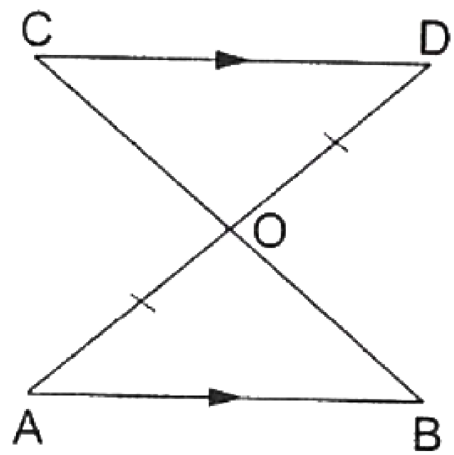
Congruence Of Triangles And Inequalities in a Triangle
CHAPTER 9

Exercise – 9 (A)

Answer1)



Given: AB || CD

To prove: i) O is the midpoint of AD

ii) ΔAOB ≅ ΔDOC

Proof:

In ΔAOB and ΔDOC

OA = OD (Given)

∠AOB = ∠ COD (vertically opposite angles)

∠OAB = ∠ODC (alternate angles)

Therefore;

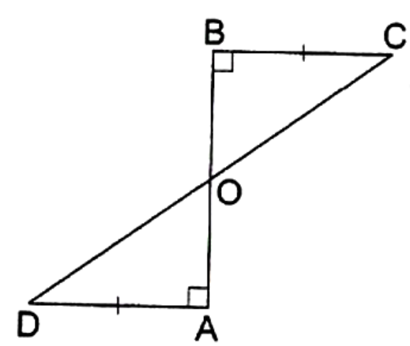
ΔAOB ≅ ΔDOC (A.A.S. criteria)

Hence;

OB = OC (c.p.c.t.)

Hence proved.

Answer2)



Given: AD=BD

To prove: CD bisects AB i.e. OA=OB

Proof:

In ΔBOC and ΔAOD

AD=BC (Given)

∠OAD=∠OBC =90 (Given)

∠AOD=∠BOC (vertically opp. Angles)

Therefore ΔBOC≅ΔAOD (A..A.S criteria)

Hence OA=OB i.e CD bisects AB

Hence proved.

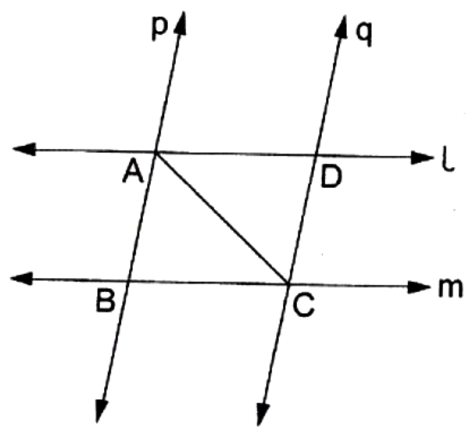
Answer3)

Given: (i) l || m

(ii) p || q

To prove: ΔABC≅ ΔCDA

Proof:



Taking l parallel to m AC is the trAnswerversal

∠ACB=∠CAD (Alternate angles)

Taking p parallel to q

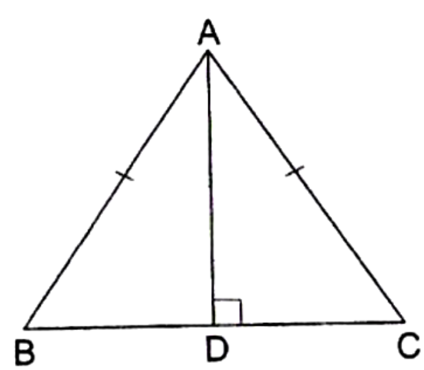
∠BAC=∠DCA (Alternate angles)

AC=AC (common)

Therefore;

ΔABC≅ ΔCDA (ASA ctiteria)

Hence Proved.



Answer4)Given: AB=AC

To prove: (i)AD bisects BC (i.e. BD=DC)

(ii) AD bisects ∠A

Prove:

In right angled ΔADB and ADC we have,

Hypotenuse AB=hypotenuse AC (Given)

AD=AD (common)

Therefore ΔADB≅ΔADC (RHS criteria)

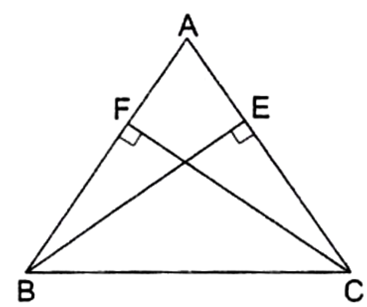
Hence BD=DC

∠BAD =∠CAD

Hence AD bisects ∠A

Answer5)

Given: BE=CF



To Prove: i) ΔABE≅ΔACF

ii)AB=AC

Proof:

In ΔABE and ΔACF

∠AEB=∠AFC=90 (Given)

∠BAE=∠CAF (common)

BE=CF (Given)

Therefore ΔABE≅ΔACF (A.A.S Criteria)

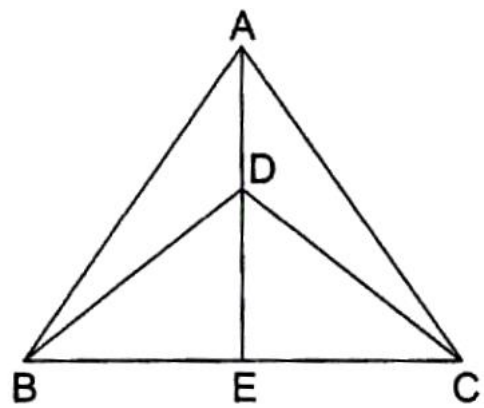
AB=AC (c.p.c.t)

Answer6)

<u>Given</u>:

(i) ΔABC and ΔDBC are two isosceles triangles in which AB=AC & BD=DC.

<u>To Prove</u>:



(i) ΔABD ≅ ΔACD

(ii) ΔABE ≅ ΔACP

(iii) AE bisects ∠A as well as ∠D.

(iv) AE is the perpendicular bisector of BC.

<u>Proof</u>:

(i) In ΔABD and ΔACD,

AD = AD (Common)

AB = AC (Given) .

BD = CD (Given)

Therefore, ΔABD ≅ ΔACD (SSS criteria)

∠BAD = ∠CAD(C.P.C.T)

∠BAE = ∠CAE

(ii) In ΔABE & ΔACE

AE= AE (Common)

∠BAE = ∠CAE

(Proved above)

AB = AC (Given)

Therefore,

$\Delta ABE \cong \Delta ACE$ (SAS criteria).

(iii) $\angle BAD = \angle CAD$ (proved in part i)

Hence, AE bisects $\angle A$.

also,

In ΔBED and ΔCED

$ED = ED$ (Common)

$BD = CD$ (Given)

$BE = CE$

($\Delta ABE \cong \Delta ACE$ so by c.p.c.t)

Therefore, $\Delta BED \cong \Delta CED$ (SSS criteria)

Thus,

$\angle BDE = \angle CDE$ (c.p.c.t)

Hence, we can say that AE bisects $\angle A$ as well as $\angle D$.

(iv) $\angle BED = \angle CED$

(by CPCT as $\Delta BED \cong \Delta CED$)

Therefore;

$BE = CE$ (c.p.c.t)

$\angle BED + \angle CED = 180°$ (BC is a straight line)

$\Rightarrow 2\angle BPD = 180°$

$\Rightarrow \angle BED = 90°$

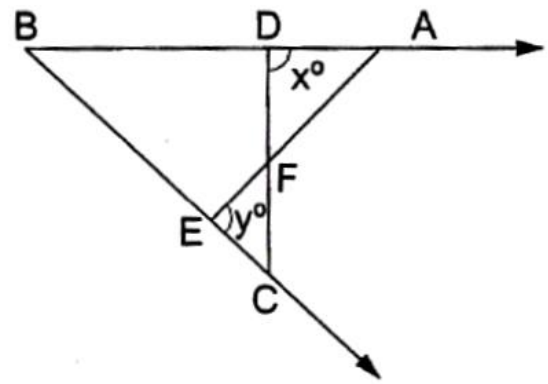
Hence, AE is the perpendicular bisector of BC.

Answer7)

Given: (i)x=y

(ii) AB=CB

To prove: AE = CD



Proof:

Consider the triangles AEB and CDB.

∠EBA=∠DBC∠EBA=∠DBC (Common angle) ...(i)

Further, we have:

∠BEA=180-y

∠BDC=180-x

Since x = y,

we have:

180-x =180-y

⇒∠BEA=∠BDC ...(ii)

AB = CB (Given) ...(iii)
From (i), (ii) and (iii),

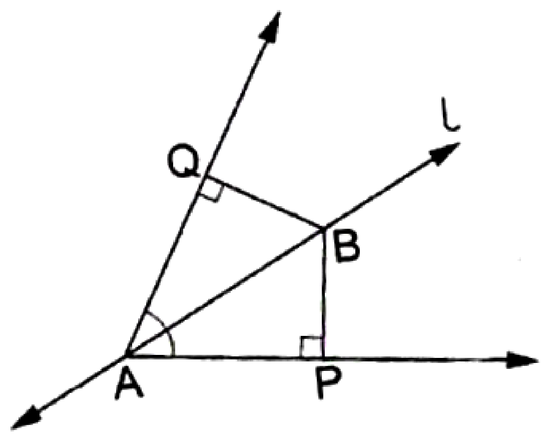
we have:
△BDC≅△BEA (AAS criterion)
∴ AE = CD (CPCT)
Hence, proved.

Answer8)

Given: (i) l is the bisector of an ∠A

(ii) BP and BQ are perpendiculars

To Prove: ΔAPB≅ΔAQB



Proof:

In ΔAPB and ΔAQB

∠P = ∠Q (Right angles)

∠BAP = ∠BAQ (l is the bisector)

AB = AB (Common)

$\Delta APB \cong \Delta AQB$ (A.A.S criteria)

Hence, Proved.

(ii) BP = BQ (By c.p.c.t)

Therefore,

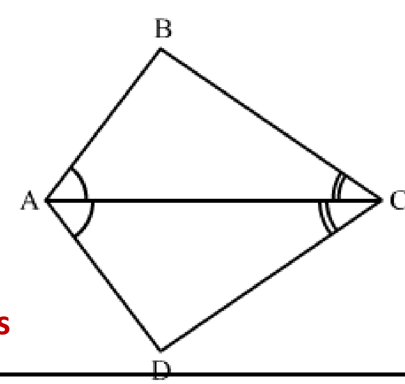
B is equidistant from the arms of ∠A

Answer9)

Given: AC bisects angles A and C.

To prove: (i) AB = AD

(ii) CB = CD



Proof:

Δ ABC and ΔADC,

we have:

∠ CAB = ∠ CAD

∠ BCA = ∠ DCA

AC = AC (common)

ΔABC≅ ΔADC

Therefore,

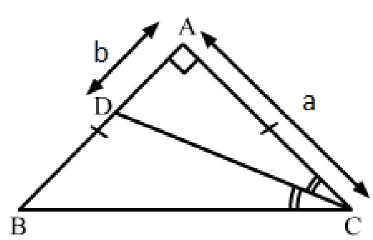
AD = AB (c.p.c.t)

CD = BC (c.p.c.t)

Answer10)Given: AB=AC

To prove: AC+AD=BC

Proof:



Let AB = AC = a and AD = b

In a right angled triangle ABC , $BC^2 = AB^2 + AC^2$

$BC^2 = a^2 + a^2$

$BC = a\sqrt{2}$

Given AD = b, we get

DB = AB – AD or DB = a – b

We have to prove that AC + AD = BC or $(a + b) = a\sqrt{2}$.

By the angle bisector theorem, we get

AD/ DB = AC / BC

$b/(a - b) = a/ a\sqrt{2}$

$b/(a - b) = 1/\sqrt{2}$

$b = (a - b)/ \sqrt{2}$

$b\sqrt{2} = a - b$

$b(1 + \sqrt{2}) = a$

$b = a/ (1 + \sqrt{2})$

Rationalizing the denominator with $(1 - \sqrt{2})$

$b = a(1 - \sqrt{2}) / (1 + \sqrt{2}) \times (1 - \sqrt{2})$

$b = a(1 - \sqrt{2})/ (-1)$

$b = a(\sqrt{2} - 1)$

$b = a\sqrt{2} - a$

$b + a = a\sqrt{2}$

or AD + AC = BC [we know that AC = a, AD = b and BC = $a\sqrt{2}$]

Hence it is proved.

Answer11)

Given: (i) OA=OB

(ii)OP=OQ

To Prove: (i) PX=QX

(ii)AX=BX

Proof:

In Δ PBO and ΔAOQ

OB=OA (Given)

OP=OQ (Given)

∠O=∠O (common)

Therefore;

Δ PBO ≅ΔQAO (S.A.S criteria)

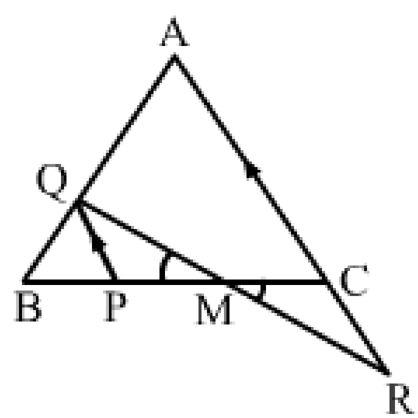
∠B=∠A (C.P.C.T)

In Δ BXQ and Δ AXP

∠B=∠A (proved above)

PX=QX (C.P.C.T.)

Hence proved



Answer12)

Given: (i) ABC is an equilateral triangle,

(ii) PQ ||AC

(iii) CR=BP

To prove: QR bisects PC or PM = MC

Proof:

Since, ΔABC is equilateral triangle,

$\angle A = \angle ACB = 60°$

Since, PQ ||AC and corresponding angles are equal,

$\angle BPQ = \angle ACB = 60°$

In ΔBPQ,

$\angle B = \angle ACB = 60°$

$\angle BPQ = 60°$

Hence, ΔBPQ is an equilateral triangle.

$\therefore PQ = BP = BQ$

Since we have BP = CR,

We say that PQ = CR ...(1)

Consider the ΔPMQ and ΔCMR,

$\angle PQM = \angle CRM$ (alternate angles)

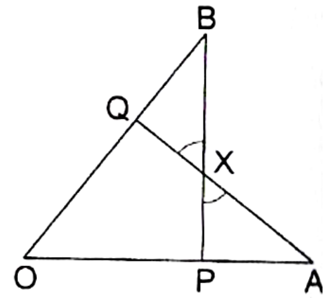
$\angle PMQ = \angle CMR$ (vertically opposite angles)

PQ = CR ... from 1

$\Delta PMQ \cong \Delta CMR$ (AAS criteria)

$\therefore PM = MC$ (c.p.c.t)

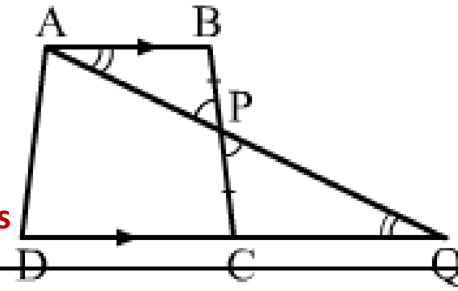
Hence proved.



Answer13)

Given: (i)AB ll DC

(ii) P is the midpoint of BC.



To Prove : (i) AB = CQ

(ii) DQ = DC + AB

so, AB ll DQ

so, ∠BAQ = ∠ DQA (alternate angles)

or ∠ BAP = ∠ CQP ------(1)

Now, in triangle ABP and triangle QCP,

∠ BAP = ∠ CQP (from (1))

∠ BPA = ∠ CPQ (vertically opposite angles)

BP = CP (since P is the midpoint of BC)

so, triangle ABP congruent triangle QCP (by AAS congruency)

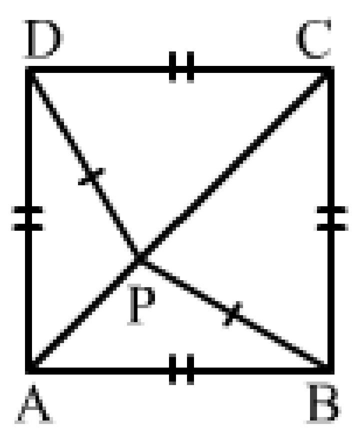
or AB = CQ (by CPCT) [proved] -----(2)

again, DQ = DC + CQ = DC + AB (from (2)) [proved]

Answer14)

Given: ABCD is a square and PB=PD

To prove: CPA is a straight line



Proof:

ΔAPD and ΔAPB,

DA = AB … (as ABCD is square)

AP = AP … (common side)

PB = PD … (Given)
ΔAPD ≅ ΔAPB (SSS criteria)

Hence, we know that, corresponding parts of the congruent triangles are equal

∠APD = ∠APB …(1)

Now consider ΔCPD and ΔCPB,

CD = CB … ABCD is square

CP = CP … common side

PB = PD … Given

Thus by SSS property of congruence,

ΔCPD ≅ ΔCPB

∠CPD = ∠CPB … (C.P.C.T.)………..(2)

Now,

Adding both sides of 1 and 2,

∠CPD +∠APD = ∠APB + ∠CPB …(3)

Angels around the point P add upto 360°

∴ ∠CPD +∠APD + ∠APB + ∠CPB = 360° ……..(4)

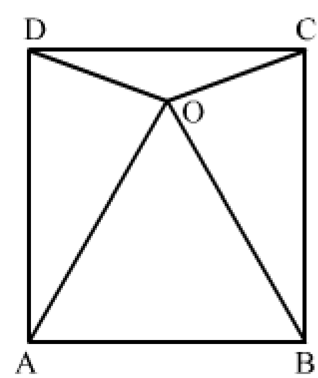
From 4,

2(∠CPD +∠APD) = 360°
∠CPD +∠APD = 180°

This proves that CPA is a straight line.

Answer15) **<u>Given:</u>** In square ABCD, ΔOAB is an equilateral triangle.

<u>To prove:</u> ΔOCD is an isosceles triangle.



<u>Proof:</u>

∵∠DAB=∠CBA=90° (Angles of square ABCD)

And, ∠OAB=OBA=60° (Angles of equilateral ΔOAB)

∴∠DAB−∠OAB=∠CBA−∠OBA=90°−60°

⇒∠OAD=∠OBC=30° .....(i)

∵∠DAB=∠CBA=90° Angles of square ABCD

And, ∠OAB=OBA=60°

Angles of equilateral ΔOAB

∴∠DAB-∠OAB=∠CBA-∠OBA=90°-60°

⇒∠OAD=∠OBC=30° .....(i)

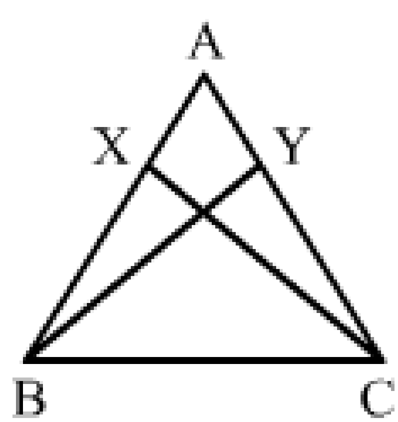
Now, in ΔDAO and ΔCBO,

AD = BC (Sides of square ABCD)
∠DAO = ∠CBO [From (i)]
AO = BO (Sides of equilateral ΔOAB)

∴ By SAS congruence criteria,
ΔDAO ≅ ΔCBO

So, OD = OC (CPCT)
Hence, ΔOCD is an isosceles triangle.

Answer16)



<u>Given:</u> AX = AY.

<u>To prove:</u> CX = BY

<u>Proof:</u>

In ΔCXA and ΔBYA,

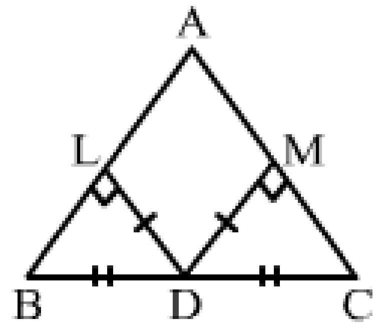
AX = AY .....Given

∠XAC = ∠YAB ... common angle

AC = AB … Given,

ΔCXA ≅ ΔBYA (S.A.S. criteria)

CX = BY (C.P.C.T.)



Answer17)

Given: BD = DC and DL ⊥ AB and DM ⊥ AC such that DL=DM

To prove: AB = AC

Proof:

In right angled triangles ΔBLD and ΔCMD,

∠BLD = ∠CMD = 90°

BD = CD … Given

DL = DM … Given

Thus by right angled hypotenuse side property of congruence,

ΔBLD ≅ ΔCMD

Hence, we know that, corresponding parts of the congruent triangles are equal

∠ABD = ∠ACD

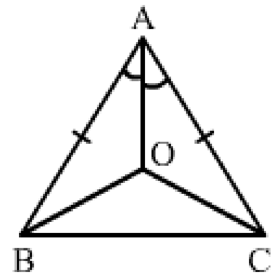
In ΔABC, we have,

∠ABD = ∠ACD

∴ AB= AC …. Sides opposite to equal angles are equal

Answer18)

Given: In ΔABC, AB=AC and the bisectors of ∠B and ∠C meet at a point O.



To prove: BO=CO and ∠BAO = ∠CAO

Proof:

In , ΔABC we have,

∠OBC = $\frac{1}{2}$ ∠B

∠OCB = $\frac{1}{2}$ ∠C

But ∠B = ∠C … Given

So, ∠OBC = ∠OCB

Since the base angles are equal, sides are equal

∴ OC = OB …(1)

Since OB and OC are bisectors of angles ∠B and ∠C respectively, we have

∠ABO = $\frac{1}{2}$ ∠B

∠ACO = $\frac{1}{2}$ ∠C

∴∠ABO = ∠ACO …(2)

Now in ΔABO and ΔACO

AB = AC … Given

∠ABO = ∠ACO … from (2)

BO = OC … from (1)

Thus by SAS property of congruence,

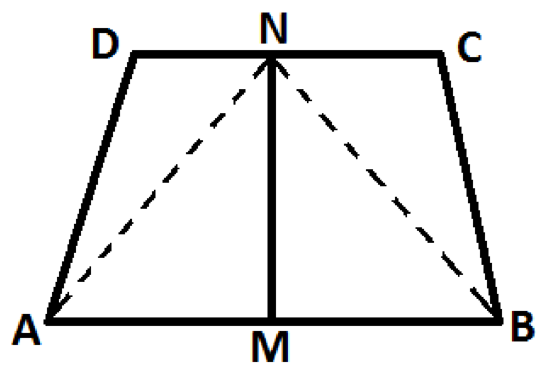
ΔABO ≅ ΔACO

Hence, we know that, corresponding parts of the congruent triangles are equal

∠BAO = ∠CAO

ie. AO bisects ∠A ; Hence proved.

Answer19)



Given: (i) ABCD is a trapezium

(ii) M is the mid point of AB

(iii) N is the mid point of CD

To Prove:AD = BC.

Construction : (i) Join B to N

(ii) Join A to N

Proof :

Consider ΔAMN and ΔBMN

∠AMN=∠BMN=90

AM=BM (M is the midpoint of AB)

MN=MN(common)

ΔAMN congruent to ΔBMN(SAS congruence rule)

Consider ΔADN and ΔBCN

DN=CN(N is the midpoint of CD)

AN=BN(CPCT)

∠MNA=∠BNM(CPCT) ....(1)

∠MNC=∠MND= 90 ....(2)

Subtracting Eq(2) from Eq(1)

∠MND-∠MNA=∠MNC-∠BNM

∠AND=∠BNC

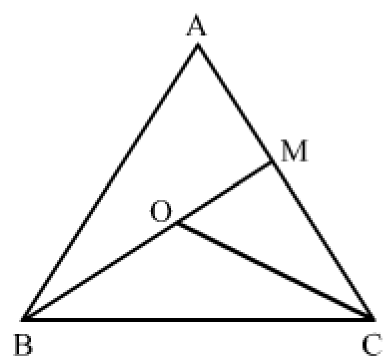
ΔAND congruent to ΔBNC

AD=BC(CPCT)

Hence proved

Answer20)

<u>**Given:**</u> Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O.



<u>**To prove:**</u>∠MOC = ∠ABC

<u>**Proof:**</u>

In ΔABC,

AB = AC (Given)

⇒ ∠ACB = ∠ABC (opposite angles to equal sides are equal)

1/2 ∠ACB = 1/2 ∠ABC (divide both sides by 2)

⇒ ∠OCB = ∠OBC ...(1) (As OB and OC are bisector of ∠B and ∠C)

Now, ∠MOC = ∠OBC + ∠OCB (as exterior angle is equal to sum of two opposite interior angle)

⇒∠MOC = ∠OBC + ∠OBC (from (1))

$\Rightarrow \angle MOC = 2\angle OBC$

$\Rightarrow \angle MOC = \angle ABC$ (because OB is bisector of $\angle B$)

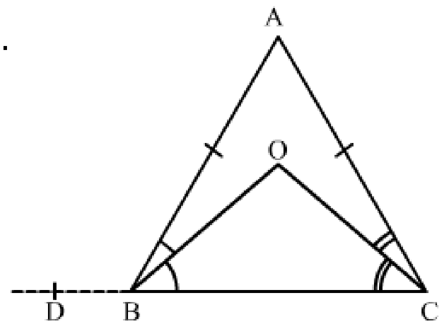
Hence proved.

Answer21)

Given: (i) In an isosceles ΔABC,

(ii) $AB = AC$,

(iii) BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$.



To prove: $\angle ABD = \angle BOC$

Construction: Produce CB to point D.

Proof:

In ΔABC,

$\because AB = AC$ (Given)

$\therefore \angle ACB = \angle ABC$ (Angle opposite to equal sides are equal)

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \angle OCB = \angle OBC$ .....(i)

(Given, BO and CO are angle bisector of $\angle ABC$ and $\angle ACB$, respectively)

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \angle OCB = \angle OBC$ .....(i)

(Given, BO and CO are angle bisector of $\angle ABC$ and $\angle ACB$, respectively)

In ΔBOC,

$\angle OBC + \angle OCB + \angle BOC = 180°$ (By angle sum property of triangle)

$\Rightarrow \angle OBC + \angle OBC + \angle BOC = 180°$ [From (i)]

$\Rightarrow 2\angle OBC+\angle BOC=180°$

$\Rightarrow \angle ABC+\angle BOC=180°$ (BO is the angle bisector of $\angle ABC$) .....(ii)

$\angle OBC+\angle OCB+\angle BOC=180°$ By angle sum property of triangle

$\Rightarrow \angle OBC+\angle OBC+\angle BOC=180°$ From (i)

$\Rightarrow 2\angle OBC+\angle BOC=180°$

$\Rightarrow \angle ABC+\angle BOC=180°$ BO is the angle bisector of $\angle ABC$ .....(ii)

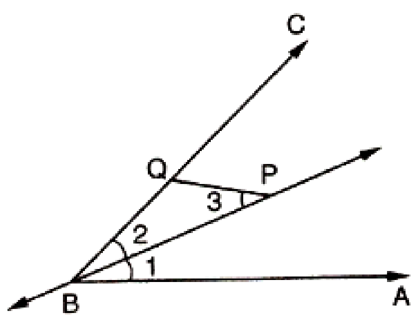
Also, DBC is a straight line.
So, $\angle ABC+\angle DBA=180°$ (Linear pair) .....(iii)

$\angle ABC+\angle DBA=180°$ (Linear pair) .....(iii)

From (ii) and (iii), we get
$\angle ABC+\angle BOC=\angle ABC+\angle DBA$

$\therefore \angle BOC=\angle DBA$

Answer22)



Given: P is the point on the bisector of an angle $\angle ABC$, and PQ || AB

To Proof: BPQ is isoscele

Since,

BP is the bisector of $\angle ABC = \angle ABP = \angle PBC$ (i)

Now,

PQ || AB

$\angle BPQ = \angle ABP$ (ii) [Alternate angles]

From (i) and (ii), we get

$\angle BPQ = \angle PBC$

Or,

∠BPQ = ∠PBQ

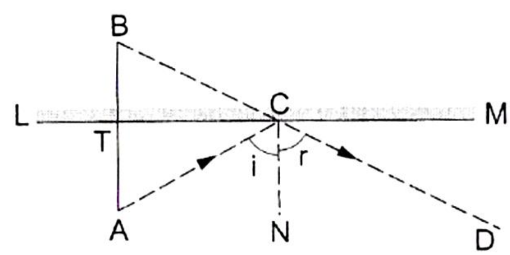
Now, in ΔBPQ

∠BPQ = ∠PBQ

ΔBPQ is an isosceles triangle

Hence Proved.

Answer23) Given: A is an object in front of mirror LM,



B is the image of A and the observer is at D

AB intersects LM at T

To Prove: A and B are equidistant from LM

AT = BT

Construction: Join BD. Let it intersect LM at C

Join AC. CN be the normal at C.

Proof:

∠i = ∠r ...(1)

AB|| NC ...[Both are perpendicular to LM]

∠CAT = ∠CAN = ∠i ...(2)[Alternate angles]

∠CBA = ∠DCN = ∠r ...(3)[Corresponding angles]

From (1), (2) and (3), we get

∠CAT = ∠CBA ...(4)

In ΔCAT and ΔCBT,

∠CAT = ∠CBT ...[From (4)]

∠ATC = ∠BTC ...[Each 90°]

CT = CT ...[Common side]

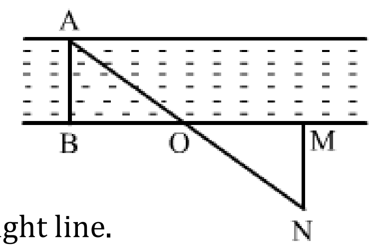
Therefore;

ΔCAT ≅ ΔCBT ...[AAA Criteria]

AT =BT ...[C.P.C.T]

Hence Proved.

Answer24)



Let AB be the breadth of the river.
M is any point situated on the bank of the river.
Let O be the mid point of BM.

Moving along perpendicular to point such that A , ,O and N are in straight line.

Then MN is the required breadth of the river.

In △OBA and △OMN,

we have:OB=OM (O is midpoint)

∠OBA=∠OMN (Each 90°)

∠AOB=∠NOM (Vertically opposite angle)

∴△OBA≅△OMN (ASA criterion)

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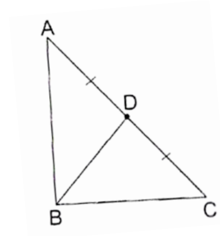
∴△OBA≅△OMN (ASA criterion)
Thus, MN = AB (CPCT)
If MN is known, one can measure the width of the river without actually crossing it.

Answer25) Given: D is the midpoint of ac
BD = ½ AC



To Prove: ∠ABC is 90°

In ΔADB, AD = BD

∠DAB = ∠DBA = ∠x (Opposite angles)

In ΔDCB, BD = CD

∠DBC = ∠DCB = ∠y

In ΔABC we will use the angle sum property

∠ABC + ∠BCA + ∠CAB = 180°

2(∠x + ∠y) = 180°

∠x + ∠y = 90°

∠ABC = 90°

This meAnswer that ABC is the right angled triangle.

Answer26) No, because in the congruent rule, the two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, that is, SAS criteria.

Answer 27) No,

Corresponding sides must be equal.