RD Sharma Solutions

Class 12 Maths

Chapter 11

Ex 11.5
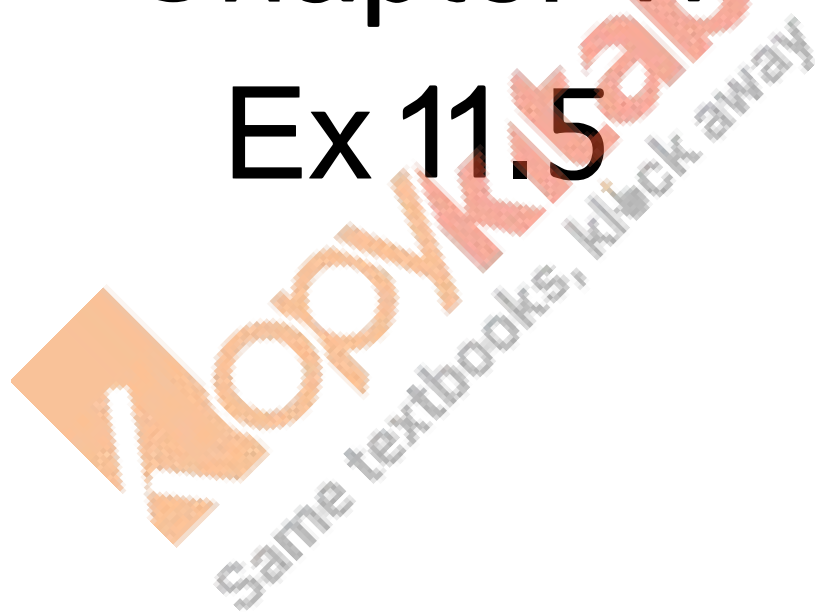
Chapter: Differentiation

Exercise: 11.5

Page Number: 11.89

Q1.

Answer :

$$Let\ y = x^{\frac{1}{x}} \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log x^{\frac{1}{x}}$$

$$\Rightarrow \log y = \frac{1}{x}\log x \qquad \left[\because \log a^b = b\log a\right]$$

Differentiating with respect to x,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}(\log\ x) + \log\ x\frac{d}{dx}(x^{-1}) \qquad \left[Using\ product\ rule\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\times\frac{1}{x} + (\log\ x)\times\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{(1-\log x)}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}}\left[\frac{1-\log x}{x}\right]$$

Q2.

Answer :

$$Let\ y = x^{\sin x} \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log x^{\sin x} \Rightarrow \log y = \sin x\ \log x$$

$$\left[\because \log a^b = b\ \log a\right]$$

Differentiating with respect to x,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x \frac{d}{dx}\log\ x + \log\ x \frac{d}{dx}\sin x \qquad \left[using\ product\ rule\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log\ x\left(\cos x\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\sin x}{x} + \left(\log\ x\right)\left(\cos x\right)\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \left(\log x\right)\left(\cos x\right)\right]$$

Q3.

Answer :

$$Let\ y = \left(1+\cos x\right)^x \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log\left(1+\cos x\right)^x$$

$$\Rightarrow \log y = x\ \log\left(1+\cos x\right)$$

Differentiating with respect to x, we get,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log\left(1+\cos x\right) + \log\left(1+\cos x\right)\frac{d}{dx}\left(x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\times\frac{1}{\left(1+\cos x\right)}\frac{d}{dx}\left(1+\cos x\right) + \log\left(1+\cos x\right)\left(1\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\left(1+\cos x\right)}\left(0-\sin\ x\right) + \log\left(1+\cos x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log\left(1+\cos x\right) - \frac{x\sin x}{\left(1+\cos x\right)}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\log\left(1+\cos x\right) - \frac{x\sin x}{\left(1+\cos x\right)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(1+\cos x\right)^x\left[\log\left(1+\cos x\right) - \frac{x\sin x}{\left(1+\cos x\right)}\right] \qquad \left[using\ equation\ \left(i\right)\right]$$

Q4.

Answer :

$$Let\ y = x^{\cos^{-1}x} \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log x^{\cos^{-1}x}$$

$$\Rightarrow \log y = \cos^{-1}x\ \log x$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x\,\frac{d}{dx}\left(\log\ x\right) + \log\ x\,\frac{d}{dx}\cos^{-1}x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos^{-1}x\left(\frac{1}{x}\right) + \log\ x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1}x}{x} - \frac{\log\ x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\cos^{-1}x}{x} - \frac{\log\ x}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{\cos^{-1}x}\left[\frac{\cos^{-1}x}{x} - \frac{\log\ x}{\sqrt{1-x^2}}\right] \qquad \left[Using\ equation\ (i)\right]$$

Q5.

Answer :

$Let\ y = \left(\log x\right)^x \qquad ...(i)$

Taking log both sides,

$$\log y = \log\left(\log x\right)^x \Rightarrow \log y = x\log\left(\log x\right)$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = x\,\frac{d}{dx}\log\left(\log x\right) + \log\left(\log x\right)\frac{d}{dx}\left(x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\,\frac{1}{\log x}\,\frac{d}{dx}\left(\log x\right) + \log\left(\log x\right)(1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\log x}\left(\frac{1}{x}\right) + \log\left(\log x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{\log x} + \log\left(\log x\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{\log x} + \log\left(\log x\right)\right]$$

$$\Rightarrow \frac{dy}{dx} = \left(\log x\right)^x\left[\frac{1}{\log x} + \log\left(\log x\right)\right] \qquad \left[using\ equation\ (i)\right]$$

Q6.

Answer :

$Let \; y = (\log x)^{\cos x} \qquad ...(i)$

Taking log on both sides,

$\log y = \log(\log x)^{\cos x}$

$\Rightarrow \log y = \cos x \, \log(\log x)$

Differentiating with respect to x using chain rule,

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \cos x \, \dfrac{d}{dx}\log(\log x) + \log(\log x)\dfrac{d}{dx}(\cos x)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos x}{\log x}\dfrac{d}{dx}(\log x) + \log(\log x)\times(-\sin x)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos x}{\log x}\times\left(\dfrac{1}{x}\right) - \sin x \, \log(\log x)$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{\cos x}{x \log x} - \sin x \, \log(\log x)\right]$

$\Rightarrow \dfrac{dy}{dx} = (\log x)^{\cos x}\left[\dfrac{\cos x}{x \log x} - \sin x \, \log(\log x)\right] \qquad \left[using \; equation \;(i)\right]$

Q7.

Answer :

$Let \; y = (\sin x)^{\cos x} \qquad ...(i)$

$Taking \; \log \; on \; both \; sides,$

$\log y = \log(\sin x)^{\cos x}$

$\Rightarrow \log y = \cos x \, \log \sin x$

$Differentiating \; with \; respect \; to \; x,$

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos x \, \dfrac{d}{dx}(\log \sin x) + \log \sin x \, \dfrac{d}{dx}(\cos x)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \cos x \, \dfrac{1}{\sin x}\dfrac{d}{dx}(\sin x) + \log \sin x \, (-\sin x)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos x}{\sin x}(\cos x) - \sin x \, \log \sin x$

$\Rightarrow \dfrac{dy}{dx} = y\left[\cos x \cot x - \sin x \, \log \sin x\right]$

$\Rightarrow \dfrac{dy}{dx} = (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \, \log \sin x\right]$

Q8.

Answer :

Let $y = e^{x \log x}$

Taking $\log$ on both sides,

$\log y = x \log x \log e$

$\Rightarrow \log y = x \log x$

Differentiating with respect to x,

$\dfrac{1}{y} \dfrac{dy}{dx} = x \dfrac{d}{dx} \log x + \log x \dfrac{d}{dx} x$

$\Rightarrow \dfrac{1}{y} \dfrac{dy}{dx} = x \left(\dfrac{1}{x} \right) + \log x (1)$

$\Rightarrow \dfrac{1}{y} \dfrac{dy}{dx} = 1 + \log x$

$\Rightarrow \dfrac{dy}{dx} = y [1 + \log x]$

$\Rightarrow \dfrac{dy}{dx} = e^{x \log x} (1 + \log x)$ $\qquad [using\ equation\ (i)]$

$\Rightarrow \dfrac{dy}{dx} = e^{\log x^x} (1 + \log x)$

$\Rightarrow \dfrac{dy}{dx} = x^x (1 + \log x)$

Q9.

Answer :

Let $y = (\sin x)^{\log x}$ $\qquad ...(i)$

Taking $\log$ on both sides,

$\log y = \log (\sin x)^{\log x}$

$\Rightarrow \log y = \log x \log \sin x$

Differentiating with respect to x,

$\dfrac{1}{y} \dfrac{dy}{dx} = \log x \dfrac{d}{dx} (\log \sin x) + \log \sin x \dfrac{d}{dx} (\log x)$

$\Rightarrow \dfrac{1}{y} \dfrac{dy}{dx} = \log x \dfrac{1}{\sin x} \dfrac{d}{dx} (\sin x) + \log \sin x \left(\dfrac{1}{x} \right)$

$\Rightarrow \dfrac{1}{y} \dfrac{dy}{dx} = \dfrac{\log x}{\sin x} (\cos x) + \dfrac{\log \sin x}{x}$

$\Rightarrow \dfrac{dy}{dx} = y \left[\log x \cot x + \dfrac{\log \sin x}{x} \right]$

$\Rightarrow \dfrac{dy}{dx} = (\sin x)^{\log x} \left[\log x \cot x + \dfrac{\log \sin x}{x} \right]$

Q10.

Answer :

$Let\ y = 10^{\log \sin x}$ $\qquad ...(i)$

$Taking\ \log\ on\ both\ sides,$

$\log y = \log 10^{\log \sin x}$

$\Rightarrow \log y = \log \sin x \log 10$

$Differentiating\ with\ respect\ to\ x,$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log 10 \dfrac{d}{dx} \log \sin x$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log 10 \dfrac{1}{\sin x}\dfrac{d}{dx}(\sin x)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log 10 \left(\dfrac{1}{\sin x}\right)(\cos x)$

$\Rightarrow \dfrac{dy}{dx} = y\left[\log 10 \times \cot x\right]$

$\Rightarrow \dfrac{dy}{dx} = 10^{\log \sin x} \times \log 10 \times \cot x \qquad \left[using\ equation\ (i)\right]$

Q11.

Answer :

$Let\ y = (\log x)^{\log x}$ $\qquad ....(i)$

$Taking\ \log\ on\ both\ sides,$

$\log y = \log(\log x)^{\log x}$

$\Rightarrow \log y = \log x \log(\log x)$

$Differentiating\ both\ side\ with\ respect\ to\ x,$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log(\log x)\dfrac{d}{dx}\log x + \log x \dfrac{d}{dx}\log(\log x)$

$\Rightarrow \dfrac{dy}{dx} = y\left[\log(\log x)\dfrac{1}{x} + \log x \dfrac{1}{\log x}\dfrac{d}{dx}(\log x)\right]$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{1}{x}\log(\log x) + \dfrac{1}{x}\right]$

$\therefore \dfrac{dy}{dx} = (\log x)^{\log x}\left[\dfrac{1 + \log(\log x)}{x}\right] \qquad \left[using\ equation\ (i)\right]$

Q12.

Answer :

$Let\ y = 10^{(10^x)}$ ...(i)

$Taking\ \log\ on\ both\ sides,$

$\log y = \log_e 10^{(10x)}$

$\log y = 10^x \log_e 10$

$Differentiating\ with\ respect\ to\ x,$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log_e 10 \times 10^x \log_e 10$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = 10^x \times (\log_e 10)^2$

$\Rightarrow \dfrac{dy}{dx} = y\left[10^x \times (\log_e 10)^2\right]$

$\therefore \dfrac{dy}{dx} = 10^{(10x)} \times 10^x \times (\log_e 10)^2$ $\qquad \left[using\ equation\ (i)\right]$

Q13.

Answer :

$Let\ y = \sin\ x^x$

$\Rightarrow\ \sin^{-1} y = x^x$ ...(i)

$Taking\ \log\ on\ both\ sides,$

$\log(\sin^{-1} y) = \log x^x$

$\Rightarrow \log(\sin^{-1} y) = x \log x$

$Differentiating\ with\ respect\ to\ x,$

$\Rightarrow \dfrac{1}{\sin^{-1} y}\dfrac{dy}{dx}\dfrac{d}{dx}(\sin^{-1} y) = x\dfrac{d}{dx}\log x + \log x \dfrac{d}{dx}x$

$\Rightarrow \dfrac{1}{\sin^{-1} y} \times \left(\dfrac{1}{\sqrt{1-y^2}}\right)\dfrac{dy}{dx} = x\left(\dfrac{1}{x}\right) + \log x$

$\Rightarrow \dfrac{dy}{dx} = \sin^{-1} y\sqrt{1-y^2}\,(1+\log x)$

$\Rightarrow \dfrac{dy}{dx} = \sin^{-1}(\sin\ x^x)\sqrt{1-(\sin\ x^x)^2}\,(1+\log x)$

$\Rightarrow \dfrac{dy}{dx} = \sin^{-1}(\sin x^x)\sqrt{1-(\sin\ x^x)^2}\,(1+\log x)$

$\therefore \dfrac{dy}{dx} = x^x \cos x^x\,(1+\log x)$ $\qquad \left[using\ equation\ (i)\right]$

Q14.

Answer :

Let $y = \left(\sin^{-1} x\right)^x$ ...(i)

Taking log *on both sides,*

$\log y = \log\left(\sin^{-1} x\right)^x$

$\Rightarrow \log y = x \log\left(\sin^{-1} x\right)$

Differentiating with respect to x,

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{d}{dx}\left(\log \sin^{-1} x\right) + \log \sin^{-1} x \dfrac{d}{dx} x$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{1}{\sin^{-1} x}\dfrac{d}{dx}\left(\sin^{-1} x\right) + \log \sin^{-1} x$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{x}{\sin^{-1} x}\left(\dfrac{1}{\sqrt{1-x^2}}\right) + \log \sin^{-1} x$

$\Rightarrow \dfrac{dy}{dx} = y\left[\log \sin^{-1} x + \dfrac{x}{\sin^{-1} x\left(\sqrt{1-x^2}\right)}\right]$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{x}{\sin^{-1} x}\left(\dfrac{1}{\sqrt{1-x^2}}\right) + \log \sin^{-1} x$

$\Rightarrow \dfrac{dy}{dx} = y\left[\log \sin^{-1} x + \dfrac{x}{\sin^{-1} x\left(\sqrt{1-x^2}\right)}\right]$

Q15.

Answer :

Let $y = x^{\sin^{-1} x}$ ...(i)

Taking log *on both sides,*

$\log y = \log x^{\sin^{-1} x}$

$\Rightarrow \log y = \sin^{-1} x \log x$

Differentiating with respect to x,

$\dfrac{1}{y}\dfrac{dy}{dx} = \sin^{-1} x \dfrac{d}{dx}\left(\log x\right) + \left(\log x\right)\dfrac{d}{dx}\left(\sin^{-1} x\right)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \sin^{-1} x\left(\dfrac{1}{x}\right) + \left(\log x\right)\left(\dfrac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{\sin^{-1} x}{x} + \dfrac{\log x}{\sqrt{1-x^2}}\right]$

$\Rightarrow \dfrac{dy}{dx} = x^{\sin^{-1} x}\left[\dfrac{\sin^{-1} x}{x} + \dfrac{\log x}{\sqrt{1-x^2}}\right]$ $\left[using\ equation\ (i)\right]$

Q16.

Answer :

$Let\ y = (\tan x)^{\frac{1}{x}}$ $...(i)$

$Taking\ \log\ on\ both\ sides,$

$\log y = \log (\tan x)^{\frac{1}{x}}$

$\Rightarrow \log y = \dfrac{1}{x}\log (\tan x)$

$Differentiating\ with\ respect\ to\ x,$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x}\dfrac{d}{dx}\{\log (\tan x)\} + \log (\tan x)\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x} \times \dfrac{1}{\tan x}\dfrac{d}{dx}(\tan x) + \log (\tan x)\left(-\dfrac{1}{x^2}\right)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x\tan x}(\sec^2 x) - \dfrac{\log (\tan x)}{x^2}$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{\sec^2 x}{x\tan x} - \dfrac{\log (\tan x)}{x^2}\right]$

$\Rightarrow \dfrac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left[\dfrac{\sec^2 x}{x\tan x} - \dfrac{\log (\tan x)}{x^2}\right]$ $\left[using\ equation\ (i)\right]$

Q17.

Answer :

$Let\ y = x^{\tan^{-1} x}$ $...(i)$

$Taking\ \log\ on\ both\ sides,$

$\log y = \log\ x^{\tan^{-1} x}$

$\Rightarrow \log y = \tan^{-1} x \log x$

$Differentiating\ with\ respect\ to\ x,$

$\dfrac{1}{y}\dfrac{dy}{dx} = \tan^{-1} x\dfrac{d}{dx}\log x + \log x\dfrac{d}{dx}(\tan^{-1} x)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \tan^{-1} x\left(\dfrac{1}{x}\right) + \log x\left(\dfrac{1}{1+x^2}\right)$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{\tan^{-1} x}{x} + \dfrac{\log x}{1+x^2}\right]$

$\Rightarrow \dfrac{dy}{dx} = x^{\tan^{-1} x}\left[\dfrac{\tan^{-1} x}{x} + \dfrac{\log x}{1+x^2}\right]$ $\left[using\ equation\ (i)\right]$

Q18.

Answer :

(i)

Let $y = x^x \sqrt{x}$ $\qquad ...(i)$

Taking $\log$ on both sides,

$\log y = \log \left(x^x \sqrt{x} \right)$

$\Rightarrow \log y = \log x^x + \log x^{\frac{1}{2}}$

$\Rightarrow \log y = x \log x + \dfrac{1}{2} \log x$

$\dfrac{1}{y} \dfrac{dy}{dx} = x \dfrac{d}{dx} \left(\log x \right) + \log x \dfrac{d}{dx} \left(x \right) + \dfrac{1}{2} \dfrac{d}{dx} \left(\log x \right)$

$\Rightarrow \dfrac{1}{y} \dfrac{dy}{dx} = x \left(\dfrac{1}{x} \right) + \log x (1) + \dfrac{1}{2} \left(\dfrac{1}{x} \right)$

$\Rightarrow \dfrac{1}{y} \dfrac{dy}{dx} = 1 + \log x + \dfrac{1}{2x}$

$\Rightarrow \dfrac{dy}{dx} = y \left[1 + \log x + \dfrac{1}{2x} \right]$

$\Rightarrow \dfrac{dy}{dx} = x^x \sqrt{x} \left[1 + \log x + \dfrac{1}{2x} \right]$ $\qquad \left[using\ equation\ (i) \right]$

$\Rightarrow \dfrac{dy}{dx} = x^{x+\frac{1}{2}} \left[\left(\dfrac{2x+1}{2x} \right) + \log x \right]$

(ii) Let $y = x^{(\sin x - \cos x)} + \left(\dfrac{x^2 - 1}{x^2 + 1} \right)$

$\Rightarrow y = e^{\log x^{(\sin x - \cos x)}} + \left(\dfrac{x^2 - 1}{x^2 + 1} \right)$

$\Rightarrow y = e^{(\sin x - \cos x)\log x} + \left(\dfrac{x^2 - 1}{x^2 + 1} \right)$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left[e^{(\sin x - \cos x)\log x} \right] + \dfrac{d}{dx}\left[\dfrac{x^2 - 1}{x^2 + 1} \right]$

$= e^{(\sin x - \cos x)\log x} \dfrac{d}{dx}\{(\sin x - \cos x)\log x\} + \left[\dfrac{(x^2 + 1)\dfrac{d}{dx}(x^2 - 1) - (x^2 - 1)\dfrac{d}{dx}(x^2 + 1)}{(x^2 + 1)^2} \right]$

$= e^{\log x^{(\sin x - \cos x)}} \left[(\sin x - \cos x)\dfrac{d}{dx}(\log x) + (\log x)\dfrac{d}{dx}(\sin x - \cos x) \right] + \left[\dfrac{(x^2 + 1)(2x) - (x^2 - 1)(2x)}{(x^2 + 1)^2} \right]$

$= x^{(\sin x - \cos x)} \left[(\sin x - \cos x)\left(\dfrac{1}{x}\right) + \log x (\sin x + \cos x) \right] + \left[\dfrac{2x^3 + 2x - 2x^3 + 2x}{(x^2 + 1)^2} \right]$

$= x^{(\sin x - \cos x)} \left[\dfrac{(\sin x - \cos x)}{x} + (\sin x + \cos x)\log x \right] + \dfrac{4x}{(x^2 + 1)^2}$

Q19.

Answer :

We have, $y = e^x + 10^x + x^x$

$\Rightarrow y = e^x + 10^x + e^{\log x^x}$

$\Rightarrow y = e^x + 10^x + e^{x \log x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(10^x\right) + \frac{d}{dx}\left(e^{x \log x}\right)$$

$$= e^x + 10^x \log 10 + e^{x \log x} \frac{d}{dx}\left(x \log x\right)$$

$$= e^x + 10^x \log 10 + e^{x \log x}\left[x \frac{d}{dx}\left(\log x\right) + \log x \frac{d}{dx}\left(x\right)\right]$$

$$= e^x + 10^x \log 10 + e^{\log x^x}\left[x\left(\frac{1}{x}\right) + \log x\right]$$

$$= e^x + 10^x \log 10 + x^x \left[1 + \log x\right]$$

$$= e^x + 10^x \log 10 + x^x \left[\log e + \log x\right] \qquad \left[\because \log_e e = 1\right]$$

$$= e^x + 10^x \log 10 + x^x \left(\log ex\right)$$

Q20.

Answer :

We have, $y = x^n + n^x + x^x + n^n$

$\Rightarrow y = x^n + n^x + e^{\log x^x} + n^n$

$\Rightarrow y = x^n + n^x + e^{x \log x} + n^n$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^n\right) + \frac{d}{dx}\left(n^x\right) + \frac{d}{dx}\left(e^{x \log x}\right) + \frac{d}{dx}\left(n^n\right)$$

$$= nx^{n-1} + n^x \log n + e^{\log x^x}\left[x \frac{d}{dx}\log x + \log x \frac{d}{dx}\left(x\right)\right]$$

$$= nx^{n-1} + n^x \log n + x^x\left[x\left(\frac{1}{x}\right) + \log x\right]$$

$$= nx^{n-1} + n^x \log n + x^x \left[1 + \log x\right]$$

$$= nx^{n-1} + n^x \log n + x^x \left[\log e + \log x\right] \qquad \left[\because \log_e e = 1 \text{ and } \log A + \log B = \log\left(AB\right)\right]$$

$$= nx^{n-1} + n^x \log n + x^x \log\left(ex\right)$$

Q21.

Answer :

We have, $y = \dfrac{\left(x^2 - 1\right)^3 (2x - 1)}{\sqrt{(x-3)(4x-1)}}$ ...(i)

$\Rightarrow y = \dfrac{\left(x^2 - 1\right)^3 (2x - 1)}{(x-3)^{\frac{1}{2}} (4x-1)^{\frac{1}{2}}}$

Taking log on both sides,

$\log y = \log\left[\dfrac{\left(x^2 - 1\right)^3 (2x - 1)}{(x-3)^{\frac{1}{2}} (4x-1)^{\frac{1}{2}}}\right]$

$\Rightarrow \log y = \log\left(x^2 - 1\right)^3 + \log(2x - 1) - \log(x-3)^{\frac{1}{2}} - \log(4x-1)^{\frac{1}{2}}$

$\Rightarrow \log y = 3\log\left(x^2 - 1\right) + \log(2x - 1) - \dfrac{1}{2}\log(x - 3) - \dfrac{1}{2}\log(4x - 1)$

Differentiating with respect to x using chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = 3\dfrac{d}{dx}\left\{\log\left(x^2 - 1\right)\right\} + \dfrac{d}{dx}\left\{\log(2x - 1)\right\} - \dfrac{1}{2}\dfrac{d}{dx}\left\{\log(x - 3)\right\} - \dfrac{1}{2}\left\{\log(4x - 1)\right\}$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = 3\left(\dfrac{1}{x^2 - 1}\right)\dfrac{d}{dx}\left(x^2 - 1\right) + \dfrac{1}{(2x-1)}\dfrac{d}{dx}(2x - 1) - \dfrac{1}{2}\left(\dfrac{1}{x-3}\right)\dfrac{d}{dx}(x - 3) - \dfrac{1}{2}\dfrac{1}{(4x-1)}\dfrac{d}{dx}(4x - 1)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = 3\left(\dfrac{1}{x^2 - 1}\right)(2x) + \dfrac{1}{2x-1}(2) - \dfrac{1}{2}\left(\dfrac{1}{x-3}\right)(1) - \dfrac{1}{2}\left(\dfrac{1}{4x-1}\right)(4)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \left[\dfrac{6x}{x^2 - 1} + \dfrac{2}{2x-1} - \dfrac{1}{2(x-3)} - \dfrac{2}{4x-1}\right]$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{6x}{x^2 - 1} + \dfrac{2}{2x-1} - \dfrac{1}{2(x-3)} - \dfrac{2}{4x-1}\right]$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\left(x^2 - 1\right)^3 (2x - 1)}{\sqrt{(x-3)(4x-1)}}\left[\dfrac{6x}{x^2 - 1} + \dfrac{2}{2x-1} - \dfrac{1}{2(x-3)} - \dfrac{2}{4x-1}\right]$ $\left[using\ equation\ (i)\right]$