Exercise 5.4

Q1

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(2A)^T = 2A^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(2A)^T = 2 \times A^T$

$$\Rightarrow \quad \left(2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}\right)^T = 2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

LHS = RHS

So,

$$(2A)^T = 2A^T$$

Q2

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(A+B)^T = A^T + B^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(A+B)^T = A^T + B^T$

$$\left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 2+1 & -7+2 \\ -3+0 & 5-4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$$(A+B)^T = A^T + B^T$$

Q3

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(A-B)^T = A^T - B^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(A-B)^T = A^T - B^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 2-1 & -7-2 \\ -3-0 & 5+4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$$(A-B)^T = A^T - B^T$$

Q4

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $b = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

$\Rightarrow$ HS = RHS

So,

$(AB)^T = B^T A^T$

Q5

If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 4 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}\begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$(AB)^T = B^T A^T$

Q6

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that $(A+B)^T = A^T + B^T$

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix},\ B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$(A+B)^T = A^T + B^T$

$$\Rightarrow \quad \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$$(A+B)^T = A^T + B^T$$

Q7

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix},\ B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \quad \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$(AB)^T = B^T A^T$

Q8

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that $(2A)^T = 2A^T$.

Solution

Given,

$$A=\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix},\ B=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$$

$$\Rightarrow \quad A^T=\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix},\ B^T=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 1\\ 3 & 3 & 1\end{bmatrix}$$

$(2A)^T = 2A^T$

$$\Rightarrow \quad \left(2\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}\right)^T = 2\begin{bmatrix}1 & -1 & 0\\ 2 & 1 & 3\\ 1 & 2 & 1\end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix}2 & -2 & 0\\ 4 & 2 & 6\\ 2 & 4 & 2\end{bmatrix}^T = 2\begin{bmatrix}1 & 2 & 1\\ -1 & 1 & 2\\ 0 & 3 & 1\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}2 & 4 & 2\\ -2 & 2 & 4\\ 0 & 6 & 2\end{bmatrix} = \begin{bmatrix}2 & 4 & 2\\ -2 & 2 & 4\\ 0 & 6 & 2\end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$(2A)^T = 2 \times A^T$

Q9

If $A=\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}$, $B=\begin{bmatrix}1 & 3 & -6\end{bmatrix}$, verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A=\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix},\ B=\begin{bmatrix}1 & 3 & -6\end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \quad \left(\begin{bmatrix}2\\ 4\\ 5\end{bmatrix}\begin{bmatrix}1 & 3 & -6\end{bmatrix}\right)^T = \begin{bmatrix}1 & 3 & -6\end{bmatrix}^T\begin{bmatrix}-2\\ 4\\ 5\end{bmatrix}^T$$

$$\Rightarrow \quad \begin{bmatrix}-2 & -6 & 12\\ 4 & 12 & -24\\ 5 & 15 & -30\end{bmatrix}^T = \begin{bmatrix}1\\ 3\\ -6\end{bmatrix}\begin{bmatrix}-2 & 4 & 5\end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix}-2 & 4 & 5\\ -6 & 12 & 15\\ 12 & -24 & -30\end{bmatrix} = \begin{bmatrix}-2 & 4 & 5\\ -6 & 12 & 15\\ 12 & -24 & -30\end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$(2A)^T = 2 \times AT$

Q10

If $A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$, find $(AB)^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, \ B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

$(AB)^T$

$$= \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix} \right) \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6-4-2 & 8+8-1 \\ -3+0+4 & -4+0+2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

Q11

For two matrices A and B, $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$ verify that

$$(AB)^T = B^T A^T$$

Solution

Given,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+0+15 & -2+20 \\ 4+0+0 & -4+2+0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}^T = \begin{bmatrix} 2+0+15 & 4+0+0 \\ -2+2+0 & -4+2+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$(AB)^T = B^T A^T$

Q12

For the matrices A and B, verify that $(AB)^T = B^T A^T$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

Solution

Given,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$(AB)^T = B^T A^T$

$$\Rightarrow \left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}^T = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$\Rightarrow$ LHS = RHS

So,

$(AB)^T = B^T A^T$

Q13

If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

Solution

Given that $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

We need to find $A^T - B^T$.

Given that, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\Rightarrow B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

Let us find $A^T - B^T$:

$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$

$\Rightarrow A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

Q14

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then verify that $A'A = I$

Solution

$$A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha)\\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Q15

$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$, then verify that $A'A=I$

Solution

$$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\therefore A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$$

$$A'A=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$

$$\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$$
$$=\begin{bmatrix}(\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha)\\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha)\end{bmatrix}$$
$$=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, we have verified that $A'A=I$.

Q16

If l_i, m_i, n_i ; i = 1, 2, 3 denote the direction cosines of three mutually perpendicular vectors in space, prove that $AA^T = I$,

Where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

Solution

Given,

l_i, m_i, n_i are direction cosines of three mutually perpendicular vectors

$$\Rightarrow \left.\begin{aligned} l_1l_2 + m_1m_2 + n_1n_2 = 0 \\ l_2l_3 + m_2m_3 + n_2n_3 = 0 \\ l_1l_3 + m_1m_3 + n_1n_3 = 0 \end{aligned}\right\} \quad \text{---(A)}$$

And,

$$\left.\begin{aligned} l_1^2 + m_1^2 + n_1^2 = 1 \\ l_2^2 + m_2^2 + n_2^2 = 1 \\ l_3^2 + m_3^2 + n_3^2 = 1 \end{aligned}\right\} \quad \text{---(B)}$$

Given,

$$A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}^T$$

$$= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_1 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix}$$

$$= \begin{bmatrix} l_1^2 + m_1^2 + n_1^2 & l_1l_2 + m_1m_2 + n_1n_2 & l_1l_3 + m_1m_3 + n_1n_3 \\ l_1l_2 + m_1m_2 + n_1n_2 & l_2^2 + m_2^2 + n_2^2 & l_2l_3 + m_2m_3 + n_2n_3 \\ l_1l_3 + m_1m_3 + n_1n_3 & l_3l_2 + m_3m_2 + n_3n_2 & l_3^2 + m_3^2 + n_3^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \{\text{Using (A) and (B)}\}$$

$$= I$$

Hence,

$$AA^T = I$$