

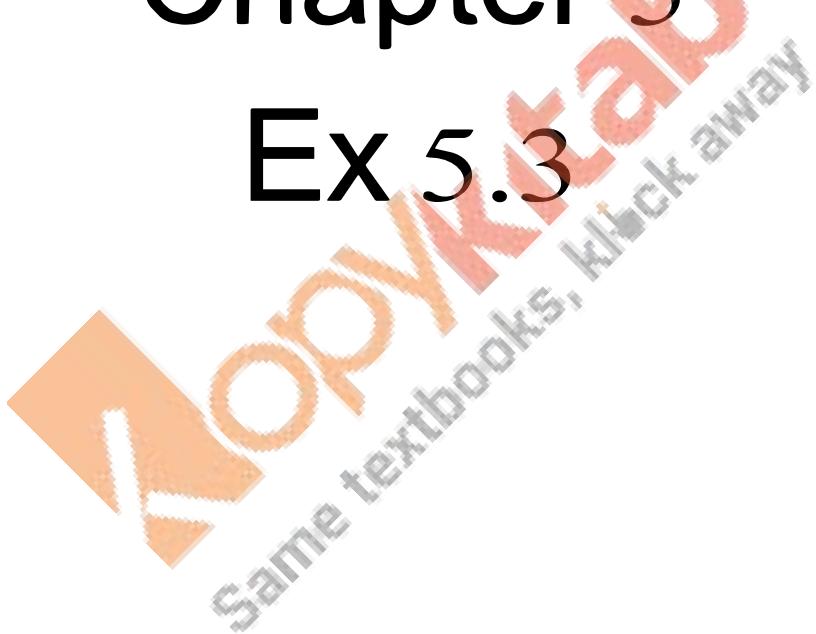
# RD Sharma

## Solutions

### Class 11 Maths

#### Chapter 5

##### Ex 5.3



### Chapter 5 Trigonometric Functions Ex 5.3 Q 1 i

$$\begin{aligned}\sin \frac{5\pi}{3} &= \sin \left(2\pi - \frac{\pi}{3}\right) \\&= -\sin \frac{\pi}{3} \quad (\because \sin(2\pi - \theta) = -\sin \theta) \\&= -\frac{\sqrt{3}}{2}\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1 ii

$$\begin{aligned}3060^\circ &= 17\pi \quad (\because \pi = 180^\circ) \\ \therefore \sin 3060^\circ &= \sin 17\pi \\&= 0 \quad (\because \sin n\pi = 0 \text{ for all } n \in \mathbb{Z})\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1 iii

$$\begin{aligned}\tan \frac{11\pi}{6} &= \tan \left(2\pi - \frac{\pi}{6}\right) \\&= -\tan \frac{\pi}{6} \quad (\because \tan(2\pi - \theta) = -\tan \theta) \\&= -\frac{1}{\sqrt{3}}\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1.iv

$$\begin{aligned}1125^\circ &= 6\pi + \frac{\pi}{4} \quad (\pi = 180^\circ) \\ \cos(-1125^\circ) &= \cos \left(-\left(6\pi + \frac{\pi}{4}\right)\right) \\&= \cos \left(6\pi + \frac{\pi}{4}\right) \quad (\because \cos(-\theta) = \cos \theta) \\&= \cos \left(2 \times 3\pi + \frac{\pi}{4}\right) \\&= \cos \frac{\pi}{4} \quad (\because \cos(2k\pi + \theta) = \cos \theta, k \in \mathbb{Z}) \\&= \frac{1}{\sqrt{2}}\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1.v**

$$\begin{aligned}\tan 315^\circ &= \tan\left(2\pi - \frac{\pi}{4}\right) \\&= -\tan\frac{\pi}{4} \quad (\because \tan(2\pi - \theta) = -\tan\theta) \\&= -1\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1.vi**

$$\begin{aligned}\sin 510^\circ &= \sin\left(3\pi - \frac{\pi}{6}\right) \\&= \sin\frac{\pi}{6} \quad \left(\because 3\pi - \frac{\pi}{6} \text{ lies in second quadrant}\right) \\&= \frac{1}{2}\end{aligned}$$

Alternative solution

$$\begin{aligned}\sin 510^\circ &= \sin\left(3\pi - \frac{\pi}{6}\right) \\&= \sin\left(2\pi + \left(\pi - \frac{\pi}{6}\right)\right) \\&= \sin\left(\pi - \frac{\pi}{6}\right) \quad (\because \sin(2\pi + \theta) = \sin\theta, \text{ as sine is periodic with period } 2\pi) \\&= \sin\frac{\pi}{6} \quad (\because \sin(\pi - \theta) = \sin\theta) \\&= \frac{1}{2}\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1.vii**

$$\begin{aligned}\cos 570^\circ &= \cos\left(3\pi + \frac{\pi}{6}\right) \\&= \cos\left(2\pi + \left(\pi + \frac{\pi}{6}\right)\right) \\&= \cos\left(\pi + \frac{\pi}{6}\right) \quad (\because \cos(2\pi + \theta) = \cos\theta, \text{ as cosine is periodic with period } 2\pi) \\&= -\cos\frac{\pi}{6} \quad (\because \cos(\pi + \theta) = -\cos\theta) \\&= -\frac{\sqrt{3}}{2}\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1.viii**

$$\begin{aligned}\sin(-330^\circ) &= \sin\left(-\left(2\pi - \frac{\pi}{6}\right)\right) \\&= \sin\left(2\pi - \frac{\pi}{6}\right) \quad (\because \sin(-\theta) = -\sin\theta) \\&= -\left(-\sin\frac{\pi}{6}\right) \quad (\because \sin(2\pi - \theta) = -\sin\theta) \\&= \sin\frac{\pi}{6} \\&= \frac{1}{2}\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1. ix**

$$\begin{aligned}\cosec(-1200^\circ) &= \cosec\left(-\left(7\pi - \frac{\pi}{3}\right)\right) \\&= \cosec\left(7\pi - \frac{\pi}{3}\right) \quad (\because \cosec(-\theta) = -\cosec\theta) \\&= -\cosec\left(2 \times 3\pi + \left(\pi - \frac{\pi}{3}\right)\right) \\&= -\cosec\left(\pi - \frac{\pi}{3}\right) \quad (\because \cosec \text{ is periodic of period } 2\pi, \\&\qquad\qquad\qquad \therefore \cosec(2\pi + \theta) = \cosec(2n\pi + \theta) \\&\qquad\qquad\qquad = \cosec\theta \text{ for all } n \in \mathbb{N}) \\&= -\cosec\frac{\pi}{3} \quad (\therefore \cosec(\pi - \theta) = \cosec\theta) \\&= \frac{-2}{\sqrt{3}}\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1.x**

$$\begin{aligned}\tan(-585^\circ) &= -\tan(585^\circ) \quad (\because \tan(-\theta) = -\tan\theta) \\&= -\tan\left(3\pi + \frac{\pi}{4}\right) \\&= -\tan\left(2\pi + \left(\pi + \frac{\pi}{4}\right)\right) \quad (\because \tan(2\pi + \theta) = \tan\theta) \\&= -\tan\frac{\pi}{4} \quad (\because \tan(\pi + \theta) = \tan\theta) \\&= -1\end{aligned}$$

**Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xi**

$$\begin{aligned}
 \cos(855^\circ) &= \cos\left(5\pi - \frac{\pi}{4}\right) \\
 &= \cos\left(2 \times 2\pi + \left(\pi - \frac{\pi}{4}\right)\right) \\
 &= \cos\left(\pi - \frac{\pi}{4}\right) \quad (\because \cos(2k\pi + \theta) = \cos \theta \text{ for all } k \in N) \\
 &= -\cos\frac{\pi}{4} \quad (\because \cos(\pi - \theta) = -\cos \theta) \\
 &= \frac{-1}{\sqrt{2}}
 \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xii

$$\begin{aligned}
 \sin 1845^\circ &= \sin\left(10\pi + \frac{\pi}{4}\right) \\
 &= \left(2 \times 5\pi + \frac{\pi}{4}\right) \\
 &= \sin \pi \quad (\because \sin(2k\pi + \theta) = \sin \theta, \text{ for all } k \in N) \\
 &= \frac{1}{\sqrt{2}}
 \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xiii

$$\begin{aligned}
 \cos 1755^\circ &= \cos\left(10\pi - \frac{\pi}{4}\right) \\
 &= \cos\left(2 \times 5\pi - \frac{\pi}{4}\right) \\
 &= \cos\frac{\pi}{4} \quad (\because \cos(2k\pi - \theta) = \cos \theta, k \in N) \\
 &= \frac{1}{\sqrt{2}}
 \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1.xiv

$$\begin{aligned}
 4530^\circ &= \left(25\pi + \frac{\pi}{6}\right) \\
 \therefore \sin 4530^\circ &= \sin\left(25\pi + \frac{\pi}{6}\right) \\
 &= \sin\left(2 \times 12\pi + \left(\pi + \frac{\pi}{6}\right)\right) \\
 &= \sin\left(\pi + \frac{\pi}{6}\right) \quad (\because \sin(2k\pi + \theta) = \sin \theta, k \in N) \\
 &= -\sin\frac{\pi}{6} \quad (\because \sin(\pi + \theta) = -\sin \theta) \\
 &= \frac{-1}{2}
 \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 2.i

$$\begin{aligned}
 \text{LHS} &= \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ \\
 &= \tan\left(\pi + \frac{\pi}{4}\right) \cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right) \cot\left(4\pi - \frac{\pi}{4}\right) \\
 &= \tan\frac{\pi}{4} \cdot \cot\frac{\pi}{4} + \tan\frac{\pi}{4} \times \left(-\cot\frac{\pi}{4}\right) \quad (\because \cot(4\pi - \frac{\pi}{4}) = -\cot\frac{\pi}{4}) \\
 &= 1 \cdot 1 + 1 \cdot (-1) \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 2.ii

$$\begin{aligned}
 \text{LHS} &= \sin\frac{8\pi}{3} \cos\frac{23\pi}{6} + \cos\frac{13\pi}{3} \sin\frac{35\pi}{6} \\
 &= \sin\left(3\pi - \frac{\pi}{3}\right) \cos\left(4\pi - \frac{\pi}{6}\right) + \cos\left(4\pi + \frac{\pi}{3}\right) \sin\left(6\pi - \frac{\pi}{6}\right) \\
 &= \sin\frac{\pi}{3} \cos\frac{\pi}{6} + \cos\frac{\pi}{3} \left(-\sin\frac{\pi}{6}\right) \quad (\because \sin(6\pi - \theta) = -\sin \theta) \\
 &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \left(-\frac{1}{2}\right) \\
 &= \frac{3}{4} - \frac{1}{4} \\
 &= \frac{2}{4} \\
 &= \frac{1}{2} \\
 &= \text{RHS}
 \end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 2.iii

$$\text{LHS} = \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 204^\circ + \cos 300^\circ$$

$$\begin{aligned}
&= \cos 24^\circ + \cos 24^\circ + \cos 55^\circ + \cos 125^\circ + \cos 300^\circ \\
&= \cos 24^\circ + \cos(\pi + 24^\circ) + \cos 55^\circ + \cos(\pi - 55^\circ) + \cos\left(2\pi - \frac{\pi}{3}\right) \\
&= \cos 24^\circ - \cos 24^\circ + \cos 55^\circ - \cos 55^\circ + \cos\left(\frac{\pi}{3}\right) \\
&= \cos\left(\frac{\pi}{3}\right) \\
&= \frac{1}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 1.2.iv

$$\begin{aligned}
\text{LHS} &= \tan(-225^\circ)\cot(-405^\circ) - \tan(-765^\circ)\cot(675^\circ) \\
&= -\tan 225^\circ(-\cot 405^\circ) + \tan 765^\circ \cot 765^\circ \quad \left( \because \tan(-\theta) = -\tan \theta \right. \\
&\quad \left. \& \cot(-\theta) = -\cot \theta \right) \\
&= \tan\left(\pi + \frac{\pi}{4}\right)\cot\left(2\pi - \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right)\cot\left(4\pi - \frac{\pi}{4}\right) \\
&= \tan\frac{\pi}{4}\cot\frac{\pi}{4} + \tan\frac{\pi}{4} \times (-\cot\frac{\pi}{4}) \quad (\because \cot(4\pi - \theta) = -\cot \theta) \\
&= 1 \cdot 1 + 1(-1) \\
&= 1 - 1 \\
&= 0 \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 2.v

$$\begin{aligned}
\text{LHS} &= \cos 570^\circ \sin 510^\circ + \sin(-330^\circ) \cos(-390^\circ) \\
&= \cos\left(3\pi + \frac{\pi}{6}\right) \sin\left(3\pi - \frac{\pi}{6}\right) - \sin 330^\circ \cos 390^\circ \quad \left( \because \sin(-\theta) = -\sin \theta \text{ and } \cos(-\theta) = \cos \theta \right) \\
&= -\cos\frac{\pi}{6} \sin\frac{\pi}{6} - \sin\left(2\pi - \frac{\pi}{6}\right) \cos\left(2\pi + \frac{\pi}{6}\right) \\
&= -\sin\frac{\pi}{6} \cos\frac{\pi}{6} + \sin\frac{\pi}{6} \cdot \cos\frac{\pi}{6} \quad (\because \sin(2\pi - \theta) = -\sin \theta) \\
&= 0 \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 2.vi

$$\begin{aligned}
\text{LHS} &= \tan\frac{11\pi}{3} - 2\sin\frac{4\pi}{6} - \frac{3}{4}\cos\sec^2\frac{\pi}{4} + 4\cos^2\frac{17\pi}{6} \\
&= \tan\left(4\pi - \frac{\pi}{3}\right) - 2\sin\frac{2\pi}{3} - \frac{3}{4} \times (\sqrt{2})^2 + 4\cos^2\left(3\pi - \frac{\pi}{6}\right) \\
&= -\tan\frac{\pi}{3} - 2\sin\left(\pi - \frac{\pi}{3}\right) - \frac{3}{4} \times 2 + 4\cos^2\frac{\pi}{6} \\
&\quad \left( \because \tan\left(4\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3}, \cos\left(3\pi - \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} \right) \\
&= -\sqrt{3} - 2\sin\frac{\pi}{3} - \frac{3}{2} + 4 \times \left(\frac{\sqrt{3}}{2}\right)^2 \\
&= -\sqrt{3} - 2 \times \frac{\sqrt{3}}{2} - \frac{3}{2} + 4 \times \frac{3}{4} \\
&= -\sqrt{3} - \sqrt{3} - \frac{3}{2} + 3 \\
&= -2\sqrt{3} \frac{-3+6}{2} \\
&= -2\sqrt{3} + \frac{3}{2} \\
&= \frac{3 - 4\sqrt{3}}{2} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 2.vii

$$\begin{aligned}
\text{LHS} &= 3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} \\
&= 3 \times \frac{1}{2} \times 2 - 4\sin\left(\pi - \frac{\pi}{6}\right) \times 1 \quad (\because \sin(\pi - \theta) = \sin \theta) \\
&= 3 - 4\sin\frac{\pi}{6} \\
&= 3 - 4 \times \frac{1}{2}
\end{aligned}$$

2  
 = 3 - 2  
 = 1  
 = RHS  
 Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 3.i

$$\begin{aligned} \text{LHS} &= \frac{\cos(2\pi + \theta) \cos \operatorname{ec}(2\pi + \theta) \tan\left(\frac{\pi}{2} + \theta\right)}{\sec\left(\frac{\pi}{2} + \theta\right) \cos \theta \cot(\pi + \theta)} \\ &= \frac{\cos \theta \times \cos \operatorname{ec} \theta (-\cot \theta)}{-\cos \operatorname{ec} \theta \cos \theta \cot \theta} \quad \left( \begin{array}{l} \because \tan\left(\frac{\pi}{2} + \theta\right) = -\cot \theta \\ \& \sec\left(\frac{\pi}{2} + \theta\right) = -\cos \operatorname{ec} \theta \end{array} \right) \\ &= 1 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 3.ii

$$\begin{aligned} \text{LHS} &= \frac{\cos \operatorname{ec}(90^\circ + \theta) + \cot(450^\circ + \theta)}{\cos \operatorname{ec}(90^\circ - \theta) + \tan(180^\circ - \theta)} + \frac{\tan(180^\circ + \theta) + \sec(180^\circ - \theta)}{\tan(360^\circ + \theta) - \sec(-\theta)} \\ &= \frac{\sec \theta + \cot\left(2\pi + \frac{\pi}{2} + \theta\right)}{\sec \theta - \tan \theta} + \frac{\tan \theta - \sec \theta}{\tan \theta - \sec \theta} \\ &\quad \left( \because \cos \operatorname{ec}(90^\circ + \theta) = \sec \theta, \cos \operatorname{ec}(90^\circ + \theta) = \sec \theta, \tan(180^\circ - \theta) = -\tan \theta \sec(-\theta) = \sec \theta \right) \\ &= \frac{\sec \theta + \cot\left(\frac{\pi}{2} + \theta\right)}{\sec \theta - \tan \theta} + 1 \quad \left( \because \cot(2\pi + \theta) = \cot \theta \right) \\ &= \frac{\sec \theta - \tan \theta}{\sec \theta - \tan \theta} + 1 \quad \left( \because \cot\left(\frac{\pi}{2} + \theta\right) = -\tan \theta \right) \\ &= 1 + 1 \\ &= 2 \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 3.iii

$$\begin{aligned} \text{LHS} &= \frac{\sin(180^\circ + \theta) \cos(90^\circ + \theta) \tan(270^\circ - \theta) \cot(360^\circ - \theta)}{\sin(360^\circ - \theta) \cos(360^\circ + \theta) \operatorname{cosec}(-\theta) \sin(270^\circ + \theta)} \\ &= \frac{-\sin \theta (-\sin \theta) \cot \theta (-\cot \theta)}{-\sin \theta \cos \theta (-\operatorname{cosec} \theta) (-\cos \theta)} \quad \left( \begin{array}{l} \because \tan(270^\circ - \theta) = \cot \theta \\ \& \& \sin(270^\circ + \theta) = -\cos \theta \end{array} \right) \\ &= \frac{-\sin \theta \times \sin \theta \times \cos \theta \times \cos \theta \times \sin \theta}{-\sin \theta \times \cos \theta \times \sin \theta \times \sin \theta \times \cos \theta} \quad \left( \begin{array}{l} \because \cot \theta = \frac{\cos \theta}{\sin \theta} \\ \& \& \operatorname{cosec} \theta = \frac{1}{\sin \theta} \end{array} \right) \\ &= 1 \end{aligned}$$

= RHS  
 Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 3.iv

$$\begin{aligned} \text{LHS} &= \left\{ 1 + \cot \theta - \sec\left(\frac{\pi}{2} + \theta\right) \right\} \cdot \left\{ 1 + \cot \theta + \sec\left(\frac{\pi}{2} + \theta\right) \right\} \\ &= \{1 + \cot \theta - (-\cos \operatorname{ec} \theta)\} \{1 + \cot \theta - \cos \operatorname{ec} \theta\} \\ &\quad \left( \because \sec\left(\frac{\pi}{2} + \theta\right) = -\cos \operatorname{ec} \theta \right) \\ &= \{(1 + \cot \theta) + \cos \operatorname{ec} \theta\} \{(1 + \cot \theta) - \cos \operatorname{ec} \theta\} \\ &= (1 + \cot \theta)^2 - \cos \operatorname{ec}^2 \theta \\ &= 1 + \cot^2 \theta + 2 \cot \theta - \cos \operatorname{ec}^2 \theta \\ &= \cos \operatorname{ec}^2 + 2 \cot \theta - \cos \operatorname{ec}^2 \theta \quad (\because 1 + \cot^2 \theta = \cos \operatorname{ec}^2 \theta) \\ &= 2 \cot \theta \\ &= \text{RHS} \\ &\text{Proved} \end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 3 v

$$\text{LHS} = \frac{\tan(90^\circ - \theta) \sec(180^\circ - \theta) \sin(-\theta)}{\sin(180^\circ + \theta) \cot(360^\circ - \theta) \csc(90^\circ - \theta)}$$

$$= \frac{\cot \theta \times (-\sec \theta) \times (-\sin \theta)}{-\sin \theta \times (-\cot \theta) \times \sec \theta}$$

$$= 1$$

$$= \text{RHS}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 4

$$\begin{aligned}\text{LHS} &= \sin^2 \frac{\pi}{18} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} + \sin^2 \frac{4\pi}{9} \\&= \sin^2 \frac{\pi}{18} + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \sin^2 \frac{7\pi}{18} \\&= \sin^2 \left( \frac{\pi}{2} - \frac{4\pi}{9} \right) + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \sin^2 \left( \frac{\pi}{2} - \frac{\pi}{9} \right) && \left( \because \frac{\pi}{18} = \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9} \right) \\&= \cos^2 \frac{4\pi}{9} + \sin^2 \frac{4\pi}{9} + \sin^2 \frac{\pi}{9} + \cos^2 \frac{\pi}{9} && \left( \because \sin \left( \frac{\pi}{2} - \theta \right) = \cos \theta \right) \\&= 1 + 1 && \left( \because \sin^2 \theta + \cos^2 \theta = 1 \right) \\&= 2 \\&= \text{RHS}\end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 5

$$\begin{aligned}\text{LHS} &= \sec \left( \frac{3\pi}{2} - \theta \right) \sec \left( \theta - \frac{5\pi}{2} \right) + \tan \left( \frac{5\pi}{2} + \theta \right) \tan \left( \theta - \frac{3\pi}{2} \right) \\&= \sec \left( \frac{3\pi}{2} - \theta \right) \sec \left( -\left( \frac{5\pi}{2} - \theta \right) \right) + \tan \left( \frac{5\pi}{2} + \theta \right) \tan \left( -\left( \frac{3\pi}{2} - \theta \right) \right) \\&= -\csc \theta \sec \left( \frac{5\pi}{2} - \theta \right) - \cot \theta \times (-) \tan \left( \frac{3\pi}{2} - \theta \right) \\&\quad \left[ \begin{array}{l} \left( \sec \left( \frac{3\pi}{2} - \theta \right) \right) = -\csc \theta, \sec \left( -\theta \right) = \sec \theta, \tan \left( \frac{5\pi}{2} + \theta \right) = -\cot \theta \\ \& \tan \left( -\theta \right) = -\tan \theta \end{array} \right] \\&= -\csc \theta \times \csc \theta - \cot \theta \times (-1) \times \cot \theta \\&\quad \left[ \begin{array}{l} \left( \sec \left( \frac{5\pi}{2} - \theta \right) \right) = \csc \theta \\ \& \tan \left( \frac{3\pi}{2} - \theta \right) = \cot \theta \end{array} \right] \\&= -\csc^2 \theta + \cot^2 \theta \\&= -\csc^2 \theta + \csc^2 \theta - 1 \\&= -1 \\&= \text{RHS}\end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 6

$$\begin{aligned}\text{We have } A + B + C &= \pi && \left( \because \text{sum of 3 angles of a triangle is } \pi = 180^\circ \right) \\&\Rightarrow A + B = \pi - C \\&\Rightarrow \cos(A + B) = \cos(\pi - C) \\&\Rightarrow = -\cos C && \left( \because \cos(\pi - \theta) = -\cos \theta \right) \\&\Rightarrow \cos(A + B) + \cos C = 0 \\&\quad \text{Proved}\end{aligned}$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 6 ii

$$\begin{aligned}\text{We have } A + B + C &= \pi && \left( \because \text{sum of 3 angles of a triangle is } \pi = 180^\circ \right) \\&\Rightarrow A + B = \pi - C \\&\Rightarrow \frac{A + B}{2} = \frac{\pi - C}{2} \\&\Rightarrow \frac{A + B}{2} = \frac{\pi}{2} - \frac{C}{2} \\&\Rightarrow \cos \left( \frac{A + B}{2} \right) = \cos \left( \frac{\pi}{2} - \frac{C}{2} \right) \\&\Rightarrow = \sin \frac{C}{2} && \left( \because \cos \left( \frac{\pi}{2} - \theta \right) = \sin \theta \right)\end{aligned}$$

$$\text{Hence } \cos \left( \frac{A + B}{2} \right) = \sin \frac{C}{2}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 6 iii

$$\text{We have } A + B + C = \pi && \left( \because \text{sum of 3 angles of a triangle is } \pi = 180^\circ \right)$$

$$\begin{aligned} \Rightarrow A + B &= \pi - C \\ \Rightarrow \frac{A+B}{2} &= \frac{\pi-C}{2} \\ \Rightarrow \frac{A+B}{2} &= \frac{\pi}{2} - \frac{C}{2} \\ \Rightarrow \tan\left(\frac{A+B}{2}\right) &= \tan\left(\frac{\pi}{2} - \frac{C}{2}\right) \\ &= \cot\frac{C}{2} \quad \left(\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right) \end{aligned}$$

Hence  $\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$   
Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 7

$\therefore A, B, C, D$  are the angles of a cyclic quadrilateral in order,  
 $\therefore A + C = \pi$  &  $B + D = \pi$   
 $\Rightarrow \pi - A = C$  &  $\pi - D = B$   
 $\Rightarrow \cos(\pi - A) = \cos C \dots \text{(i)}$   
 $\quad \& \quad \cos(\pi - D) = \cos B \dots \text{(ii)}$

Now,  $\cos(180^\circ - A) + \cos(180^\circ + B) + (180^\circ + C) - \sin(90^\circ + D)$   
 $= \cos C + (-\cos B) - \cos C - \cos D$   
 $\quad (\because \cos(180^\circ + B) = -\cos B, \cos(180^\circ + C) = -\cos C \text{ & using (i)})$   
 $= -\cos B - \cos D$   
 $= -\cos B - (-\cos B) \quad (\text{using (ii)})$   
 $= -\cos B + \cos B$   
 $= 0$   
 Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 8i.

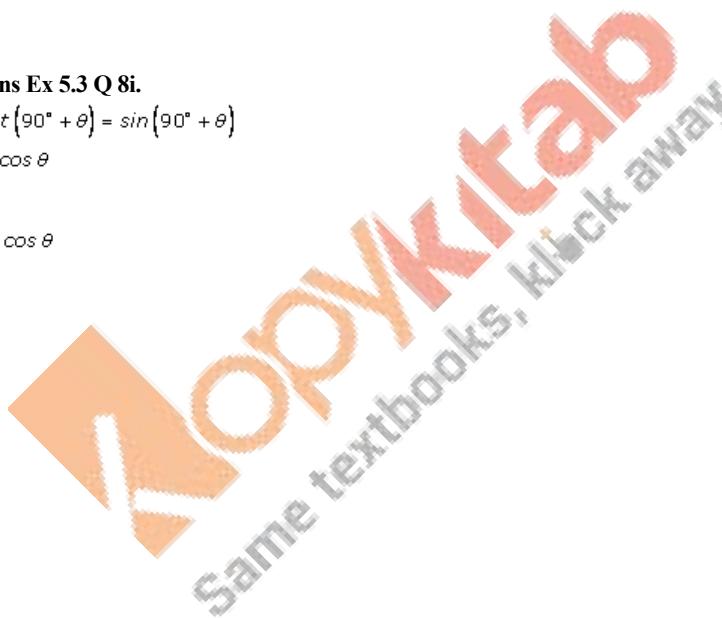
$$\begin{aligned} \csc(90^\circ + \theta) + x \cos\theta \cot(90^\circ + \theta) &= \sin(90^\circ + \theta) \\ \Rightarrow \sec\theta + x \cos\theta \times (-\tan\theta) &= \cos\theta \\ \Rightarrow \frac{1}{\cos\theta} + x \cos\theta \times \frac{(-\sin\theta)}{\cos\theta} &= \cos\theta \\ \Rightarrow \frac{1}{\cos\theta} - x \sin\theta &= \cos\theta \\ \Rightarrow \frac{1 - x \sin\theta \cos\theta}{\cos\theta} &= \cos\theta \\ \Rightarrow 1 - x \sin\theta \cos\theta &= \cos^2\theta \\ \Rightarrow 1 - \cos^2\theta &= x \sin\theta \cos\theta \\ \Rightarrow \sin^2\theta &= x \sin\theta \cos\theta \\ \Rightarrow \sin\theta &= x \cos\theta \\ \Rightarrow x &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \end{aligned}$$

Hence  $x = \tan\theta$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 8. ii.

We have  $x \cot(90^\circ + \theta) + \tan(90^\circ + \theta) \sin\theta + \csc(90^\circ + \theta) = 0$   
 $\Rightarrow x(-\tan\theta) - \cot\theta \times \sin\theta + \sec\theta = 0$

$$\begin{aligned} \Rightarrow -x \tan\theta - \frac{\cos\theta}{\sin\theta} \times \sin\theta + \frac{1}{\cos\theta} &= 0 \\ \Rightarrow -x \frac{\sin\theta}{\cos\theta} - \cos\theta + \frac{1}{\cos\theta} &= 0 \\ \Rightarrow \frac{-x \sin\theta - \cos^2\theta + 1}{\cos\theta} &= 0 \\ \Rightarrow -x \sin\theta + 1 - \cos^2\theta &= 0 \\ \Rightarrow -x \sin\theta + \sin^2\theta &= 0 \\ \Rightarrow x \sin\theta &= \sin^2\theta \end{aligned}$$



$$\Rightarrow x = \frac{\sin^2 \theta}{\sin \theta}$$

$$\Rightarrow x = \sin \theta$$

### Chapter 5 Trigonometric Functions Ex 5.3 Q 9. i.

$$\text{LHS} = \tan 720^\circ - \cos 270^\circ - \sin 150^\circ \cos 120^\circ$$

$$\begin{aligned} &= \tan 4\pi - \cos\left(\frac{3\pi}{2}\right) - \sin\left(\pi - \frac{\pi}{6}\right) \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \quad (\because \pi = 180^\circ) \\ &= 0 - 0 - \sin\frac{\pi}{6} \left(-\sin\frac{\pi}{6}\right) \quad \left(\because \tan n\pi = 0 \text{ for all } n \in \mathbb{Z} \text{ & } \cos \frac{3\pi}{2} = 0\right) \\ &= \sin^2 \frac{\pi}{6} \\ &= \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4} \\ &= \text{RHS} \end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 9. ii.

$$\text{LHS} = \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ$$

$$\begin{aligned} &= \sin\left(4\pi + \frac{\pi}{3}\right) \sin\left(3\pi - \frac{\pi}{3}\right) + \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \sin\left(\pi - \frac{\pi}{6}\right) \quad (\because \pi = 180^\circ) \\ &= \sin\frac{\pi}{3} \times \sin\frac{\pi}{3} + \left(-\sin\frac{\pi}{6}\right) \sin\frac{\pi}{6} \quad \left(\because \sin\left(4\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} \text{ & } \sin\left(3\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3}\right) \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \\ &= \text{RHS} \end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 9. iii.

$$\text{LHS} = \sin 780^\circ \sin 120^\circ + \cos 240^\circ \sin 390^\circ$$

$$\begin{aligned} &= \sin\left(4\pi + \frac{\pi}{3}\right) \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) + \cos\left(\pi + \frac{\pi}{6}\right) \sin\left(2\pi + \frac{\pi}{6}\right) \\ &= \sin\frac{\pi}{3} \times \cos\frac{\pi}{6} - \cos\frac{\pi}{3} \times \left(+\sin\frac{\pi}{6}\right) \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{4} - \frac{1}{4} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \\ &= \text{RHS} \end{aligned}$$

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 9.iv.

$$\text{LHS} = \sin 600^\circ \cos 390^\circ + \cos 480^\circ \sin 150^\circ$$

$$\begin{aligned} &= \sin\left(3\pi + \frac{\pi}{3}\right) \cos\left(2\pi + \frac{\pi}{6}\right) + \cos\left(3\pi - \frac{\pi}{3}\right) \sin\left(\pi - \frac{\pi}{6}\right) \\ &= -\sin\frac{\pi}{3} \cos\frac{\pi}{6} - \cos\frac{\pi}{3} - \sin\frac{\pi}{6} \quad \left(\because \sin\left(3\pi + \frac{\pi}{3}\right) = -\sin\frac{\pi}{3} \text{ & } \cos\left(3\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3}\right) \\ &= -\frac{\sqrt{3}}{2} \times \frac{-\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \\ &= \frac{-3}{4} - \frac{1}{4} \end{aligned}$$

$$= \frac{-4}{4}$$

$$= -1$$

= RHS

Proved

### Chapter 5 Trigonometric Functions Ex 5.3 Q 9.v.

$$\text{LHS} = \tan 225^\circ \cot 405^\circ + \tan 765^\circ \cot 675^\circ$$

$$= \tan\left(\pi + \frac{\pi}{4}\right) \cot\left(2\pi + \frac{\pi}{4}\right) + \tan\left(4\pi + \frac{\pi}{4}\right) \cot\left(4\pi - \frac{\pi}{4}\right)$$

$$= \tan\frac{\pi}{4} \cot\frac{\pi}{4} + \tan\frac{\pi}{4} \left(-\cot\frac{\pi}{4}\right)$$

$$= 1 \cdot 1 + 1 \cdot (-1)$$

$$= 1 - 1$$

$$= 0$$

= RHS

Proved

