac{abc}{16}$

and

$a * (b * c) = a * \left(\frac{bc}{4}\right) = \frac{a.\left(\frac{bc}{4}\right)}{4} = \frac{abc}{16}$

$\therefore (a * b) * c = a * (b * c)$, where $a, b, c \in Q$

Hence, the operation $*$ is associative.

(vi) On Q, the binary operation $*$ is defined as $a * b = ab^2$

It can be found that

$\frac{1}{2} * \frac{1}{3} = \frac{1}{2} . \left(\frac{1}{3}\right)^2 = \frac{1}{2} . \frac{1}{9} = \frac{1}{18}$

and

$\frac{1}{3} * \frac{1}{2} = \frac{1}{3} . \left(\frac{1}{2}\right)^2 = \frac{1}{3} . \frac{1}{4} = \frac{1}{12}$

$\therefore \frac{1}{2} * \frac{1}{3} \neq \frac{1}{3} * \frac{1}{2}$, where $\frac{1}{2}$ and $\frac{1}{3} \in Q$

Hence, the operation $*$ is not commutative.

It is also seen that

$\left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} = \left[\frac{1}{2}\left(\frac{1}{3}\right)^2\right] * \frac{1}{4} = \frac{1}{18} * \frac{1}{4} = \frac{1}{18} . \left(\frac{1}{4}\right)^2 = \frac{1}{18 \times 16} = \frac{1}{288}$

and

$\frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right) = \frac{1}{2} * \left[\frac{1}{3}\left(\frac{1}{4}\right)^2\right] = \frac{1}{2} * \frac{1}{48} = \frac{1}{2}\left(\frac{1}{48}\right)^2 = \frac{1}{2 \times 2304} = \frac{1}{4608}$

$\therefore \left(\frac{1}{2} * \frac{1}{3}\right) * \frac{1}{4} \neq \frac{1}{2} * \left(\frac{1}{3} * \frac{1}{4}\right)$, where $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} \in Q$

Hence, the operation $*$ is not associative.

Hence, the operations defined in (ii), (iv), (v) are commutative and the operation defined in (v) is associative.

10. Find which of the operations given above has identity.

Solution:

An element $e \in Q$ will be the identity element for the binary operation $*$ if $a * e = a = e * a$, for all $a \in Q$.

However, there is no such element $e \in Q$ with respect to any of the six operations satisfying the above condition.

Hence, none of the six operations has identity.

11. Let $A = N \times N$ and $*$ be the binary operation on A defined by

$(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

Solution:

Given:

$A = N \times N$ and $*$ is a binary operation on A and is defined by

$(a, b) * (c, d) = (a + c,\ b + d)$

Suppose $(a, b), (c, d) \in A$

Then, $a, b, c, d \in N$

We have:

$(a, b) * (c, d) = (a + c, b + d)$

$(c, d) * (a, b) = (c + a, d + b) = (a + c, b + d)$

[Since, addition is commutative in the set of natural numbers]

$\therefore (a, b) * (c, d) = (c, d) * (a, b)$

Hence, the operation $*$ is commutative.

Now, suppose $(a, b), (c, d), (e, f) \in A$

Then, $a, b, c, d, e, f \in N$

We have

$[(a, b) * (c, d)] * (e, f) = (a + c, b + d) * (e, f) = (a + c + e, b + d + f)$

and

$(a, b) * [(c, d) * (e, f)] = (a, b) * (c + e, d + f) = (a + c + e, b + d + f)$

$\therefore [(a, b) * (c, d)] * (e, f) = (a, b) * [(c, d) * (e, f)]$

Hence, the operation $*$ is associative.

Suppose an element $e = (e_1, e_2) \in A$ will be an identity element for the operation $*$ if $a * e = a = e * a$ for all $a = (a_1, a_2) \in A$

i.e., $(a_1 + e_1, a_2 + e_2) = (a_1, a_2) = (e_1 + a_1, e_2 + a_2)$

Which is not true for any element in A.

Hence, the operation $*$ does not have any identity element.

12. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation $*$ on a set $N, a * a = a \ \forall \ a \in N$.

(ii) If $*$ is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

Solution:

(i) Defining an operation $*$ on N as $a * b = a + b \ \forall \ a, b \in N$

Then, in particular, for $b = a = 3$, we have

$3 * 3 = 3 + 3 = 6 \neq 3$

Thus, statement (i) is false.

(ii) R.H.S. $= (c * b) * a$

$= (b * c) * a$ [Since, $*$ is commutative]

$= a * (b * c)$ [Again, as $*$ is commutative]

$= \text{L.H.S.}$

$\therefore a * (b * c) = (c * b) * a$

Hence, statement (ii) is true.

13. Consider a binary operation $*$ on N defined as $a * b = a^3 + b^3$. Choose the correct answer.

(A) Is $*$ both associative and commutative?

(B) Is $*$ commutative but not associative?

(C) Is $*$ associative but not commutative?

(D) Is $*$ neither commutative nor associative?

Solution:

On N, the binary operation $*$ is defined as $a * b = a^3 + b^3$.

For, $a, b, \in N$, we have

$a * b = a^3 + b^3 = b^3 + a^3 = b * a$ [Since, addition is commutative in N]

Hence, the operation $*$ is commutative.

It is also observed that

$(1 * 2) * 3 = (1^3 + 2^3) * 3 = (1 + 8) * 3 = 9 * 3 = 9^3 + 3^3 = 729 + 27 = 756$ and

$1 * (2 * 3) = 1 * (2^3 + 3^3) = 1 * (8 + 27) = 1 * 35 = 1^3 + 35^3 = 1 + 42875 = 42876$

$\therefore (1 * 2) * 3 \neq 1 * (2 * 3)$, where $1, 2, 3 \in N$

Hence, the operation $*$ is not associative.

Hence, the operation $*$ is commutative, but not associative.

Hence, the correct answer is B.

Miscellaneous Exercise on Chapter 1

1. Let $f: R \to R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \to R$ such that $gof = fog = I_R$.

Solution:

Given that $f: R \to R$ is defined as $f(x) = 10x + 7$.

For one – one

Suppose $f(x) = f(y)$, where $x, y \in R$.

$\Rightarrow 10x + 7 = 10y + 7$

$\Rightarrow x = y$

Hence, f is a one-one function.

For onto