Ratio and Proportion
Exercise 8A

Ratio and Proportion

- A ratio is a comparison of two values expressed as a quotient
  - Example: A class has 12 girls and 18 boys. The ratio of girls to boys is $\frac{12}{18}$
  - This ratio can also be expressed as an equivalent fraction $\frac{2}{3}$
- A proportion is an equation stating that two ratios are equal.
  - Example: $\frac{12}{18} = \frac{2}{3}$

1. **Ratio:**

   The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.

   In the ratio $a : b$, we call a as the first term or **antecedent** and b, the second term or **consequent**.

   Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.

   If $a : b = c : d$, we write $a : b :: c : d$ and we say that a, b, c, d are in proportion.

   Here a and d are called **extremes**, while b and c are called **mean terms**.

   Product of means = Product of extremes.

   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. **Fourth Proportional:**

   If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

   Third Proportional:

   $a : b = c : d$, then c is called the third proportion to a and b.

   Mean Proportional:

   Mean proportional between a and b is $\sqrt{ab}$.

4. **Comparison of Ratios:**

   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

   Compounded Ratio:

   The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

5. **Duplicate Ratios:**

   Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

   Sub-duplicate ratio of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

   Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

   Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. [componendo and dividendo]

6. **Variations:**

   We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

   We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

Properties of proportions:

Convertendo: If a : b :: c : d ,then a : (a - b) :: c : (c – d) .

Invertendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{b}{a}=\frac{d}{c}$.

Alternendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a}{c}=\frac{b}{d}$.

Componendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{b}=\frac{c+d}{d}$.

Dividendo: $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a-b}{b}=\frac{c-d}{d}$

Componendo and Dividendo: If $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a+b}{a-b}=\frac{c+d}{c-d}$.

Q1

Answer :

(i) HCF of 24 and 40 is 8.

$\therefore 24 : 40 = \frac{24}{40} = \frac{24\div 8}{40\div 8} = \frac{3}{5} = 3 : 5$

Hence, 24 : 40 in its simplest form is 3 : 5.

(ii) HCF of 13.5 and 15 is 1.5.

$\frac{13.5}{15}=\frac{135}{150}$

The HCF of 135 and 150 is 15.

$=\frac{135\div 15}{150\div 15}=\frac{9}{10}$

Hence, 13.5 : 15 in its simplest form is 9 : 10.

(iii) $\frac{20}{3} : \frac{15}{2} = 40 : 45$

The HCF of 40 and 45 is 5.

$\therefore 40 : 45 = \frac{40}{45} = \frac{40\div 5}{45\div 5} = \frac{8}{9} = 8 : 9$

Hence, $6\frac{2}{3} : 7\frac{1}{2}$ in its simplest form is 8 : 9

(iv) 9 : 6

The HCF of 9 and 6 is 3.

$\therefore 9 : 6 = \frac{9}{6} = \frac{9\div 3}{6\div 3} = 3 : 2$

Hence, $\frac{1}{6} : \frac{1}{9}$ in its simplest form is 3 : 2.

(v) LCM of the denominators is 2.

$\therefore 4 : 5 : \frac{9}{2} = 8 : 10 : 9$

The HCF of these 3 numbers is 1.

∴ 8 : 10 : 9 is the simplest form.

(vi) 2.5 : 6.5 : 8 = 25 : 65 : 80

The HCF of 25, 65 and 80 is 5.

$\therefore 25 : 65 : 80 = \frac{25}{\frac{65}{80}} = \frac{25\div 5}{\frac{65\div 5}{80\div 5}} = \frac{5}{\frac{13}{16}} = 5 : 13 : 16$

Q2

Answer :

(i) Converting both the quantities into the same unit, we have:
75 paise : (3 × 100) paise = 75 : 300

$= \frac{75}{300} = \frac{75 \div 75}{300 \div 75} = \frac{1}{4}$ (∵ HCF of 75 and 300 = 75)
= 1 paise : 4 paise

(ii) Converting both the quantities into the same unit, we have:
105 cm : 63 cm = $\frac{105}{63} = \frac{105 \div 21}{63 \div 21} = \frac{5}{3}$ (∵ HCF of 105 and 63 = 21)
= 5 cm : 3 cm

(iii) Converting both the quantities into the same unit
65 min : 45 min = $\frac{65}{45} = \frac{65 \div 5}{45 \div 5} = \frac{13}{9}$ (∵ HCF of 65 and 45 = 5)
= 13 min : 9 min

(iv) Converting both the quantities into the same unit, we get:
8 months : 12 months = $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ (∵ HCF of 8 and 12 = 4)
= 2 months : 3 months

(v) Converting both the quantities into the same unit, we get:

2250g : 3000 g = $\frac{2250}{3000} = \frac{2250 \div 750}{3000 \div 750} = \frac{3}{4}$ (∵ HCF of 2250 and 3000 = 750)

= 3 g : 4 g

(vi) Converting both the quantities into the same unit, we get:
1000 m : 750 m = $\frac{1000}{750} = \frac{1000 \div 250}{750 \div 250} = \frac{4}{3}$ (∵ HCF of 1000 and 750 = 250)
= 4 m : 3 m

Q3

Answer :

$\frac{A}{B} = \frac{7}{5}$ and $\frac{B}{C} = \frac{9}{14}$

Therefore, we have:

$\frac{A}{B} \times \frac{B}{C} = \frac{7}{5} \times \frac{9}{14}$
$\frac{A}{C} = \frac{9}{10}$

∴ A : C = 9 : 10

Q4

Answer :

$\frac{A}{B} = \frac{5}{8}$ and $\frac{B}{C} = \frac{16}{25}$
Now, we have : $\frac{A}{B} \times \frac{B}{C} = \frac{5}{8} \times \frac{16}{25} \Rightarrow \frac{A}{C} = \frac{2}{5}$

∴ A : C = 2 : 5

Q5

Answer :

$A : B = 3 : 5$

$B : C = 10 : 13 = \frac{10 : 2}{13 : 2} = 5 : \frac{13}{2}$

Now, $A : B : C = 3 : 5 : \frac{13}{2}$

$\therefore A : B : C = 6 : 10 : 13$

Q6

Answer :

We have the following:

$A : B = 5 : 6$

$B : C = 4 : 7 = \frac{4}{7} = \frac{4 \times \frac{6}{4}}{7 \times \frac{6}{4}} = 6 : \frac{21}{2}$

$\therefore A : B : C = 5 : 6 : \frac{21}{2} = 10 : 12 : 21$

Q7

Answer :

Sum of the ratio terms = 7 + 8 = 15

Now, we have the following:

Kunal's share = Rs 360 $\times \frac{7}{15} = 24 \times 7$ = Rs 168

Mohit's share = Rs 360 $\times \frac{8}{15} = 24 \times 8$ = Rs 192

Q8

Answer :

Sum of the ratio terms = $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$

Now, we have the following:

Rajan's share = Rs 880 $\times \frac{\frac{1}{5}}{\frac{11}{30}} = \text{Rs } 880 \times \frac{6}{11} = \text{Rs } 80 \times 6$ = Rs 480

Kamal's share = Rs 880 $\times \frac{\frac{1}{6}}{\frac{11}{30}} = \text{Rs } 880 \times \frac{5}{11} = \text{Rs } 80 \times 5$ = Rs 400

Q9

Answer :

Sum of the ratio terms is (1 + 3 + 4) = 8

We have the following:

A's share = Rs 5600 × $\frac{1}{8}$ = Rs $\frac{5600}{8}$ = Rs 700

B's share = Rs 5600 × $\frac{3}{8}$ = Rs 700 × 3 = Rs 2100

C's share = Rs 5600 × $\frac{4}{8}$ = Rs 700 × 4 = Rs 2800

Q10

Answer :

Let *x* be the required number.
Then, (9 + *x*) : (16 + *x*) = 2 : 3

$\Rightarrow \frac{9+x}{16+x} = \frac{2}{3}$
$\Rightarrow 27 + 3x = 32 + 2x \Rightarrow x = 5$

Hence, 5 must be added to each term of the ratio 9 : 16 to make it 2 : 3.

Q11

Answer :

Suppose that *x* is the number that must be subtracted.
Then, (17 − *x*) : (33 − *x*) = 7 : 15

$\Rightarrow \frac{17-x}{33-x} = \frac{7}{15}$
$\Rightarrow 255 - 15x = 231 - 7x \Rightarrow 8x = 255 - 231 = 24 \Rightarrow x = 3$

Hence, 3 must be subtracted from each term of ratio 17 : 33 so that it becomes 7 : 15.

Q12

Answer :

Suppose that the numbers are 7*x* and 11*x*.

Then, (7*x* + 7) : (11*x* + 7) = 2 : 3
$\Rightarrow \frac{7x+7}{11x+7} = \frac{2}{3}$

⇒ 21*x* + 21 = 22*x* + 14

⇒ x = 7

Hence, the numbers are (7 × 7 =) 49 and (11 × 7 =) 77.

Q13

Answer :

Suppose that the numbers are 5*x* and 9*x*.
Then, (5*x* − 3) : (9*x* − 3) = 1 : 2

$\Rightarrow \frac{5x-3}{9x-3} = \frac{1}{2}$

⇒ 10*x* − 6 = 9*x* − 3
⇒ *x* = 3

Hence, the numbers are (5 × 3 =) 15 and (9 × 3 =) 27.

Q14

Answer :

Let the numbers be $3x$ and $4x$.
Their LCM is $12x$.
Then, $12x = 180$
$\Rightarrow x = 15$

$\therefore$ The numbers are $(3 \times 15 =) 45$ and $(4 \times 15 =) 60$.

Q15

Answer :

Suppose that the present ages of A and B are $8x$ yrs and $3x$ yrs.
Then, $(8x + 6) : (3x + 6) = 9 : 4$
$\Rightarrow \frac{8x+6}{3x+6} = \frac{9}{4}$
$\Rightarrow 32x + 24 = 27x + 54$
$\Rightarrow 5x = 30$
$\Rightarrow x = 6$

Now, present age of A $= 8 \times 6$ yrs $= 48$ yrs
Present age of B $= 3 \times 6$ yrs $= 18$ yrs

Q16

Answer :

Suppose that the weight of zinc is x g.

Then, $48.6 : x = 9 : 5$

$\Rightarrow x = \frac{48.6 \times 5}{9} = \frac{243}{9} = 27$

Hence, the weight of zinc in the alloy is 27 g.

Q17

Answer :

Suppose that the number of boys is x.
Then, $x : 375 = 8 : 3$

$\Rightarrow x = \frac{8 \times 375}{3} = 8 \times 125 = 1000$

Hence, the number of girls in the school is 1000.

Q18

Answer :

Suppose that the monthly income of the family is Rs x.

Then, $x : 2500 = 11 : 2$

$\Rightarrow x = \frac{11 \times 2500}{2} = 11 \times 1250$
$\Rightarrow x =$ Rs 13750

Hence, the income is Rs 13,750.
$\therefore$ Expenditure = (monthly income − savings)
=Rs (13750 − 2500)
= Rs 11250

Q19

Answer :

Let the numbers one rupee, fifty paise and twenty-five paise coins be 5*x*, 8*x* and 4*x*, respectively.

Total value of these coins = $(5x \times \frac{100}{100} + 8x \times \frac{50}{100} + 4x \times \frac{25}{100})$

$\Rightarrow 5x + \frac{8x}{2} + \frac{4x}{4}$

$= \frac{20x + 16x + 4x}{4} = \frac{40x}{4} = 10x$

However, the total value is Rs 750.

$\therefore 750 = 10x$

$\Rightarrow x = 75$

Hence, number of one rupee coins = 5 × 75 = 375

Number of fifty paise coins = 8 × 75 = 600

Number of twenty-five paise coins = 4 × 75 = 300

Q20

Answer :

(4*x* + 5) : (3*x* + 11) = 13 : 17

$\Rightarrow \frac{4x+5}{3x+11} = \frac{13}{17}$

$\Rightarrow 68x + 85 = 39x + 143 \Rightarrow 29x = 58 \Rightarrow x = 2$

Q21

Answer :

$\frac{x}{y} = \frac{3}{4}$

$\Rightarrow x = \frac{3y}{4}$

Now, we have (3*x* + 4*y*) : (5*x* + 6*y*)

$= \frac{3x+4y}{5x+6y} = \frac{3\times\frac{3y}{4}+4y}{5\times\frac{3y}{4}+6y}$

$= \frac{9y+16y}{15y+24y} = \frac{25y}{39y} = \frac{25}{39}$

= 25 : 39

Q22

Answer :

$\frac{x}{y} = \frac{6}{11}$

$\Rightarrow x = \frac{6y}{11}$

Now, we have:

$\frac{8x-3y}{3x+2y}$

$= \frac{8\times\frac{6y}{11}-3y}{3\times\frac{6y}{11}+2y}$

$= \frac{48y-33y}{18y+22y}$

$= \frac{15y}{40y} = \frac{3}{8}$

$\therefore$ (8*x* − 3*y*) : (3*x* + 2*y*) = 3 : 8

Q23

Answer :

Suppose that the numbers are 5*x* and 7*x*.
The sum of the numbers is 720.
i.e., 5*x* + 7*x* = 720
⇒ 12*x* = 720
⇒ *x* = 60

Hence, the numbers are (5 × 60 =) 300 and (7 × 60 =) 420.

Q24

Answer :

(i) The LCM of 6 and 9 is 18.

$\frac{5}{6} = \frac{5\times3}{6\times3} = \frac{15}{18}$
$\frac{7}{9} = \frac{7\times2}{9\times2} = \frac{14}{18}$ **Clearly,** $\frac{14}{18} < \frac{15}{18}$

∴ (7 : 9) < (5 : 6)

(ii) The LCM of 3 and 7 is 21.

$\frac{2}{3} = \frac{2\times7}{3\times7} = \frac{14}{21}$
$\frac{4}{7} = \frac{4\times3}{7\times3} = \frac{12}{21}$
Clearly, $\frac{12}{21} < \frac{14}{21}$

∴ (4 : 7) < (2 : 3)

(iii) The LCM of 2 and 7 is 14.

$\frac{1\times7}{2\times7} = \frac{7}{14}$
$\frac{4\times2}{7\times2} = \frac{8}{14}$

Clearly, $\frac{7}{14} < \frac{8}{14}$

∴ (1 : 2) < (4 : 7)

(iv) The LCM of 5 and 13 is 65.

$\frac{3}{5} = \frac{3\times13}{5\times13} = \frac{39}{65}$
$\frac{8}{13} = \frac{8\times5}{13\times5} = \frac{40}{65}$
Clearly, $\frac{39}{65} < \frac{40}{65}$

∴ (3 : 5) < (8 : 13)

Q25

Answer :

(i) We have $\frac{5}{6}$, $\frac{8}{9}$ **and** $\frac{11}{18}$.

$2 \mid 6, 9, 18$
$3 \mid 3, 9, 9$
$3 \mid 1, 3, 3$
$\mid 1, 1, 1$

The LCM of 6, 9 and 18 is 18. Therefore, we have:

$\frac{5}{6} = \frac{5\times3}{6\times3} = \frac{15}{18}$
$\frac{8}{9} = \frac{8\times2}{9\times2} = \frac{16}{18}$ $\frac{11}{18} = \frac{11}{18}$ **Clearly,** $\frac{11}{18} < \frac{15}{18} < \frac{16}{18}$

Hence, (11 : 18) < (5 : 6) < (8 : 9)

(ii) We have $\frac{11}{14}$, $\frac{17}{21}$, $\frac{5}{7}$ and $\frac{2}{3}$.

$2 \mid 14, 21, 7, 3$

$7 \mid 7, 21, 7, 3$,

$3 \mid 1, 3, 1, 3$

$\mid 1, 1, 1, 1$

The LCM of 14, 21, 7 and 3 is 42.

$\frac{11}{14} = \frac{11 \times 3}{14 \times 3} = \frac{33}{28}$

$\frac{17}{21} = \frac{17 \times 2}{21 \times 2} = \frac{34}{42}$

$\frac{5}{7} = \frac{5 \times 6}{7 \times 6} = \frac{30}{42}$

$\frac{2}{3} = \frac{2 \times 14}{3 \times 14} = \frac{28}{42}$

Clearly, $\frac{28}{42} < \frac{30}{42} < \frac{33}{28} < \frac{34}{42}$

Hence, $(2 : 3) < (5 : 7) < (11 : 14) < (17 : 21)$