RD Sharma Solutions Class 12 Maths Chapter 3 Ex 3.4

Binary Operations Ex 3.4 Q1

Given,

$$a * b = a + b - 4 \text{ for all } a, b \in Z$$

(i)

Commutative: Let $a, b \in Z$, then

$\Rightarrow$ $a * b = a + b - 4 = b + a - 4 = b * a$

$\Rightarrow$ $a * b = b * a$

So, '$*$' is commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (a + b - 4) * c = a + b - 4 + c - 4$$

$$= a + b + c - 8 \qquad ---(i)$$

and, $a * (b * c) = a * (b + c - 4) = a + b + c - 8 \qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Z.

(ii)

Let $e \in Z$ be the identity element with respect to $*$.

By identity property, we have

$$a * e = e * a = a \text{ for all } a \in Z$$

$\Rightarrow$ $a + e - 4 = a$

$\Rightarrow$ $e = 4$

So, $e = 4$ will be the identity element with respect to $*$

(iii)

Let $b \in Z$ be the inverse element of $a \in Z$

Then, $a * b = b * a = e$

$\Rightarrow$ $a + b - 4 = e$

$\Rightarrow$ $a + b - 4 = 4 \qquad [\because e = 4]$

$\Rightarrow$ $b = 8 - a$

Thus, $b = 8 - a$ will be the inverse element of $a \in Z$.

Binary Operations Ex 3.4 Q2

We have,

$$a * b = \frac{3ab}{5} \text{ for all } a, b \in Q_0$$

(i)

Commutative: Let $a, b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$\Rightarrow$ $a * b = b * a$

So, '$*$' is commutative on Q_0

Associativity: Let $a, b, c \in Q_0$, then

$$(a * b) * c = \frac{3ab}{5} * c$$

$$= \frac{9abc}{25} \quad ---(i)$$

and, $a * (b * c) = a * \frac{3bc}{5}$

$$= \frac{9abc}{25} \quad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$, then

$a * e = e * a = a$ for all $a \in Q_0$

$\Rightarrow$ $\frac{3ae}{5} = a$

$\Rightarrow$ $e = \frac{5}{3}$

will be the identity element with respect to $*$.

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$a * b = b * a = e$

$\Rightarrow$ $\frac{3}{5} ab = e$

$\Rightarrow$ $\frac{3}{5} ab = \frac{5}{3}$ $\left[\because e = \frac{5}{3}\right]$

$\Rightarrow$ $b = \frac{25}{9a}$

$\therefore$ $b = \frac{25}{9a}$ is the inverse of $a \in Q_0$.

Binary Operations Ex 3.4 Q3

We have,

$a * b = a + b + ab$ for all $a, b \in Q - \{-1\}$

(i)

Commutativity: Let $a, b \in Q - \{-1\}$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

$\Rightarrow \quad$ '$*$' is commutative on $Q - \{-1\}$

Associativity: Let $a, b, c \in Q - \{-1\}$, then

$\Rightarrow \quad (a * b) * c = (a + b + ab) * c$

$\qquad = a + b + ab + c + ac + bc + abc \qquad ---(i)$

and, $\quad a * (b * c) = a * (b + c + bc)$

$\qquad = a + b + c + bc + ab + ac + abc \qquad ---(ii)$

From (i) & (ii)

$(a * b) * c = a * (b * c)$

$\Rightarrow \quad *$ is associative on $Q - \{-1\}$

(ii)

Let e be identity element with respect to $*$.

By identity property,

$a * e = a = e * a$ for all $a \in Q - \{-1\}$

$\Rightarrow \quad a + e + ae = a$

$\Rightarrow \quad e(1 + a) = 0 \Rightarrow e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$

$\therefore \quad e = 0$ is the identity element with respect to $*$

(iii)

Let b be the inverse of $a \in Q - \{-1\}$

Then, $\quad a * b = b * a = e \qquad$ [e is the identity element]

$\Rightarrow \quad a + b + ab = e$

$\Rightarrow \quad a + b + ab = 0$

$\Rightarrow \quad b(1 + a) = -a$

$\Rightarrow \quad b = \frac{-a}{1 + a} \qquad \left[\because \frac{-a}{1+a} \neq -1, \text{ because if } \frac{-a}{1+a} = -1 \Rightarrow a = 1 + a \Rightarrow 1 = 0 \text{ Not possible}\right]$

$\therefore \quad b = \frac{-a}{1 + a}$ is the inverse of a with respect to $*$

Binary Operations Ex 3.4 Q4

We have,

$$(a,b)\odot(c,d)=(ac,bc+d) \text{ for all } (a,b),(c,d)\in R_0\times R$$

(i)

Commutativity: Let $(a,b),(c,d)\in R_0\times R$, then

$\Rightarrow \quad (a,b)\odot(c,d)=(ac,bc+d) \qquad ---(i)$

and, $\quad (c,d)\odot(a,b)=(ca,da+b) \qquad ---(ii)$

From (i) & (ii)

$$(a,b)\odot(c,d)\neq(c,d)\odot(a,b)$$

$\Rightarrow \quad$ '$\odot$' is not commutative on $R_0\times R$.

Associativity: Let $(a,b),(c,d),(e,f)\in R_0\times R$, then

$\Rightarrow \quad ((a,b)\odot(c,d))\odot(e,f)=(ac,bc+d)\odot(e,f)$

$= (ace,bce,de+f) \qquad ---(i)$

and, $\quad (a,b)\odot(c,d\odot(e,f))=(a,b)\odot(ce,de+f)$

$= (ace,bce+de+f) \qquad ---(ii)$

$\Rightarrow \quad ((a,b) \odot (c,d)) \odot (e,f) = (a,b) \odot ((c,d) \odot (e,f))$

$\Rightarrow \quad$ '$\odot$' is associative on $R_0 \times R$.

(ii)

Let $(x,y) \in R_0 \times R$ be the identity element with respect to $\odot$, then

$$(a,b) \odot (x,y) = (x,y) \odot (a,b) = (a,b) \text{ for all } (a,b) \in R_0 \times R$$

$\Rightarrow \quad (ax, bx + y) = (a,b)$

$\Rightarrow \quad ax = a$ and $bx + y = b$

$\Rightarrow \quad x = 1,$ and $y = 0$

$\therefore \quad (1,0)$ will be the identity element with respect to $\odot$.

(iii)

Let $(c,d) \in R_0 \times R$ be the inverse of $(a,b) \in R_0 \times R$, then

$$(a,b) \odot (c,d) = (c,d) \odot (a,b) = e$$

$\Rightarrow \quad (ac, bc + d) = (1,0) \qquad [\because e = (1,0)]$

$\Rightarrow \quad ac = 1$ and $bc + d = 0$

$\Rightarrow \quad c = \frac{1}{a}$ and $d = -\frac{b}{a}$

$\therefore \quad \left(\frac{1}{a}, -\frac{b}{a}\right)$ will be the inverse of (a,b).

Binary Operations Ex 3.4 Q5

We have,

$$a * b = \frac{ab}{2} \text{ for all } a, b \in Q_0$$

(i)

Commutativity: Let $a, b \in Q_0$, then

$$\Rightarrow \quad a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$$

$$\Rightarrow \quad a * b = b * a$$

Hence, '*' is commutative on Q_0.

Associativity: Let $a, b, c \in Q_0$, then

$$\Rightarrow \quad (a * b) * c = \frac{ab}{2} * c = \frac{abc}{4} \qquad ---(i)$$

$$\text{and,} \quad a * (b * c) = a * \frac{bc}{2} = \frac{abc}{4} \qquad ---(ii)$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on Q_0.

(ii)

Let $e \in Q_0$ be the identity element with respect to $*$.

By identity property, we have,

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ae}{2} = a \quad \Rightarrow e = 2$$

Thus, the required identity element is 2.

(iii)

Let $b \in Q_0$ be the inverse of $a \in Q_0$ with respect to $*$, then,

$$a * b = b * a = e \text{ for all } a \in Q_0$$

$$\Rightarrow \quad \frac{ab}{2} = e \quad \Rightarrow \frac{ab}{2} = 2$$

$$\Rightarrow b = \frac{4}{a}$$

Thus, $b = \frac{4}{a}$ is the inverse of a with respect to $*$.

Binary Operations Ex 3.4 Q6

We have,

$$a * b = a + b - ab \text{ for all } a, b \in R - \{+1\}$$

(i)

Commutative: Let $a, b \in R - \{+1\}$, then,

$$\Rightarrow \quad a * b = a + b - ab = b + a - ba = b * a$$
$$\Rightarrow \quad a * b = b * a$$

So, '$*$' is commutative on $R - \{+1\}$.

Associativity: Let $a, b, c \in R - \{+1\}$, then

$$(a * b) * c = (a + b - ab) * c$$
$$= a + b - ab + c - ac - bc + abc$$
$$= a + b + c - ab - ac - bc + abc \qquad \text{---(i)}$$

and, $$a * (b * c) = a * (b + c - bc)$$
$$= a + b + c - bc - ab - ac + abc \qquad \text{---(ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '$*$' is associative on $R - \{+1\}$.

(ii)

Let $e \in R - \{+1\}$ be the identity element with respect to $*$, then

$$a * e = e * a = a \text{ for all } a \in R - \{+1\}$$

$$\Rightarrow \quad a + e - ae = a$$
$$\Rightarrow \quad e(1 - a) = 0$$
$$\Rightarrow \quad e = 0 \qquad [\because a \neq 1 \Rightarrow 1 - a \neq 0]$$

$\therefore$ $e = 0$ will be the identity element with respect to $*$.

(iii)

Let $b \in R - \{1\}$ be the inverse element of $a \in R - \{1\}$, then

$$a * b = b * a = e$$

$$\Rightarrow \quad a + b - ab = 0 \qquad [\because e = 0]$$
$$\Rightarrow \quad b(1 - a) = -a$$
$$\Rightarrow \quad b = \frac{-a}{1 - a} \neq 1 \qquad \left[\begin{array}{l} \because \text{if } \frac{-a}{1-a} = 1 \\ \Rightarrow -a = 1 - a \quad \Rightarrow 1 = 0 \\ \quad \text{Not possible} \end{array}\right]$$

$\therefore$ $b = \frac{-a}{1 - a}$ is the inverse of $a \in R - \{1\}$ with respect to $*$.

Binary Operations Ex 3.4 Q7

We have,

$$(a,b) * (c,d) = (ac, bd) \text{ for all } (a,b),(c,d) \in A$$

(i)

Let $(a,b),(c,d) \in A$, then

$$(a,b) * (c,d) = (ac, bd)$$
$$= (ca, db) \qquad [\because ac = ca \text{ and } bd = db]$$
$$= (c,d) * (a,b)$$

$\Rightarrow \quad (a,b) * (c,d) = (c,d) * (a,b)$

So, '$*$' is commutative on A

Associativity: Let $(a,b),(c,d),(e,f) \in A$, then

$\Rightarrow \quad ((a,b) * (c,d)) * (e,f) = (ac, bd) * (e,f)$
$$= (ace, bdf) \qquad ---(i)$$

and, $\quad (a,b) * ((c,d) * (e,f)) = (a,b) * (ce, df)$
$$= (ace, bdf) \qquad ---(ii)$$

From (i) & (ii)

$\Rightarrow \quad ((a,b) * (c,d)) * (e,f) = (a,b) * ((c,d) * (e,f))$

So, '$*$' is associative on A.

(ii)

Let $(x,y) \in A$ be the identity element with respect to $*$.

$$(a,b) * (x,y) = (x,y) * (a,b) = (a,b) \text{ for all } (a,b) \in A$$

$\Rightarrow \quad (ax, by) = (a,b)$

$\Rightarrow \quad ax = a$ and $by = b$

$\Rightarrow \quad x = 1$, and $y = 1$

$\therefore \quad (1,1)$ will be the identity element

(iii)

Let $(c,d) \in A$ be the inverse of $(a,b) \in A$, then

$$(a,b) * (c,d) = (c,d) * (a,b) = e$$

$\Rightarrow \quad (ac, bd) = (1,1) \qquad [\because e = (1,1)]$

$\Rightarrow \quad ac = 1$ and $bd = 1$

$\Rightarrow \quad c = \frac{1}{a}$ and $d = \frac{1}{b}$

$\therefore \quad \left(\frac{1}{a}, \frac{1}{b}\right)$ will be the inverse of (a, b) with respect to $*$.

Binary Operations Ex 3.4 Q8

The binary operation * on **N** is defined as:

$a * b$ = H.C.F. of a and b

It is known that:

H.C.F. of a and b = H.C.F. of b and a, $a, b \in$ **N**.

Therefore, $a * b = b * a$

Thus, the operation * is commutative.

For $a, b, c \in$ **N**, we have:

$(a * b) * c$ = (H.C.F. of a and b) $* c$ = H.C.F. of a, b, and c

$a * (b * c) = a *$(H.C.F. of b and c) = H.C.F. of a, b, and c

Therefore, $(a * b) * c = a * (b * c)$

Thus, the operation * is associative.

Now, an element $e \in$ **N** will be the identity for the operation * if $a * e = a = e * a$, $\forall\ a \in$ **N**.

But this relation is not true for any $a \in$ **N**.

Thus, the operation * does not have any identity in **N**.