We have to check the continuity of function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{-h}{|-h|} = \lim_{h\to 0} \frac{-h}{h} = -1$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{h}{|h|} = 1$$

Thus, LHL $\neq$ R.H.L

So, the given function in discontinuous and the discontinuity is of first kind.

Continuity Ex 9.1 Q2

We have, to check the continuity at $x = 3$.

$$\text{L.H.L} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\frac{(3-h)^2-(3-h)-6}{(3-h)-3} = \lim_{h\to 0}\frac{h^2-5h}{-h} = \lim_{h\to 0} -h+5 = 5$$

$$\text{R.H.L} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\frac{(3+h)^2-(3+h)-6}{(3+h)-3} = \lim_{h\to 0}\frac{h^2+5h}{h} = \lim_{h\to 0} h+5 = 5$$

$f(3) = 5$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 5$

So, The function is continuous at $x = 3$

Continuity Ex 9.1 Q3

We have, to check the continuity of the function at $x = 3$.

$$\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0}\frac{(3-h)^2-9}{(3-h)-3} = \lim_{h\to 0}\frac{h^2-6h}{-h} = \lim_{h\to 0} -h+6 = 6$$

$$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0}\frac{(3+h)^2-9}{(3+h)-3} = \lim_{h\to 0}\frac{h^2+6h}{h} = \lim_{h\to 0} h+6 = 6$$

$f(3) = 6$

Thus, we have, $\text{LHL} = \text{RHL} = f(3) = 6$

So, the given function is continuous at $x = 3$.

Continuity Ex 9.1 Q4

We want, to check the continuity of the function at $x = 1$.

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0}\frac{(1-h)^2-1}{(1-h)-1} = \lim_{h\to 0}\frac{h^2-2h}{-h} = \lim_{h\to 0} -h+2 = 2$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0}\frac{(1+h)^2-1}{(1+h)-1} = \lim_{h\to 0}\frac{h^2+2h}{h} = \lim_{h\to 0} h+2 = 2$$

$f(1) = 2$

we find that $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, $f(x)$ is continuous at $x = 1$.

Continuity Ex 9.1 Q5

We have, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 3(-h)}{-h} = \lim_{h\to 0} \frac{-\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\sin 3h}{h} = 3$$

$f(0) = 1$

$\text{LHL} = \text{RHL} \neq f(0)$

$\Rightarrow$ Function is discontinuous at $x = 0$. It is removable discontinuty.

Continuity Ex 9.1 Q6

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} e^{1/-h} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} e^{1/h} = e^{\infty} = \infty$$

So, LHL $\neq$ RHL

Hence, the function is discontinuous at $x = 0$. This is discontinuty of I[st] kind.

Continuity Ex 9.1 Q7

We want, to check the continuity of the given function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{1-\cos(-h)}{(-h)^2}$$

$$= \lim_{h\to 0}\frac{1-\cos h}{h^2} \quad [\because \cos(-\theta) = \cos\theta]$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2} \quad \left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$$

$$= \lim_{h\to 0} 2.\left(\frac{\sin\frac{h}{2}}{h}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{1-\cos h}{h^2} = \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2} = \lim_{h\to 0} 2\left(\frac{\sin^2\frac{h}{2}}{h}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$f(0) = 1$

$\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is discontinuous at $x = 0$

This is removable discontinuty.

Continuity Ex 9.1 Q8

We want, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h - |-h|}{2} = \lim_{h \to 0} \frac{-h-h}{2} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h - (|h|)}{2} = 0$$

$$f(0) = 2$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, The function is discontinuous at $x = 0$

This is removable discontinuty.

Continuity Ex 9.1 Q9

We want, to check the continuty of the function at $x = a$.

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h) = \lim_{h \to 0} \frac{|a-h-a|}{a-h-a} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h) = \lim_{h \to 0} \frac{|a+h-a|}{a+h-a} = \lim_{h \to 0} \frac{h}{h} = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, function is discontinuous at $x = a$. And the discontinuty is of first kind.

Continuity Ex 9.1 Q10(i)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} |-h| \cos\left(\frac{1}{-h}\right) = \lim_{h \to 0} h \cos\left(\frac{1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} |h| \cos\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(ii)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} h^2 \sin\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, LHL = RHL = $f(0) = 0$

Hence, the function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(iii)

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h) = \lim_{h\to 0} (a-h-a)\sin\left(\frac{1}{a-h-a}\right) = \lim_{h\to 0} -h\sin\left(\frac{-1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h) = \lim_{h\to 0} (a+h-a)\sin\left(\frac{1}{a+h-a}\right) = \lim_{h\to 0} h\sin\left(\frac{1}{h}\right) = 0$$

$$f(a) = 0$$

Thus, LHL $\neq$ RHL = $f(a) = 0$

Hence, the function is continuous at $x = a$.

Continuity Ex 9.1 Q10(iv)

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{e^{-h}-1}{\log(1+2(-h))} = \lim_{h\to 0} \frac{e^{-h}-1}{\log(1-2h)} = DNE$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{e^{h}-1}{\log(1+2h)} = DNE$$

Thus, Both LHL and RHL do not exist

$\therefore$ Function is discontinuous and the discontinutiy is of II^{nd} kind.

Continuity Ex 9.1 Q10(v)

We want, to check the continuity at $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{1-(1-h)^n}{1-(1-h)} = \lim_{h \to 0} \frac{1-\left[1-nh+\frac{n(n-1)}{2!}h^2+\dots\right]}{h}$$

$$= \lim_{h \to 0} n - \frac{n(n-1)}{2!}h + \dots$$

$$= n$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{1-(1+h)^n}{1-(1+h)} = \lim_{h \to 0} \frac{1-\left[1+nh+\frac{n(n-1)}{2!}h^2+\dots\right]}{-h}$$

$$= \lim_{h \to 0} n + \frac{n(n-1)}{2!}h + \dots$$

$$= n$$

$f(1) = n - 1$

Thus, LHL = RHL $\neq f(1)$
Hence, function is discontinuous at $x = 1$
This is removable discontinuity.

Continuity Ex 9.1 Q10(vi)

We want, to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{\left|(1-h)^2-1\right|}{(1-h)-1} = \lim_{h \to 0} \frac{\left|h^2-2h\right|}{-h} = \lim_{h \to 0} -(h-2) = 2$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{\left|(1+h)^2-1\right|}{1+h-1} = \lim_{h \to 0} \frac{h^2+2h}{h} = 2$$

$f(1) = 2$

$\therefore$ LHL = RHL = $f(1) = 2$

Hence, function is continuous.

Continuity Ex 9.1 Q10(vii)

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{2(|-h|)+(-h)^2}{-h} = \lim_{h \to 0} \frac{2h+h^2}{-h} = -2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2 \times |h| + h^2}{h} = 2$$

Thus, LHL $\neq$ RHL
Function is not continuous at $x = 0$
This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q11

We want to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} 1 + (1-h)^2 = \lim_{h\to 0} 1 + 1 - 2h + h^2 = 2$$

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} 2 - (1+h) = 1$$

$\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 1$

This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q12

We want to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin(3\times(-h))}{\tan(2\times(-h))} = \lim_{h\to 0} \frac{-\sin 3h}{-\tan 2h} = \lim_{h\to 0} \frac{\frac{\sin 3h}{3h}\times 3h}{\frac{\tan 2h}{2h}\times 2h} = \frac{3}{2}$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{\log(1+3h)}{e^{2h}-1} = \lim_{h\to 0} \frac{\frac{\log(1+3h)}{3h}\times 3h}{\frac{e^{2h}-1}{2h}\times 2h} = \frac{3}{2}$$

$f(0) = \frac{3}{2}$

Thus, $\text{LHL} = \text{RHL} = f(0) = \frac{3}{2}$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q13

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 2(-h) - |-h| = \lim_{h\to 0} -2h - h = 0$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 2h - |h| = 0$$

$f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x = 0$

Continuity Ex 9.1 Q14

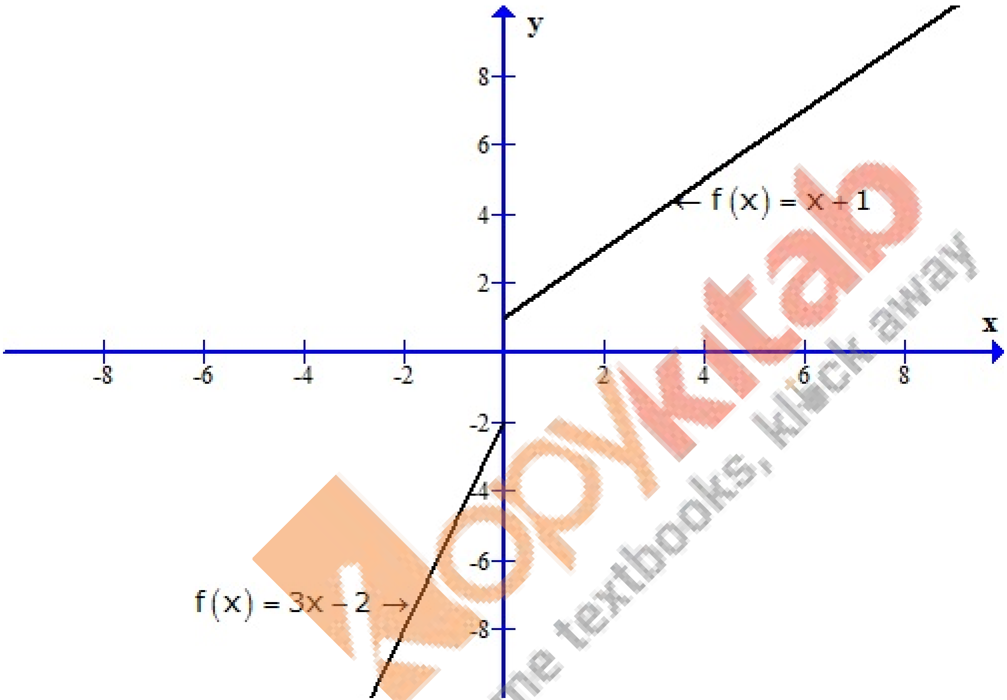
We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} 3(-h) - 2 = \lim_{h \to 0} -3h - 2 = -2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h + 1 = 1 = 0$$

LHL $\neq$ RHL

So, the function is discontinuous



Continuity Ex 9.1 Q15

We want to discuss the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(-h) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h = 0$$

$$f(0) = 1$$

Thus, LHL = RHL $\neq f(0)$

Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

Continuity Ex 9.1 Q16

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

LHL $= \lim_{x \to \frac{1}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} - h\right) = \lim_{h \to 0} \frac{1}{2} - h = \frac{1}{2}$

RHL $= \lim_{x \to \left(\frac{1}{2}\right)^+} = \lim_{h \to 0} f\left(\frac{1}{2} + h\right) = \lim_{h \to 0} 1 - \left(\frac{1}{2} + h\right) = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{1}{2}$

Thus, LHL = RHL $= f\left(\frac{1}{2}\right) = \frac{1}{2}$

Hence, the function is continuous at $x = \frac{1}{2}$.

Continuity Ex 9.1 Q17

We want to check the continuity of the function at $x = 0$.

LHL $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} 2(-h) - 1 = -1$

RHL $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} 2h + 1 = 1$

Thus, LHL $\neq$ RHL

Hence, the function is discontinuous at $x = 0$. This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q18

We have given that the function is continuous at $x = 1$

LHL = RHL $= f(1)$ ..... (1)

Now, LHL $= \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h) = \lim_{h \to 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h \to 0} \frac{h^2 - 2h}{-h} = 2$

$f(1) = k$

From (1), LHL $= f(1)$

$\therefore 2 = k$

Continuity Ex 9.1 Q19

We have that the function is continuous at $x = 1$

$\therefore$ LHL = RHL $= f(1)$ .... (1)

Now,

LHL $= \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h) = \lim_{h \to 0} \frac{(1-h)^2 - 3(1-h) + 2}{(1-h) - 1} = \lim_{h \to 0} \frac{h^2 + h}{-h} \quad = \lim_{h \to 0} -h - 1 = -1$

$f(1) = k$

From (1), we get,

$k = -1$

Continuity Ex 9.1 Q20

We know that a function is continuous at 0 if

LHL = RHL = $f(0)$ ....(1)

Now,

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 5(-h)}{3(-h)} = \lim_{h\to 0} \frac{-\sin 5h}{-3h} = \lim_{h\to 0} \frac{\sin 5h}{5h} \times \frac{5h}{3h} = \frac{5}{3}$$

$f(0) = k$

Thus, from (1),

$k = \frac{5}{3}$

Continuity Ex 9.1 Q21

The given function is $f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$

The given function f is continuous at $x = 2$, if f is defined at $x = 2$ and if the value of f at $x = 2$ equals the limit of f at $x = 2$

It is evident that f is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\Rightarrow \lim_{x\to 2^-} (kx^2) = \lim_{x\to 2^+} (3) = 4k$

$\Rightarrow k \times 2^2 = 3 = 4k$

$\Rightarrow 4k = 3 = 4k$

$\Rightarrow 4k = 3$

$\Rightarrow k = \frac{3}{4}$

Therefore, the required value of k is $\frac{3}{4}$.

Continuity Ex 9.1 Q22

We have given that the function is continuous at $x = 0$

So, LHL = RHL = $f(0)$.....(1)

Now,

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 2(-h)}{5(-h)} = \lim_{h\to 0} \frac{-\sin 2h}{-5h} = \lim_{h\to 0} \frac{\sin 2h}{2h} \times \frac{2h}{5h} = \frac{2}{5}$$

$f(0) = k$

Using (1), $k = \frac{2}{5}$

Continuity Ex 9.1 Q23

We have given that the function is continuous at $x = 2$

LHL = RHL = $f(2)$.....(1)

Now,

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} a(2-h) + 5 = 2a + 5$$

$$f(2) = 2a + 5$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} 2 + h - 1 = 1$$

$\therefore$ Using (1),

$2a + 5 = 1 \Rightarrow a = -2$

Continuity Ex 9.1 Q24

We have, at $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h}{|-h| + 2(-h)^2} = \lim_{h \to 0} \frac{-h}{h + 2h^2} = \lim_{h \to 0} \frac{-1}{1 + 2h} = -1$$

$$f(0) = k$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h}{|h| + 2h^2} = \lim_{h \to 0} \frac{1}{1 + 2h} = 1$$

Since, LHL $\neq$ RHL, function will remain discontinuous at $x = 0$, regardless the choice of k.

Continuity Ex 9.1 Q25

Since $f(x)$ is continuous at $x = \frac{\pi}{2}$, L.H.Limit = R.H.Limit.

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at $x = 0$

$\text{LHL} = \text{RHL} = f(0) \ldots (1)$

$f(0) = c$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin(ah + h) - \sin h}{-h}$$

$$= \lim_{h \to 0} \frac{\sin(a+h)h}{h} + \lim_{h \to 0} \frac{\sin h}{h}$$

$$= a + 1 + 1 = a + 2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sqrt{h + bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}}$$

$$= \lim_{h \to 0} \frac{\sqrt{h + bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h + bh^2} + \sqrt{h}}{\sqrt{h + bh^2} + \sqrt{h}}$$

$$= \lim_{h \to 0} \frac{h + bh^2 - h}{bh^{\frac{3}{2}}\left(\sqrt{h + bh^2} + \sqrt{h}\right)} = \lim_{h \to 0} \frac{bh^2}{bh^2\left(\sqrt{1 + bh} + 1\right)} = \frac{1}{2}$$

$\therefore$ from (1),

$a + 2 = \frac{1}{2} \Rightarrow a = \frac{-3}{2}$

$c = \frac{1}{2}$ and

$b \in R - \{0\}$

Hence, $a = \frac{-3}{2}$, $b \in R - \{0\}$, $c = \frac{1}{2}$

Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)\ldots\ldots(1)$

$$f(0) = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1-\cos k(-h)}{-h\sin(-h)} = \lim_{h \to 0} \frac{1-\cos kh}{+h\sinh}$$

$$= \lim_{h \to 0} \frac{2\sin^2\frac{kh}{2}}{h.2\sin\frac{h}{2}.\cos\frac{h}{2}}$$

$$= \lim_{h \to 0} \left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \times \frac{\frac{k^2h^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{h}{2}} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2}}$$

$$= \frac{k^2}{2}$$

$\therefore$ Using (1) we get,

$$\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$\therefore$ LHL = RHL = $f(4)\ldots.(1)$

$f(4) = a + b\ldots..(A)$

$$\text{LHL} = \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0} \frac{(4-h)-4}{|(4-h)-4|} + a = \lim_{h\to 0} \frac{-h}{h} + a = a - 1 \qquad \ldots.(B)$$

$$\text{RHL} = \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} \frac{(4+h)-4}{|(4+h)-4|} + b = \lim_{h\to 0} \frac{h}{h} + b = b + 1 \qquad \ldots.(C)$$

$\therefore$ from (1)

$a - 1 = b + 1 \Rightarrow a - b = 2 \;\; \ldots..(D)$

from (A) and (B)

$a + b = a - 1 \Rightarrow b = -1$

from (A) and (C)

$a + b = b + 1 \Rightarrow a = 1$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)\ldots..(1)$

$f(0) = k$

$$\text{LHL} = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin 2(0-h)}{-h} = \lim_{h\to 0} \frac{-\sin 2h}{-h} = 2$$

$\therefore$ using (1), we get $k = 2$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

LHL = RHL = $f(0)\ldots.(1)$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\log\left(1-\frac{h}{a}\right) - \log\left(1+\frac{h}{b}\right)}{(-h)} = \lim_{h\to 0} \frac{\log\left(1+\left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right)\times a} + \frac{\log\left(1+\frac{h}{b}\right)}{h}$$

$$= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

from (1),

$$f(0) = \frac{a+b}{ab}$$

Continuity Ex 9.1 Q31

We are given that the function is continuous at $x = 2$

$\therefore$ LHL = RHL = $f(2)$ .....(1)

Now,

$f(2) = k$ ....(A)

$$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0}\frac{2^{(2-h)+2}-16}{4^{(2-h)}-16} = \lim_{h\to 0}\frac{2^{4-h}-16}{4^{2-h}-16}$$

$$= \lim_{h\to 0}\frac{2^4.2^{-h}-16}{4^2.4^{-h}-16}$$

$$= \lim_{h\to 0}\frac{16.2^{-h}-16}{16.4^{-h}-16}$$

$$= \lim_{h\to 0}\frac{16\left(2^{-h}-1\right)}{16\left(4^{-h}-1\right)}$$

$$= \lim_{h\to 0}\frac{2^{-h}-1}{\left(2^{-h}\right)-1^2} \quad \left[\because 2^{-2h} = \left(2^{-h}\right)^2 = 4^{-h}\right]$$

$$= \lim_{h\to 0}\frac{2^{-h}-1}{\left(2^{-h}-1\right)\left(2^{-h}+1\right)} = \frac{1}{2} \quad ......(B)$$

$\therefore$ Using (1) from (A) & (B)

$k = \frac{1}{2}$

Continuity Ex 9.1 Q33

We know that a function is said to be continuous at $x = \pi$ if

LHL = RHL = value of the function at $x = \pi$ .....(1)

$$\text{LHL} = \lim_{x\to \pi^-} f(x) = \lim_{h\to 0} f(\pi-h) = \lim_{h\to 0}\frac{1-\cos 7(\pi-h-\pi)}{5\left((\pi-h)-\pi\right)^2} = \lim_{h\to 0}\frac{1-\cos 7h}{5h^2}$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{7}{2}h}{5h^2}$$

$$= \lim_{h\to 0}\frac{2}{5}\left(\frac{\sin\frac{7}{2}h}{\frac{7}{2}h}\right)^2\times\left(\frac{7}{2}\right)^2$$

$$= \frac{2}{5}\times\frac{49}{4} = \frac{49}{10} \quad ....(B)$$

Thus, using (1) we get,

$f(\pi) = \frac{49}{10}$

Continuity Ex 9.1 Q34

It is given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)\ldots.(1)$

$$\text{LHL} = \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h) = \lim_{h\to0} \frac{2(-h)+3\sin(-h)}{3(-h)+2\sin(-h)} = \lim_{h\to0} \frac{-2h-3\sin h}{-3h-2\sin h}$$

$$= \lim_{h\to0} \frac{\frac{2h+3\sin h}{h}}{\frac{3h+2\sin h}{h}}$$

$$= \lim_{h\to0} \frac{2+3\frac{\sin h}{h}}{3+2\frac{\sin h}{h}} = \frac{2+3}{3+2} = 1 \quad \left[\because \lim_{\theta\to0} \frac{\sin\theta}{\theta} = 1\right]$$

Using (1) we get,

$f(0) = 1$

Continuity Ex 9.1 Q35

It is given that the function is continuous at $x = 0$.

LHL = RHL = $f(0)\ldots.(1)$

$f(0) = k \ldots.(A)$

$$\text{LHL} = \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h) = \lim_{h\to0} \frac{1-\cos 4(-h)}{8(-h)^2} = \lim_{h\to0} \frac{1-\cos 4h}{8h^2} = \lim_{h\to0} \frac{2\sin^2 2h}{8h^2} = \lim_{h\to0} \left(\frac{\sin 2h}{2h}\right)^2 = 1$$

Thus, using (1) we get,

$k = 1$

Continuity Ex 9.1 Q36

The given function will be continuous at $x = 0$ if

LHL = RHL = $f(0)$....(1)

$f(0) = 8$.....(A)

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1-\cos 2k(-h)}{(-h)^2} = \lim_{h \to 0} \frac{1-\cos 2kh}{h^2} = \lim_{h \to 0} \frac{2\sin^2 kh}{h^2}$$

$$= \lim_{h \to 0} 2\left(\frac{\sin kh}{kh}\right)^2 .k^2$$

$$= 2k^2$$

Thus, using (1) we get,

$2k^2 = 8 \quad \Rightarrow k^2 = 4 \quad \Rightarrow k = \pm 2$

Hence, $k = \pm 2$

Let $x - 1 = y$

$\Rightarrow x = y + 1$

Thus,

$$\lim_{x \to 1} (x-1)\tan\frac{\pi x}{2} = \lim_{y \to 0} y\tan\frac{\pi(y+1)}{2}$$

$$= \lim_{y \to 0} y\tan\left(\frac{\pi y}{2} + \frac{\pi}{2}\right)$$

$$= -\lim_{y \to 0} y\cot\frac{\pi y}{2}$$

$$= -\lim_{y \to 0} y\frac{\cos\frac{\pi y}{2}}{\sin\frac{\pi y}{2}}$$

$$= -\lim_{y \to 0} y\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi}{2}}}$$

$$= -\lim_{y \to 0} \frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi y}{2}}}$$

$$= -\lim_{y \to 0} \frac{2}{\pi} \frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)}{\frac{\pi y}{2}}}$$

$$= -\frac{2}{\pi}\lim_{y \to 0} \cos\frac{\pi y}{2}$$

$$= -\frac{2}{\pi}$$

Since the function is continuous, *L.H.Limit* = *R.H.Limit*

Thus, $k = -\frac{2}{\pi}$

Since the function is continuous at every point, therefore

$$LHL = RHL = f(0)$$

Now

$$f(0) = \cos 0$$
$$= 1$$

Again

$$LHL = \lim_{x\to 0} k(x^2 - 2x)$$
$$= \lim_{h\to 0^+} k(h^2 - 2h)$$
$$= 0$$

Therefore there is no value of k

Since the function is continuous at every point, therefore

$$LHL = RHL = f(\pi)$$

Now

$$f(\pi) = k\pi + 1$$

Again

$$RHL = \lim_{x\to \pi^+} \cos x$$
$$= \lim_{h\to 0^+} \cos(\pi - h)$$
$$= -\lim_{h\to 0^+} \cosh$$
$$= -1$$

Therefore we can write

$$k\pi + 1 = -1$$
$$k = -\frac{2}{\pi}$$

We are given that function is continuous at $x = 5$.

$\therefore$ LHL = RHL = $f(5)$.... (1)

$f(5) = 5k + 1$

LHL = $\lim_{x\to 5^+} f(x) = \lim_{h\to 0} f(5+h) = \lim_{h\to 0} 3(5+h) - 5 = 10$

Thus, using (1), we get,

$5k + 1 = 10$

$5k = 9$

$k = \frac{9}{5}$

We know that the function will be continuous at $x = 5$. if

LHL = RHL = $f(5)$ ..... (1)

$f(5) = k$

$$\text{LHL} = \lim_{x\to 5^-} f(x) = \lim_{h\to 0} f(5-h) = \lim_{h\to 0} \frac{(5-h)^2 - 25}{(5-h)-5} = \lim_{h\to 0} \frac{h^2 - 10h}{-h} = \lim_{h\to 0} -h + 10 = 10$$

Thus, using (1), we get,

$k = 10$

We know that a function will be continuous at $x = 1$. if

LHL = RHL = $f(1)$ .... (1)

$f(1) = k.1^2 = k$

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} 4 = 4$$

Thus, using (1), we get,

$k = 4$

We know that a function will be continuous at $x = 0$. if

LHL = RHL = $f(0)$ ..... (1)

$f(0) = k(0+2) = 2k$

$$\text{LHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 3(h) + 1 = 1$$

Thus, using (1), we get,

$2k = 1$

$k = \frac{1}{2}$

Continuity Ex 9.1 Q37

It is given that the function is continuous at $x = 3$ and at $x = 5$

$\therefore$ LHL = RHL = $f(3)$.....(1) and
LHL = RHL = $f(5)$.....(2)

Now,
$f(3) = 1$
RHL = $\lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3+h) = \lim_{h \to 0} a(3+h) + b = 3a + b$

Thus, using (1), we get,
$3a + b = 1$....(3)

$f(5) = 7$
LHL = $\lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h) = \lim_{h \to 0} a(5-h) + b = 5a + b$

Thus, using (2), we get
$5a + b = 7$ .....(4)

Now, solving (3) and (4) we get,
$a = 3$ and $b = -8$

Continuity Ex 9.1 Q38

We want to discuss the continuity of the function at $x = 1$

We need to prove that

LHL = RHL = $f(1)$

$$f(1) = \frac{1^2}{2} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{(1-h)^2}{2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} 2(1+h)^2 - 3(1+h) + \frac{3}{2} = 2 - 3 + \frac{3}{2} = \frac{1}{2}$$

Thus, LHL = RHL = $f(1) = \frac{1}{2}$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q39

We want to discuss the continuity at $x = 0$ and $x = 1$

Now,

$f(0) = 1$

$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} |-h| + |-h - 1| = 1.$

$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} |h| + |h - 1| = 1$

$\because$ LHL = RHL = $f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$f(1) = 1$

$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h) = \lim_{h \to 0} |1 - h| + |1 - h - 1| = 1$

$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h) = \lim_{h \to 0} |1 + h| + |1 + h - 1| = 1$

$\because$ LHL = RHL = $f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$f(-1) = |-1 - 1| + |-1 + 1| = 2$

$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(-1 - h) = \lim_{h \to 0} |-1 - h - 1| + |-1 - h + 1| = 2$

$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(-1 + h) = \lim_{h \to 0} |-1 + h - 1| + |-1 + h + 1| = 2$

Thus, LHL = RHL = $f(-1) = 2$

Hence, function is continuous at $x = -1$

For $x = 1$

$f(1) = |1 - 1| + |1 + 1| = 2$

$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h) = \lim_{h \to 0} |1 - h - 1| + |1 - h + 1| = 2$

$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h) = \lim_{h \to 0} |1 + h - 1| + |1 + h + 1| = 2$

Thus, LHL = RHL = $f(1) = 2$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q40

Since f(x) is continuous at x = 0, L.H.Limit = R.H.Limit.

Thus, we have

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \lim_{x \to 0^-} a\sin\frac{\pi}{2}(x+1) = \lim_{x \to 0^+} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a \times 1 = \lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\frac{\sin x}{x}\left(\frac{1}{\cos x} - 1\right)}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\frac{\sin x}{x}\left(\frac{1-\cos x}{\cos x}\right)}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{1}{\cos x} \times \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$\Rightarrow a = 1 \times 1 \times \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \lim_{x \to 0} \frac{1}{1+\cos x}$$

$$\Rightarrow a = 1 \times \lim_{x \to 0} \frac{1}{1+\cos x}$$

$$\Rightarrow a = 1 \times \frac{1}{1+1}$$

$$\Rightarrow a = \frac{1}{2}$$

Continuity Ex 9.1 Q41

It is given that function is continuous at $x = 0$. then,

LHL = RHL = $f(0)$ .... (1)

Now,

$f(0) = 2 \cdot 0 + k = k$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -2(-h)^2 + k = k$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} 2(h^2) + k = k$

Thus, the function will be continuous for any $k \in R$.

Continuity Ex 9.1 Q42

The given function f is $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \le 0 \\ 4x+1, & \text{if } x > 0 \end{cases}$

If f is continuous at $x = 0$, then

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0^-} \lambda(x^2 - 2x) = \lim_{x \to 0^+} (4x+1) = \lambda(0^2 - 2 \times 0)$

$\Rightarrow \lambda(0^2 - 2 \times 0) = 4 \times 0 + 1 = 0$

$\Rightarrow 0 = 1 = 0$, which is not possible

Therefore, there is no value of λ for which f is continuous at $x = 0$

At $x = 1$,

$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$

$\lim_{x \to 1} (4x+1) = 4 \times 1 + 1 = 5$

$\therefore \lim_{x \to 1} f(x) = f(1)$

Therefore, for any values of λ, f is continuous at $x = 1$

Continuity Ex 9.1 Q43

The function will be continuous at $x = 2$

if LHL = RHL = $f(2)$ ....(1)

Now,

$f(2) = k$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} 2(2-h) + 1 = 5.$$

Thus, using (1) we get,

$k = 5$

Continuity Ex 9.1 Q44

It is given that the function is continuous at $x = \frac{\pi}{2}$

$\therefore$ LHL = RHL = $f\left(\frac{\pi}{2}\right)$ .....(1)

Now,

$f\left(\frac{\pi}{2}\right) = a$

$$\text{LHL} = \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \to 0} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{1 - \cos^3 h}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{(1 - \cos h)(1 + \cos^2 h + \cos h)}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}(1 + \cos^2 h + \cos h)}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{2\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h^2}{4}(1 + \cos^2 h + \cos h)}{3\left(\frac{\sin h}{h}\right)^2 .h^2}$$

$$= \lim_{h \to 0} \frac{2.\frac{1}{4}(1 + \cos^2 h + \cos h)}{3} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \to 0} \frac{b\left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} = \lim_{h \to 0} \frac{b(1 - \cos h)}{(\pi - \pi - 2h)^2}$$

$$= \lim_{h \to 0} \frac{b.2\sin^2 \frac{h}{2}}{(2h)^2}$$

$$= \lim_{h \to 0} \frac{b}{2}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4}$$

$$= \lim_{h \to 0} \frac{b}{8} = \frac{b}{8}$$

Thus, using (1) we get,

$a = \frac{1}{2}$

And

$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$

Thus, $a = \frac{1}{2}$ and $b = 4$

Continuity Ex 9.1 Q45

It is given that the function is continuous at $x = 0$, then

LHL = RHL = $f(0) \ldots\ldots (1)$

Now,

$f(0) = k$

$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h}{|h|} = 1 \ldots\ldots (B)$

Thus, using (1) we get,

$k = 1$

Continuity Ex 9.1 Q46

Since the function is continuous at $x = 3$, therefore

$$LHL = RHL = f(3)$$

Now

$$RHL = \lim_{x \to 3^+} f(x)$$
$$= \lim_{h \to 0} f(3+h)$$
$$= \lim_{h \to 0} b(3+h) + 3$$
$$= \lim_{h \to 0} 3b + 3h + 3$$
$$= 3b + 3$$

Again

$$f(3) = a(3) + 1$$
$$= 3a + 1$$

Thus we can write

$$f(3) = RHL$$
$$3a + 1 = 3b + 3$$
$$3a - 3b = 2$$