EXERCISE – 3G

Formula Used : $(x^3+y^3+z^3-3xyz)=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Answer.1. $(x+y-z)(x^2+y^2+z^2-xy+yz+zx)$

$.(x+y-z)(x^2+y^2+z^2-xy+yz+zx)=\{x+y+(-z)\}\{x^2+y^2+(-z)^2-xy-y(-z)-(-z)x\}$

$=\{x^3+y^3+(-z)^3-3xy(-z)\}$

$=(x^3+y^3-z^3+3xyz)$

Answer.2. $(x-y-z)(x^2+y^2+z^2+xy-yz+zx)$

$.(x-y-z)(x^2+y^2+z^2+xy-yz+zx)=\{x+(-y)+(-z)\}\{x^2+(-y)^2+(-z)^2-x(-y)-(-y)(-z)-(-z)x\}$

$=\{x^3+(-y)^3+(-z)^3-3x(-y)(-z)\}$

$=(x^3-y^3-z^3-3xyz)$

Answer.3. $(x-2y+3)(x^2+4y^2+2xy+6y-3x+9)$

$.(x-2y+3)(x^2+4y^2+9+2xy+6y-3x)=\{x+(-2y)+3\}\{x^2+(-2y)^2+3^2-x(-2y)-(-2y)(3)-3(x)\}$

$=\{x^3+(-2y)^3+(3)^3-3x(-2y)(3)\}$

$=(x^3-8y^3+27+18xy)$

Answer.4. $(3x-5y+4)(9x^2+25y^2+15xy+20y-12x+16)$

$.(3x-5y+4)(9x^2+25y^2+15xy+20y-12x+16)=\{3x+(-5y)+4\}\{(3x)^2+(-5y)^2+4^2-3x(-5y)-(-5y)(4)-4(3x)\}$

$=\{(3x)^3+(-5y)^3+(4)^3-3(3x)(-5y)(4)\}$

$=(27x^3-125y^3+64+180xy)$

Answer.5. $125a^3+b^3+64c^3-60abc$

$125a^3+b^3+64c^3-60abc=(5a)^3+(b)^3+(4c)^3-3\times(5a)\times(b)\times(4c)$

$=(5a+b+4c)[(5a)^2+(b)^2+(4c)^2-(5a)(b)-(b)(4c)-(4c)(5a)]$

$=(5a+b+4c)(25a^2+b^2+16c^2-5ab-4bc-20ca)$

Answer.6. $a^3+8b^3+64c^3-24abc$

$a^3+8b^3+64c^3-24abc=(a)^3+(2b)^3+(4c)^3-3\times(a)\times(2b)\times(4c)$

$=(a+2b+4c)[(a)^2+(2b)^2+(4c)^2-(a)(2b)-(2b)(4c)-(4c)(a)]$

$=(a+2b+4c)(a^2+4b^2+16c^2-2ab-8bc-4ca)$

Answer.7. $1+b^3+8c^3-6abc$

$1+b^3+8c^3-6abc=(1)^3+(b)^3+(2c)^3-3\times(1)\times(b)\times(2c)$

$=(1+b+2c)[(1)^2+(b)^2+(2c)^2-(1)(b)-(b)(2c)-(2c)(1)]$

$=(1+b+2c)(1+b^2+4c^2-b-2b-2c)$

Answer.8. $216+27b^3+8c^3-108abc$

$216+27b^3+8c^3-108abc=(6)^3+(3b)^3+(2c)^3-3\times(6)\times(3b)\times(2c)$

$=(6+3b+2c)[(6)^2+(3b)^2+(2c)^2-(6)(3b)-(3b)(2c)-(2c)(6)]$

$=(6+3b+2c)(36+9b^2+4c^2-18b-6bc-12c)$

Answer.9. $27a^3-b^3+8c^3+18abc$

$27a^3-b^3+8c^3+18abc=(3a)^3+(-b)^3+(2c)^3-3\times(3a)\times(-b)\times(2c)$

$=(3a-b+2c)[(3a)^2+(-b)^2+(2c)^2-(3a)(-b)-(-b)(2c)-(2c)(3a)]$

$=(3a-b+2c)(9a^2+b^2+4c^2+3ab+2bc-6ca)$

Answer.10. $\boldsymbol{8a^3+125b^3-64c^3+120abc}$

$$8a^3+125b^3-64c^3+120abc = (2a)^3+(5b)^3+(-4c)^3-3\times(2a)\times(5b)\times(-4c)$$
$$=(2a+5b-4c)[(2a)^2+(5b)^2+(-4c)^2-(2a)(5b)-(5b)(-4c)-(-4c)(2a)]$$
$$=(2a+5b-4c)(4a^2+25b^2+16c^2-10ab+20bc+8ca)$$

Answer.11. $\boldsymbol{8-27b^3-343c^3-126abc}$

$$8-27b^3-343c^3-126abc = (2)^3+(-3b)^3+(-7c)^3-3\times(2)\times(-3b)\times(-7c)$$
$$=(2-3b-7c)[(2)^2+(-3b)^2+(-7c)^2-(2)(-3b)-(-3b)(-7c)-(-7c)(2)]$$
$$=(2-3b-7c)(4+9b^2+49c^2+6b-21bc+14c)$$

Answer.12. $\boldsymbol{125-8x^3-27y^3-90xy}$

$$125-8x^3-27y^3-90xy = (5)^3+(-2x)^3+(-3y)^3-3\times(5)\times(-2x)\times(-3y)$$
$$=(5-2x-3y)[(5)^2+(-2x)^2+(-3y)^2-(5)(-2x)-(-2x)(-3y)-(-3y)(5)]$$
$$=(5-2x-3y)(25+4x^2+9y^2+10x-6xy+15y)$$

Answer.13. $\boldsymbol{2\sqrt{2}a^3+16\sqrt{2}b^3+c^3-12abc}$

$$2\sqrt{2}a^3+16\sqrt{2}b^3+c^3-12abc = (\sqrt{2}a)^3+(2\sqrt{2}b)^3+(c)^3-3(\sqrt{2}a)(2\sqrt{2}b)(c)$$
$$=(\sqrt{2}a+2\sqrt{2}b+c)\left[(\sqrt{2}a)^2+(2\sqrt{2}b)^2+(c)^3-(\sqrt{2}a)(2\sqrt{2}b)-(2\sqrt{2}b)c-c(\sqrt{2}a)\right]$$
$$=(\sqrt{2}a+2\sqrt{2}b+c)(2a^2+8b^2+c^2-4ab-2\sqrt{2}bc-\sqrt{2}ac)$$

Answer.14. $\boldsymbol{27x^3-y^3-z^3-9xyz}$

$$27x^3-y^3-z^3-9xyz = (3x)^3+(-y)^3+(-z)^3-3\times(3x)\times(-y)\times(-z)$$
$$=(3x-y-z)[(3x)^2+(-y)^2+(-z)^2-(3x)(-y)-(-y)(-z)-(-z)(3x)]$$
$$=(3x-y-z)(9x^2+y^2+z^2+3xy-xy+3zx)$$

Answer.15. $\boldsymbol{2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc}$

$$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc = (\sqrt{2}a)^3+(\sqrt{3}b)^3+(c)^3-3(\sqrt{2}a)(\sqrt{3}b)(c)$$
$$=(\sqrt{2}a+\sqrt{3}b+c)\left[(\sqrt{2}a)^2+(\sqrt{3}b)^2+(c)^3-(\sqrt{2}a)(\sqrt{3}b)-(\sqrt{3}b)c-c(\sqrt{2}a)\right]$$
$$=(\sqrt{2}a+\sqrt{3}b+c)(2a^2+9b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ac)$$

Answer.16. $\boldsymbol{3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc}$

$$3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc = (\sqrt{3}a)^3+(-b)^3+(-\sqrt{5}c)^3-3(\sqrt{3}a)(-b)(-\sqrt{5}c)$$
$$=(\sqrt{3}a-b-\sqrt{5}c)\left[(\sqrt{3}a)^2+(-b)^2+(-\sqrt{5}c)^3-(\sqrt{3}a)(-b)-(-b)(-\sqrt{5}c)-(-\sqrt{5}c)(\sqrt{3}a)\right]$$
$$=(\sqrt{3}a-b-\sqrt{5}c)(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ac)$$

Answer.17. $\boldsymbol{(a-b)^3+(b-c)^3+(c-a)^3}$

$Let (a-b)=x, (b-c)=y \text{ and } (c-a)=z,$

$(a-b)^3+(b-c)^3+(c-a)^3$

$= x^3+y^3+z^3, where\ (x+y+z)=(a-b)+(b-c)+(c-a)=0$

$= 3xyz\ [\because (x+y+z)=0 \Rightarrow (x^3+y^3+z^3)=3xyz]$

$= 3(a-b)(b-c)(c-a)$

Answer.18. $\boldsymbol{(a-3b)^3+(3b-c)^3+(c-a)^3}$

$Let (a-3b)=x, (3b-c)=y \text{ and } (c-a)=z,$

$(a-3b)^3+(3b-c)^3+(c-a)^3$

$= x^3+y^3+z^3, where\ (x+y+z)=(a-3b)+(3b-c)+(c-a)=0$

$= 3xyz\ [\because (x+y+z)=0 \Rightarrow (x^3+y^3+z^3)=3xyz]$

$= 3(a-3b)(3b-c)(c-a)$

Answer.19. $(3a-2b)^3+(2b-5c)^3+(5c-3a)^3$

$Let (3a-2b) = x, (2b-5c) = y\ and\ (5c-3a) = z,$

$(3a-2b)^3+(2b-5c)^3+(5c-3a)^3$

$= x^3+y^3+z^3, where\ (x+y+z) = (3a-2b)+(2b-5c)+(5c-3a) = 0$

$= 3xyz\ [\because (x+y+z) = 0 \Rightarrow (x^3+y^3+z^3) = 3xyz]$

$= 3(3a-2b)(2b-5c)(5c-3a)$

Answer.20. $(5a-7b)^3+(7b-9c)^3+(9c-5a)^3$

$Let (5a-7b) = x, (7b-9c) = y\ and\ (9c-5a) = z,$

$(5a-7b)^3+(7b-9c)^3+(9c-5a)^3$

$= x^3+y^3+z^3, where\ (x+y+z) = (5a-7b)+(7b-9c)+(9c-5a) = 0$

$= 3xyz\ [\because (x+y+z) = 0 \Rightarrow (x^3+y^3+z^3) = 3xyz]$

$= 3(5a-7b)(7b-9c)(9c-5a)$

Answer.21. $a^3(b-c)^3+b^3(c-a)^3+c^3(a-b)^3$

$Let a(b-c) = x, b(c-a) = y\ and\ c(a-b) = z,$

$[a(b-c)]^3+[b(c-a)]^3+[c(a-b)]^3$

$= x^3+y^3+z^3, where\ (x+y+z) = a(b-c)+b(c-a)+c(a-b) = 0$

$= 3xyz\ [\because (x+y+z) = 0 \Rightarrow (x^3+y^3+z^3) = 3xyz]$

$= 3abc(b-c)(c-a)(a-b)$

Answer.22.

(i) $(-12)^3+7^3+5^3$

$$Let\ x = (-12), \quad y = 7\ and\ z = 5$$
$$(x+y+z) = 0$$
$$\Rightarrow (x^3+y^3+z^3) = 3xyz$$
$$\Rightarrow (-12)^3+7^3+5^3 = 3\times(-12)\times(7)\times(5)$$
$$\Rightarrow (-12)^3+7^3+5^3 = -108$$

(ii) $(28)^3+(-15)^3+(-13)^3$

$$Let\ x = 28, \quad y = (-15)\ and\ z = (-13)$$
$$(x+y+z) = 0$$
$$\Rightarrow (x^3+y^3+z^3) = 3xyz$$
$$\Rightarrow (28)^3+(-15)^3+(-13)^3 = 3\times(28)\times(-15)\times(-13)$$
$$\Rightarrow (28)^3+(-15)^3+(-13)^3 = 16380$$

Answer.23. $(a+b+c)^3-a^3-b^3-c^3 = 3(a+b)(b+c)(c+a)$

$L.H.S \Rightarrow (a+b+c)^3-a^3-b^3-c^3$

$= [(a+b)+c]^3-a^3-b^3-c^3$

$= (a+b)^3+c^3+3(a+b)c\times[(a+b)+c]-a^3-b^3-c^3$

$= a^3+b^3+3ab(a+b)+c^3+3(a+b)c\times[(a+b)+c]-a^3-b^3-c^3$

$= 3ab(a+b)+3(a+b)c\times[(a+b)+c]$

$= 3(a+b)[ab+c(a+b)+c^2]$

$= 3(a+b)[ab+ac+bc+c^2]$

$= 3(a+b)[a(b+c)+c(b+c)]$

$= 3(a+b)(b+c)(a+c)$

$= R.H.S$

Answer.24. $Given, a+b+c = 0\ and\ a, b, c\ are\ all\ non-zero$

$\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab} = 3$

$L.H.S \Rightarrow \frac{(a^3+b^3+c^3)}{abc}$

$\Rightarrow \frac{3abc}{abc}$ $\quad [\because (a+b+c) = 0 \Rightarrow (a^3 + b^3 + c^3) = 3abc]$

$\Rightarrow 3$

$\Rightarrow R.H.S$

Answer.25. *Given* $a + b + c = 9$ *and* $a^2 + b^2 + c^2 = 35$

$(a + b + c) = 9 \Rightarrow (a + b + c)^2 = 81$

$\Rightarrow (a^2 + b^2 + c^2) + 2(ab + bc + ca) = 81$

$\Rightarrow 35 + 2(ab + bc + ca) = 81$

$\Rightarrow (ab + bc + ca) = 23$

$\therefore (a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$

$= 9 \times (35 - 23)$

$= 9 \times 12$

$= 108$