RD Sharma Solutions Class 11 Maths Chapter 31 Ex 31.6

Mathematical Reasoning Ex 31.6 Q1

The statement is:

"100 is multiple of 4 and 5"

We know that 100 is a multiple of 4 as well as 5. So, p is true statement.
Hence, the statement is true i.e. the statement "p"
is a valid statement.

The statement is:

"125 is multiple of 5 and 7"

Since 125 is a multiple of 5 but it is not a multiple of 7. So, q is not a true statement i.e. the statement "q" is not a valid statement.

The statement is

r: 60 is multiple of 3 or 5

is a compound statement of the following statements:
p: 60 is multiple of 3
q: 60 is multiple of 5

Suppose q is false. That is, 60 is not a multiple of 5. Clearly p is true.

Thus, if we assume that q is false, then p is true.
Hence, the statement is true i.e. the statement "r" is a valid statement.

Mathematical Reasoning Ex 31.6 Q2

Let q and r be the statements given by
q : x and y are odd integers.
r : $x+y$ is an even integer.
Then, the given statement is

if q, then r.

Direct Method: Let q be true. Then,

q is true.

$\Rightarrow$ x and y are odd integers

$\Rightarrow$ $x = 2m+1$, $y = 2n+1$ for some integers m, n

$\Rightarrow$ $x+y = (2m+1)+(2n+1)$

$\Rightarrow$ $x+y = (2m+2n+2)$

$\Rightarrow$ $x+y = 2(m+n+1)$

$\Rightarrow$ $x+y$ is an even integer

$\Rightarrow$ r is true.

Thus, q is true $\Rightarrow$ r is true.
Hence, "if q, then r" is a true statement.

Let r and s be two statements given by
r : xy is an even integer.
s : At least one of x and y is an even integer

Let s be not true. Then,
s is not true
$\Rightarrow$ Both x and y are odd integers

Let $x = 2n+1$ and $y = 2m+1$ for some integers n and m. Then,
$\Rightarrow$ $xy = (2n+1)(2m+1)$ for some integers n and m.
$\Rightarrow$ $xy = 4nm + 2(n+m) + 1$ for some integers n and m.
$\Rightarrow$ xy is an odd integer
$\Rightarrow$ xy is not an even integer
$\Rightarrow$ $-r$ is true
Thus, $-s$ is true $\Rightarrow$ $-r$ is true
Hence, the given statement is true.

Mathematical Reasoning Ex 31.6 Q3

Let q and r be the statements given
q : x is a real number such that $x^3 + x = 0$.
r : x is 0.
Then, p : if q, then r.
(i) *Direct Method* : Let q be true. Then,
q is true
$\Rightarrow$ x is a real number such that $x^3 + x = 0$
$\Rightarrow$ x is a real number such that $x(x^2+1) = 0$
$\Rightarrow$ $x = 0$
$\Rightarrow$ r is true.
Thus, q is true $\Rightarrow$ r is true.
Hence, p is true.

(ii) *Method of contrapositive* : Let r be not true. Then,

r is not true.

$\Rightarrow \quad x \neq 0, x \in R$

$\Rightarrow \quad x(x^2+1) \neq 0,\ x \in R$

$\Rightarrow \quad q$ is not true

Thus, $-r \Rightarrow -q$.

Hence, $p : q \Rightarrow r$ is true.

(iii) *Method of contradiction* : If possible, let p be not true. Then,

p is not true

$\Rightarrow \quad -p$ is true

$\Rightarrow \quad -(p \Rightarrow r)$ is true

$\Rightarrow \quad q$ and $-r$ is true

$\Rightarrow \quad x$ is a real number such that $x^3 + x = 0$ and $x \neq 0$

$\Rightarrow \quad x = 0$ and $x \neq 0$

This a contradiction.

Hence, p is true.

Mathematical Reasoning Ex 31.6 Q4

Let q and r be the statements given by

q : If x is an integer and x^2 is odd

r : x is an odd integer.

Then, p : "If q, then r."

If possible, let r be false. Then,

r is false

$\Rightarrow \quad x$ is not an odd integer

$\Rightarrow \quad x$ is an even integer

$\Rightarrow \quad x = (2n)$ for some integer n

$\Rightarrow \quad x^2 = 4n^2$

$\Rightarrow \quad x^2$ is an even integer

$\Rightarrow \quad q$ is false.

Thus, r is false $\Rightarrow$ q is false.
Hence, p: "if q, then r" is a true statement.

Mathematical Reasoning Ex 31.6 Q5

The given statement can be re-written as
"*The necessary and sufficient condition that the integer n is even is n^2 must be even*"

Let p and q be the statements given by

p: the integer n is even.

q: n^2 is even.

The given statement is

"p if and only if q"

In order to check its validity, we have to check the validity of the following statements.
(i) *"If p, then q"*
(ii) *"if q, then p"*

Checking the validity of "if p, then q":

The statement "if p, then q" is given by:

"if the integer n is even, then n^2 is even"

Let us assume that n is even. Then,

$n = 2m$, where m is an integer

$\Rightarrow \quad n^2 = (2m)^2$

$\Rightarrow \quad n^2 = 4m^2$

$\Rightarrow \quad n^2$ is an even integer

Thus, n is even $\Rightarrow$ n^2 is even

$\therefore \quad$ "if p, then q" is true.

Checking the validity of "if q, then p":

"if n is an integer and n^2 is even, then n is even"

To check the validity of this statemens, we will use contrapositive method.
So, let n be an odd integer. Then,

n is odd

$\Rightarrow \quad n = 2k + 1$ for some integer k

$\Rightarrow \quad n^2 = (2k+1)^2$

$\Rightarrow \quad n^2 = 4k^2 + 4k + 1$

$\Rightarrow \quad n^2$ is not an even integer.

Thus, n is not even $\Rightarrow n^2$ is not even

$\therefore$ "if q, then p" is true.

Hence, "p if and only if q" is true.

Mathematical Reasoning Ex 31.6 Q6

Consider a triangle ABC with all angles equal. Then each angle of the triangle is equal to 60°.
Hence, ABC is not an obtuse angle triangle.
Therefore the following statement is false.
p : *"if all the angles of a triangle are equal, then the triangle is an obtuse angled triangle"*.

Mathematical Reasoning Ex 31.6 Q7

(i) False. Because, no radius of a circle is its chord.
(ii) False. Because, a chord does not have to pass through the centre.
(iii) True. Because a circle is an ellipse that has equal axes.
(iv) True. Because, for any two integers, if $x - y$ is positive then $-(x - y)$ is negative.
(v) False. Because square roots of prime numbers are irrational numbers.

Mathematical Reasoning Ex 31.6 Q8

The argument used to check the validity of the given statement
is not correct because it does not produce a contradiction.