Exercise 4.2

Question 1: Write the following in the expanded form:
(i) $(a + 2b + c)^2$

(ii) $(2a − 3b − c)^2$

(iii) $(−3x+y+z)^2$

(iv) $(m+2n−5p)^2$

(v) $(2+x−2y)^2$

(vi) $(a^2 +b^2 +c^2)^2$

(vii) $(ab+bc+ca)^2$

(viii) $(x/y+y/z+z/x)2$

(ix) $(a/bc + b/ac + c/ab)^2$

(x) $(x+2y+4z)^2$

(xi) $(2x−y+z)^2$

(xii) $(−2x+3y+2z)^2$

Solution:
Using identities:
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

(i) $(a + 2b + c)^2$
$= a^2 + (2b)^2 + c^2 + 2a(2b) + 2ac + 2(2b)c$

$= a^2 + 4b^2 + c^2 + 4ab + 2ac + 4bc$

(ii) $(2a − 3b − c)^2$

$= [(2a) + (−3b) + (−c)]^2$

$= (2a)^2 + (−3b)^2 + (−c)^2 + 2(2a)(−3b) + 2(−3b)(−c) + 2(2a)(−c)$

= 4a² + 9b² + c² − 12ab + 6bc − 4ca

(iii) (−3x+y+z)²

= [(−3x)² + y² + z² + 2(−3x)y + 2yz + 2(−3x)z

= 9x² + y² + z² − 6xy + 2yz − 6xz

(iv) (m+2n−5p)²

= m² + (2n)² + (−5p)² + 2m × 2n + (2×2n×−5p) + 2m × −5p

= m² + 4n² + 25p² + 4mn − 20np − 10pm

(v) (2+x−2y)²

= 2² + x² + (−2y)² + 2(2)(x) + 2(x)(−2y) + 2(2)(−2y)

= 4 + x² + 4y² + 4 x − 4xy − 8y

(vi) (a² +b² +c²)²

= (a²)² + (b²)² + (c²)² + 2a² b² + 2b²c² + 2a²c²

= a⁴ + b⁴ + c⁴ + 2a² b² + 2b² c² + 2c² a²

(vii) (ab+bc+ca)²

= (ab)² + (bc)² + (ca)² + 2(ab)(bc) + 2(bc)(ca) + 2(ab)(ca)

= a² b² + b²c² + c² a² + 2(ac)b² + 2(ab)(c)² + 2(bc)(a)²

(viii) (x/y+y/z+z/x)²

$$= \left(\tfrac{x}{y}\right)^2 + \left(\tfrac{y}{z}\right)^2 + \left(\tfrac{z}{x}\right)^2 + 2\tfrac{x}{y}\tfrac{y}{z} + 2\tfrac{y}{z}\tfrac{z}{x} + 2\tfrac{z}{x}\tfrac{x}{y}$$

$$= \left(\tfrac{x^2}{y^2}\right) + \left(\tfrac{y^2}{z^2}\right) + \left(\tfrac{z^2}{x^2}\right) + 2\tfrac{x}{z} + 2\tfrac{y}{x} + 2\tfrac{x}{y}$$

(ix) (a/bc + b/ac + c/ab) ^{2}

$$= \left(\frac{a}{bc}\right)^2 + \left(\frac{b}{ca}\right)^2 + \left(\frac{c}{ab}\right)^2 + 2\left(\frac{a}{bc}\right)\left(\frac{b}{ca}\right) + 2\left(\frac{b}{ca}\right)\left(\frac{c}{ab}\right) + 2\left(\frac{a}{bc}\right)\left(\frac{c}{ab}\right)$$

$$= \left(\frac{a^2}{b^2c^2}\right) + \left(\frac{b^2}{c^2a^2}\right) + \left(\frac{c^2}{a^2b^2}\right) + \frac{2}{a^2} + \frac{2}{b^2} + \frac{2}{c^2}$$

(x) (x+2y+4z) ^{2}

$= x^2 + (2y)^2 + (4z)^2 + (2x)(2y) + 2(2y)(4z) + 2x(4z)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(xi) (2x–y+z) ^{2}

$= (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(2x)(z)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(xii) (−2x+3y+2z) ^{2}

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(-2x)(2z)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

Question 2: Simplify
(i) (a + b + c)^2 + (a − b + c) ^{2}
(ii) (a + b + c)^2 − (a − b + c) ^{2}
(iii) (a + b + c)^2 + (a - b + c) ^{2} + (a + b − c) ^{2}
(iv) (2x + p − c)^2 − (2x − p + c) ^{2}
(v) (x^2 + y^2 − z^2) ^{2} − (x^2 − y^2 + z^2) ^{2}

Solution:
(i) (a + b + c)^2 + (a − b + c) ^{2}

$= (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) + (a^2 + (-b)^2 + c^2 - 2ab - 2bc + 2ca)$

$= 2a^2 + 2b^2 + 2c^2 + 4ca$

(ii) (a + b + c)^2 − (a − b + c) ^{2}

= $(a^2 + b^2 + c^2 + 2ab+2bc+2ca) - (a^2 + (-b)^2 + c^2 -2ab-2bc+2ca)$

= $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - a^2 - b^2 - c^2 + 2ab + 2bc - 2ca$

= $4ab + 4bc$

(iii) $(a + b + c)^2 + (a - b + c)^2 + (a + b - c)^2$

= $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca + (a^2 + b^2 + (c)^2 - 2bc - 2cb + 2ca) + (a^2 + b^2 + c^2 + 2ab - 2bc - 2ab)$

= $3a^2 + 3b^2 + 3c^2 + 2ab - 2bc + 2ca$

(iv) $(2x + p - c)^2 - (2x - p + c)^2$

= $[4x^2 + p^2 + c^2 + 4xp - 2pc - 4xc] - [4x^2 + p^2 + c^2 - 4xp - 2pc + 4xc]$

= $4x2 + p2 + c2 + 4xp - 2pc - 4cx - 4x2 - p2 - c2 + 4xp + 2pc - 4cx$

= $8xp - 8xc$

= $8(xp - xc)$

(v) $(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2$

= $(x^2 + y^2 + (-z)^2)^2 - (x^2 - y^2 + z^2)^2$

= $[x^4 + y^4 + z^4 + 2x^2y^2 + 2y^2z^2 + 2x^2z^2 - [x^4 + y^4 + z^4 - 2x^2y^2 - 2y^2z^2 + 2x^2z^2]$

= $4x^2y^2 - 4z^2x^2$

Question 3: If a + b + c = 0 and $a^2 + b^2 + c^2$ = 16, find the value of ab + bc + ca.
Solution:
$a + b + c = 0$ and $a^2 + b^2 + c^2 = 16$ (given)
Choose $a + b + c = 0$
Squaring both sides,
$(a + b + c)^2 = 0$
$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0$
$16 + 2(ab + bc + c) = 0$
$2(ab + bc + ca) = -16$
$ab + bc + ca = -16/2 = -8$
or $ab + bc + ca = -8$