RD Sharma Solutions Class 12 Maths Chapter 13 Ex 13.1

Derivatives as a Rate Measurer Ex 13.2 Q1

Let total suface area of the cylinder be A

$$A = 2\pi r\left(h + r\right)$$

Differentiating it with respect to r as r varies

$$\frac{dA}{dr} = 2\pi r\left(0 + 1\right) + \left(h + r\right)2\pi$$
$$= 2\pi r + 2\pi h + 2\pi r$$

$$\frac{dA}{dr} = 4\pi r + 2\pi h$$

Derivatives as a Rate Measurer Ex 13.1 Q2

Let D be the diatmeter and r be the radius of sphere,

So, volume of sphere $= \frac{4}{3}\pi r^2$

$$v = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$
$$v = \frac{4}{24}\pi D^3$$

Differentiating it with respect to D.

$$\frac{dv}{dD} = \frac{12}{24}\pi D^2$$

$$\frac{dv}{dD} = \frac{\pi D^2}{2}$$

Derivatives as a Rate Measurer Ex 13.1 Q3

Given, radius of sphere $(r) = 2$cm.

We know that,

$$v = \frac{4}{3}\pi r^2$$
$$\frac{dv}{dr} = 4\pi r^2 \qquad \text{---(i)}$$

And $A = 4\pi r^2$

$$\frac{dA}{dr} = 8\pi r^2 \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dv}{dr}}{\frac{dA}{dr}} = \frac{4\pi r^2}{8\pi r}$$

$$\frac{dv}{dA} = \frac{r}{2}$$
$$\left(\frac{dv}{dA}\right)_{r=2} = 1$$

Derivatives as a Rate Measurer Ex 13.1 Q4

Let r be two radius of circular disc.

We know that,

$$\text{Area } A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r \qquad \text{---(i)}$$

$$\text{Circumference } C = 2\pi r$$

$$\frac{dc}{dr} = 2\pi \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{dA}{dr}}{\frac{dc}{dr}} = \frac{2\pi r}{2\pi}$$

$$\frac{dA}{dc} = r$$

$$\left(\frac{dA}{dc}\right)_{r=3} = 3$$

Derivatives as a Rate Measurer Ex 13.1 Q5

Let r be the radius, v be the volume of cone and h be height

$$v = \frac{1}{3}\pi r^2 h$$

$$\frac{dv}{dr} = \frac{2}{3}\pi rh.$$

Derivatives as a Rate Measurer Ex 13.1 Q6

Let r be radius and A be area of circle, so

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 2\pi (5)$$

$$\left(\frac{dA}{dr}\right)_{r=5} = 10\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q7

Here, $r = 2$ cm

$$v = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 4\pi (2)^2$$

$$\left(\frac{dV}{dr}\right)_{r=2} = 16\pi$$

Derivatives as a Rate Measurer Ex 13.1 Q8

Marginal cost is the rate of change of total cost with respect to output.

$$\therefore \text{Marginal cost (MC)} = \frac{dC}{dx} = 0.007\left(3x^2\right) - 0.003\left(2x\right) + 15$$

$$= 0.021x^2 - 0.006x + 15$$

When $x = 17$, MC $= 0.021\ (17^2) - 0.006\ (17) + 15$

$= 0.021(289) - 0.006(17) + 15$

$= 6.069 - 0.102 + 15$

$= 20.967$

Hence, when 17 units are produced, the marginal cost is Rs. 20.967

Derivatives as a Rate Measurer Ex 13.1 Q9

Marginal revenue is the rate of change of total revenue with respect to the number of units sold.

$$\therefore \text{Marginal Revenue (MR)} = \frac{dR}{dx} = 13(2x) + 26 = 26x + 26$$

When $x = 7$,

MR $= 26(7) + 26 = 182 + 26 = 208$

Hence, the required marginal revenue is Rs 208.

Derivatives as a Rate Measurer Ex 13.1 Q10

$$R\left(x\right) = 3x^2 + 36x + 5$$

$$\frac{dR}{dx} = 6x + 36$$

$$\left.\frac{dR}{dx}\right|_{x=5} = 6 \times 5 + 36$$

$= 30 + 36$

$= 66$

This, as per the question, indicates the money to be spent
on the welfare of the employess, when the number of employees is 5.

RD Sharma Solutions Class 12 Maths Chapter 13 Ex 13.2

Let x be the side of square.

Given, $\frac{dx}{dt} = 4$ cm/min, $x = 8$ cm

We know that

$$\text{Area } (A) = x^2$$

$$\frac{dA}{dt} = 2x\frac{dx}{dt}$$

$$\left(\frac{dA}{dt}\right)_{8 \text{ cm}} = 2 \times (8)(4)$$

$$\frac{dA}{dt} = 64 \text{ cm}^2/\text{min}$$

Area increases at a rate of 64 cm^2/min.

Derivatives as a Rate Measurer Ex 13.2 Q2

Let edge of the cube is x cm.

$$\frac{dx}{dt} = 3 \text{ cm/sec}, x = 10 \text{ cm}$$

Let V be volume of cube,

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

$$= 3(10)^2 \times (3)$$

$$= 900 \text{ cm}^3/\text{sec}$$

So,

Volume increases at a rate of 900 cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q3

Let x be the side of the square.

Here, $\frac{dx}{dt} = 0.2$ cm/sec.

$$P = 4x$$

$$\frac{dP}{dt} = 4\frac{dx}{dt}$$

$$= 4 \times (0.2)$$

$$\frac{dP}{dt} = 0.8 \text{ cm/sec}$$

So, perimeter increases at the rate of 0.8 cm/sec.

Derivatives as a Rate Measurer Ex 13.2 Q4

The circumference of a circle (C) with radius (r) is given by

$C = 2\pi r$.

Therefore, the rate of change of circumference (C) with respect to time (t) is given by,

$$\frac{dC}{dt} = \frac{dC}{dr} \cdot \frac{dr}{dt} \text{ (By chain rule)}$$

$$= \frac{d}{dr}(2\pi r)\,\frac{dr}{dt}$$

$$= 2\pi \cdot \frac{dr}{dt}$$

It is given that $\frac{dr}{dt} = 0.7$ cm/s.

Hence, the rate of increase of the circumference is $2\pi(0.7) = 1.4\pi$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q5

Let r be the radius of the spherical soap bubble.

Here, $\frac{dr}{dt} = 0.2$ cm/sec, $r = 7$ cm

Surface Area $(A) = 4\pi r^2$

$$\frac{dA}{dt} = 4\pi(2r)\frac{dr}{dt}$$

$$\left(\frac{dA}{dt}\right)_{r=7} = 4\pi(2 \times 7) \times 0.$$

$$= 11.2\pi \text{ cm}^2/\text{sec}.$$

So, area of bubble increases at the rate of 11.2π cm^2/sec.

Derivatives as a Rate Measurer Ex 13.2 Q6

The volume of a sphere (V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$Rate of change of volume (V) with respect to time (t) is given by,

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} \text{ [By chain rule]}$$

$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot \frac{dr}{dt}$$

It is given that $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$.

$$\therefore 900 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2} = \frac{225}{\pi r^2}$$

Therefore, when radius = 15 cm,

$$\frac{dr}{dt} = \frac{225}{\pi(15)^2} = \frac{1}{\pi}$$

Hence, the rate at which the radius of the balloon increases when the radius is 15 cm is $\frac{1}{\pi}$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q7

Let r be the radius of the air bubble.

Here, $\frac{dr}{dt} = 0.5$ cm/sec, $r = 1$ cm

Volume $(V) = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

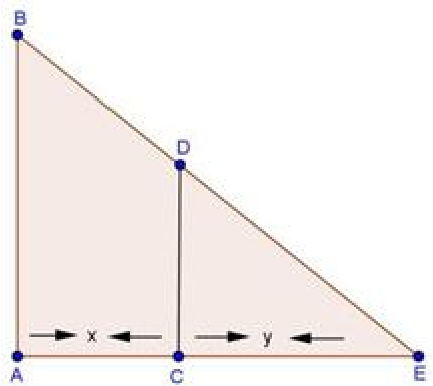
$$= 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi\left(1\right)^2 \times (0.5)$$

$$\frac{dV}{dt} = 2\pi \text{ cm}^3/\text{sec.}$$

So,volume of air bubble increases at the rate of 2π cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q8



Let AB be the lamp-post. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CB.

Here, $\frac{dx}{dt} = 5$ km/hr

$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar, so

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{6}{2} = \frac{x+y}{y}$$

$$3y = x + y$$

$$2y = x$$

$$2\frac{dy}{dt} = \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{5}{2} \text{ km/hr}$$

So, the length of his shadow increases at the rate of $\frac{5}{2}$ km/hr.

Derivatives as a Rate Measurer Ex 13.2 Q9

The area of a circle (A) with radius (r) is given by $A = \pi r^2$.

Therefore, the rate of change of area (A) with respect to time (t) is given by,

$$\frac{dA}{dt} = \frac{d}{dt}\left(\pi r^2\right) = \frac{d}{dr}\left(\pi r^2\right)\frac{dr}{dt} = 2\pi r\frac{dr}{dt} \text{ [By chain rule]}$$

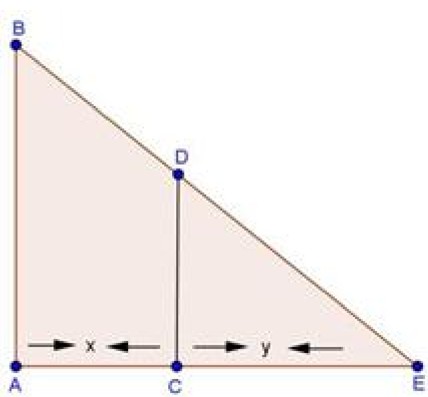
It is given that $\frac{dr}{dt} = 4$ cm/s.

Thus, when $r = 10$cm,

$$\frac{dA}{dt} = 2\pi(10)(4) = 80\pi$$

Hence, when the radius of the circular wave is 8 cm, the enclosed area is increasing at the rate of 80π cm^2/s

Derivatives as a Rate Measurer Ex 13.2 Q10



Let AB be the height of pole. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 1.1 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$
$$\frac{600}{160} = \frac{x+y}{y}$$
$$\frac{15}{4} = \frac{x+y}{y}$$
$$15y = 4x + 4y$$
$$11y = 4x$$
$$11\frac{dy}{dx} = 4\frac{dx}{dt}$$
$$\frac{dy}{dt} = \frac{4}{11}(1.1)$$
$$\frac{dy}{dt} = 0.4 \text{ m/sec}$$

Rate of increasing of shadow = 0.4 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q11

Let AB be the height of source of light. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 2 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{900}{180} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4\frac{dy}{dt} = \frac{dx}{dt}$$

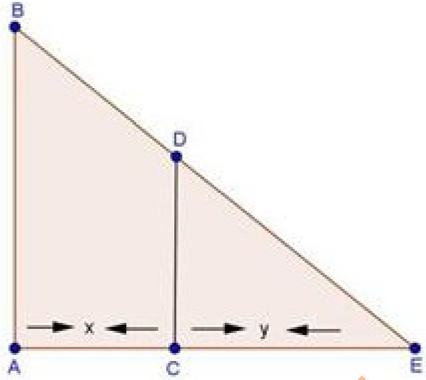
$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

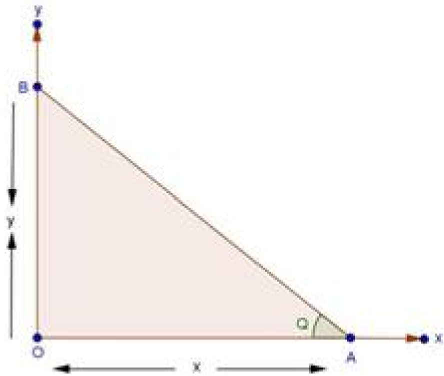
$$\frac{dy}{dt} = 0.5 \text{ m/sec}$$

So, rate of increase of shadow is 0.5 m/sec.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q12



Let AB be the position of the ladder, at time t, such that $OA = x$ and $OB = y$

Here,

$$OA^2 + OB^2 = AB^2$$

$$x^2 + y^2 = (13)^2$$

$$x^2 + y^2 = 169 \quad \text{---(i)}$$

And $\dfrac{dx}{dt} = 1.5$ m/sec

From figure, $\tan\theta = \dfrac{y}{x}$

Differentiating equation (i) with respect to t,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$2(1.5)x + 2y\frac{dy}{dt} = 0$$

$$3x + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{3x}{2y}$$

Differentiating equation (ii) with respect to t,

$$\sec^2\theta \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

$$= \frac{x \times \left(-\frac{3x}{2y}\right) - y(1.5)}{x^2}$$

$$= \frac{-1.5x^2 - 1.5y^2}{yx^2}$$

$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y \sec^2\theta}$$

$$= \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \tan^2\theta\right)}$$

$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \frac{y^2}{x^2}\right)}$$

$$= \frac{-1.5\left(x^2 + y^2\right) \times x^2}{x^2 y\left(x^2 + y^2\right)}$$

$$= \frac{-1.5}{y}$$

$$= \frac{-1.5}{\sqrt{169 - x^2}}$$

$$= \frac{-1.5}{\sqrt{169 - 144}}$$

$$= \frac{-1.5}{5}$$

$$= -0.3 \text{ radian/sec}$$

So, angle between ladder and ground is decreasing at the rate of 0.3 radian/sec.

Derivatives as a Rate Measurer Ex 13.2 Q13

Here, curve is

$$y = x^2 + 2x$$

And $\dfrac{dy}{dt} = \dfrac{dx}{dt}$ ---(i)

$$y = x^2 + 2x$$

$\Rightarrow$ $\dfrac{dy}{dt} = 2x\dfrac{dx}{dt} + 2\dfrac{dx}{dt}$

$\Rightarrow$ $\dfrac{dy}{dt} = \dfrac{dx}{dt}(2x + 2)$

Using equation (i),

$$2x + 2 = 1$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

So, $y = x^2 + 2x$

$$= \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right)$$
$$= \frac{1}{4} - 1$$
$$y = -\frac{3}{4}$$

So, required points is $\left(-\frac{1}{2}, -\frac{3}{4}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q14

Here,

$\dfrac{dx}{dt} = 4$ units/sec, and $x = 2$

And, $y = 7x - x^3$

Slope of the curve $(S) = \dfrac{dy}{dx}$

$$S = 7 - 3x^2$$
$$\frac{ds}{dt} = -6x\frac{dx}{dt}$$
$$= -6(2)(4)$$
$$= -48 \text{ units/sec}$$

So, slope is decreasing at the rate of 48 units/sec.

Derivatives as a Rate Measurer Ex 13.2 Q15

Here,

$$\frac{dy}{dt} = 3\frac{dx}{dt} \qquad \text{---(i)}$$

And, $y = x^3$

$$\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$$3\frac{dx}{dt} = 3x^2\frac{dx}{dt} \qquad \left[\text{Using equation (i)}\right]$$

$$3x^2 = 3$$

$$x^2 = 1$$

$$x = \pm 1$$

Put $x = 1 \Rightarrow y = (1)^3 = 1$

Put $x = -1 \Rightarrow y = (-1)^3 = -13$

So, the required points are $(1,1)$ and $(-1,-1)$.

Derivatives as a Rate Measurer Ex 13.2 Q16(i)

Here,

$$2\frac{d(\sin\theta)}{dt} = \frac{d\theta}{dt}$$

$$2 \times \cos\theta\frac{d\theta}{dt} = \frac{d\theta}{dt}$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}.$$

Derivatives as a Rate Measurer Ex 13.2 Q16(ii)

$$\frac{d\theta}{dt} = -2\frac{d}{dt}(\cos\theta)$$

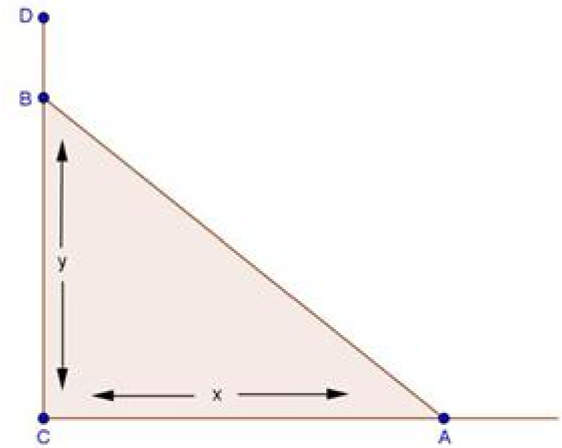
$$\frac{d\theta}{dt} = -2(-\sin\theta)\frac{d\theta}{dt}$$

$$1 = 2\sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

Derivatives as a Rate Measurer Ex 13.2 Q17



Let CD be the wall and AB is the ladder

Here, $AB = 6$ meter and $\left(\frac{dx}{dt}\right)_{x=4} = 0.5$ m/sec.

From figure,

$$AB^2 = x^2 + y^2$$
$$(6)^2 = x^2 + y^2$$
$$36 = x^2 + y^2$$

Differentiating it with respect to t,

$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} \qquad \text{---(i)}$$
$$\left(\frac{dy}{dt}\right)_{x=4} = \frac{4(0.5)}{\sqrt{36 - x^2}}$$
$$= -\frac{2}{\sqrt{36 - 16}}$$
$$= -\frac{2}{2\sqrt{5}}$$
$$= -\frac{1}{\sqrt{5}} \text{ m/sec.}$$

So, ladder top is sliding at the rate of $\frac{1}{\sqrt{5}}$ m/sec.

Now, to find x when $\frac{dx}{dt} = -\frac{dy}{dt}$

From equation (i),

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$-\frac{dx}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$x = y$$

Now,

$$36 = x^2 + y^2$$

$$36 = x^2 + x^2$$

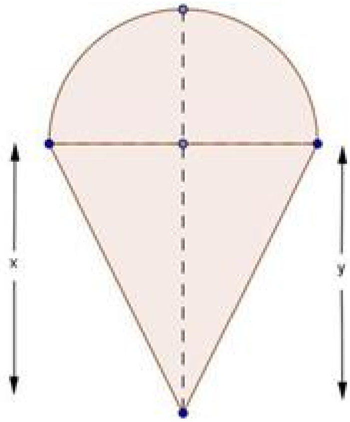
$$2x^2 = 36$$

$$x^2 = 18$$

$$x = 3\sqrt{2} \text{ m}$$

When foot and top are moving at the same rate, foot of wall is $3\sqrt{2}$ meters away from the wall

Derivatives as a Rate Measurer Ex 13.2 Q18



Let height of the cone is x cm. and radius of sphere is r cm.

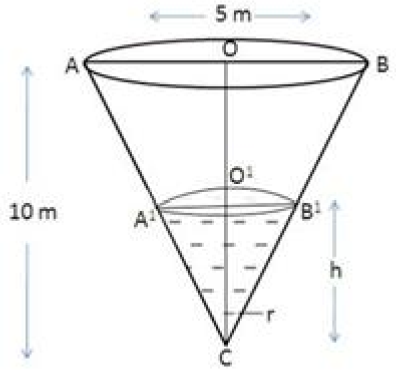
Here given,

$$x = 2r \qquad \text{---(i)}$$
$$h = x + r$$
$$h = 2r + r$$
$$h = 3r \qquad \text{---(ii)}$$

$$v = \text{volume of cone} + \text{volume of hemisphere}$$
$$= \frac{1}{3}\pi r^2 x + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi r^2 (2r) + \frac{2}{3}\pi r^3 \qquad \left[\text{Using equation (ii)}\right]$$
$$v = \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$
$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \left(\frac{h}{3}\right)^3$$
$$v = \frac{4}{81}\pi h^3$$
$$\frac{dv}{dh} = \frac{4}{81}\pi \times 3h^2$$
$$\left(\frac{dv}{dh}\right)_{h=9} = \frac{12}{81}\pi (9)^2$$
$$\left(\frac{dv}{dh}\right)_{h=9} = 12\pi \text{ cm}^2$$

Volume is changing at the rate 12π cm^2 with respect to total height.

Derivatives as a Rate Measurer Ex 13.2 Q19



Let α be the semi vertical angle of the cone CAB whose height CO is 10 m and radius OB = 5 m.

Now,

$$\tan\alpha = \frac{OB}{CO}$$
$$= \frac{5}{10}$$
$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the water in the cone, then

$$v = \frac{1}{3}\pi\left(O'B'\right)^2\left(CO'\right)$$
$$= \frac{1}{3}\pi\left(h\tan\alpha\right)^2(h)$$
$$v = \frac{1}{3}\pi h^3\tan^2\alpha$$
$$v = \frac{\pi}{12}h^2 \qquad \left[\because \tan\alpha = \frac{1}{2}\right]$$
$$\frac{dv}{dt} = \frac{\pi}{12}3h^2\frac{dh}{dt}$$
$$\pi = \frac{\pi}{4}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = m^3/\min\right]$$
$$\frac{dh}{dt} = \frac{4}{h^2}$$
$$\left(\frac{dh}{dt}\right)_{2.5} = \frac{4}{(2.5)^2} \qquad \left[\because h = 10 - 7.5 = 2.5 \text{ m}\right]$$
$$= \frac{4}{6.25}$$
$$= 0.64 \text{ m/min}$$

So, water level is rising at the rate of 0.64 m/min.

Derivatives as a Rate Measurer Ex 13.2 Q20

Let AB be the lamp-post. Suppose at time t, the man CD is at a distance x m. from the lamp-post and y m be the length of the shadow CE.

Here, $\frac{dx}{dt} = 6$ km/hr

$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar

So, $\frac{AB}{CD} = \frac{AE}{CE}$

$\frac{6}{2} = \frac{x + y}{y}$

$3y = x + y$

$2y = x$

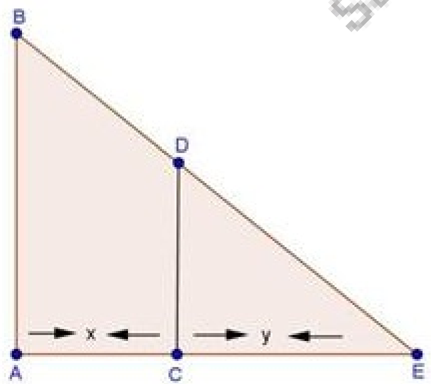
$2\frac{dy}{dt} = \frac{dx}{dt}$

$2\frac{dy}{dt} = 6$

$\frac{dy}{dt} = 3$ km/hr

So, length of his shadow increases at the rate of 3 km/hr.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q21

Here, $\frac{dA}{dt} = 2\ \text{cm}^2/\text{sec}$

To find $\frac{dV}{dt}$ at $r = 6$ cm

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$2 = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r}\ \text{cm/sec}$$

Now, $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi r^2\left(\frac{1}{4\pi r}\right)$$

$$= r$$

$$\frac{dV}{dt} = 6\ \text{cm}^3/\text{sec}$$

So, volume of bubble is increasing at the rate of 6 cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q22

Here, $\frac{dr}{dt} = 2$ cm/sec, $\frac{dh}{dt} = -3$ cm/sec

To find $\frac{dV}{dt}$ when $r = 3$ cm, $h = 5$ cm

Now, V = volume of cylinder

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi\left[2r\frac{dr}{dt} \times h + r^2\frac{dh}{dt}\right]$$

$$= \pi\left[2(3)(2)(5) + (3)^2(-3)^2\right]$$

$$= \pi[60 - 27]$$

$$\frac{dV}{dt} = 33\pi\ \text{cm}^3/\text{sec}$$

So, volume of cylinder is increasing at the rate of 33π cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q23

Let V be volume of sphere with miner radius r and onter radius R, then

$$V = \frac{4}{3}\pi\left(R^3 - r^3\right)$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3R^2\frac{dR}{dt} - 3r^2\frac{dr}{dt}\right)$$

$$0 = \frac{4\pi}{3}3\left(R^2\frac{dR}{dt} - r^2\frac{dr}{dt}\right)$$ [Since volume V is constant]

$$R^2\frac{dR}{dt} = r^2\frac{dr}{dt}$$

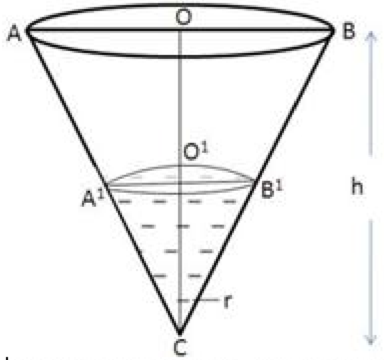
$$(8)^2\frac{dR}{dt} = (4)^2(1)$$

$$\frac{dR}{dt} = \frac{16}{64}$$

$$\frac{dR}{dt} = \frac{1}{4} \text{ cm/sec}$$

Rate of increasing of onter radius = $\frac{1}{4}$ cm/sec.

Derivatives as a Rate Measurer Ex 13.2 Q24



Let α be the semi vertical angle of the cone CAB whose height CO is half of radius OB.

Now,

$$\tan\alpha = \frac{OB}{CO}$$

$$= \frac{OB}{2OB} \qquad [\because CO = 2OB]$$

$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the sand in the cone

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$= \frac{\pi}{12}h^3$$

$$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$$

$$50 = \frac{3\pi}{12}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = 50\ \text{cm}^3/\text{min}\right]$$

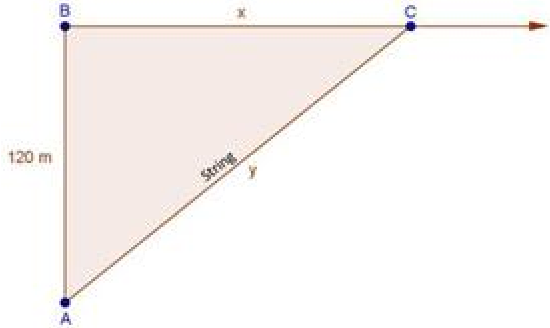
$$\frac{dh}{dt} = \frac{200}{\pi h^2}$$

$$= \frac{200}{\pi(5)^2}$$

$$\frac{dh}{dt} = \frac{8}{3.14}\ \text{cm/min}$$

Rate of increasing of height = $\frac{8}{\pi}$ cm/min

Derivatives as a Rate Measurer Ex 13.2 Q25



Let C be the position of kite and AC be the string.

Here, $y^2 = x^2 + (120)^2$ ---(i)

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$y\frac{dy}{dt} = x\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}(52) \quad \text{---(ii)} \quad \left[\because \frac{dx}{dt} = 52 \text{ m/sec}\right]$$

From equation (i),

$$y^2 = x^2 + (120)^2$$

$$(130)^2 = x^2 + (120)^2$$

$$x^2 = 16900 - 14400$$

$$x^2 = 2500$$

$$x = 50$$

Using equation (ii),

$$\frac{dy}{dt} = \frac{x}{y}(52)$$

$$= \frac{50}{130}(52)$$

$$= 20 \text{ m/sec}$$

So, string is being paid out at the rate of 20 m/sec.

Derivatives as a Rate Measurer Ex 13.2 Q26

Here,

$$\frac{dy}{dt} = 2\frac{dx}{dt} \qquad ---(i)$$

and $\quad y = \left(\frac{2}{3}\right)x^3 + 1$

$$\frac{dy}{dt} = \frac{2}{3} \times 3x^2 \frac{dx}{dt}$$

$$2\frac{dx}{dt} = 2x^2 \frac{dx}{dt} \qquad \left[\text{Using equation (i)}\right]$$

$$2 = 2x^2$$

$\Rightarrow \quad x = \pm 1$

$$y = \left(\frac{2}{3}\right)x^3 + 1$$

Put $x = 1$, $\quad y = \frac{2}{3} + 1 = \frac{5}{3}$

Put $x = -1$, $\quad y = \frac{2}{3}(-1) + 1 = \frac{1}{3}$

So, required point $\left(1, \frac{5}{3}\right)$ and $\left(-1, \frac{1}{3}\right)$.

Derivatives as a Rate Measurer Ex 13.2 Q27

Here,

$$\frac{dx}{dt} = \frac{dy}{dt} \qquad ---(i)$$

and $\quad$ curve is

$$y^2 = 8x$$

$$2y\frac{dy}{dt} = 8\frac{dx}{dt}$$

$$2y = 8 \qquad \left[\text{using equation (i)}\right]$$

$$y = 4$$

$\Rightarrow \quad (4)^2 = 8x$

$\Rightarrow \quad x = 2$

So, required point $= (2, 4)$.

Derivatives as a Rate Measurer Ex 13.2 Q28

Let edge of cube be x cm

Here,

$$\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$$

To find $\frac{dA}{dt}$ when $x = 10$ cm

We know that

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$$

$$9 = 3(10)^2\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{100} \text{ cm/sec}$$

Now, $A = 6x^2$

$$\frac{dA}{dt} = 12x\frac{dx}{dt}$$

$$= 12(10)\left(\frac{3}{100}\right)$$

$$\frac{dA}{dt} = 3.6 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q29

Given, $\frac{dV}{dt} = 25 \text{ cm}^3/\text{sec}$

To find $\frac{dA}{dt}$ when $r = 5$ cm

We know that,

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

$$25 = 4\pi(5)^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi} \text{ cm/sec}$$

Now, $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$$

$$= 8\pi(5)\left(\frac{1}{4\pi}\right)$$

$$\frac{dA}{dt} = 10 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q30

Given,

$$\frac{dx}{dt} = -5 \text{ cm/min}$$

$$\frac{dy}{dt} = 4 \text{ cm/min}$$

(i) To find $\frac{dP}{dt}$ when $x = 8$ cm, $y = 6$ cm

$$P = 2(x + y)$$

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2(-5 + 4)$$

$$\frac{dP}{dt} = -2 \text{ cm/min}$$

(ii) To find $\frac{dA}{dt}$ when $x = 8$ cm and $y = 6$ cm

$$A = xy$$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= (8)(4) + (6)(-5)$$

$$= 32 - 30$$

$$\frac{dA}{dt} = 2 \text{ cm}^2\text{/min.}$$

Derivatives as a Rate Measurer Ex 13.2 Q31

Let r be the radius of the given disc and A be its area.

Then, $A = \pi r^2$

$$\therefore \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \qquad \text{[by chain rule]}$$

Now, the approximate increase ofradius $= dr = \frac{dr}{dt}\Delta t = 0.05$ cm / sec

$\therefore$ the approximate rate of increase in areais given by

$$dA = \frac{dA}{dt}(\Delta t) = 2\pi r\left(\frac{dr}{dt}\Delta t\right) = 2\pi(3.2)(0.05) = 0.320\pi \text{ cm}^3\text{/s}$$