EXERCISE – 3F

Formulae Used: (i) $(x^3 + y^3) = (x + y)(x^2 - xy + y^2)$
(ii) $(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$

Answer.1. $x^3 + 27$

$.x^3 + 27 = (x)^3 + (3)^3$
$= (x + 3)(x^2 - 3x + 9)$

Answer.2. $27a^3 + 64b^3$

$.27a^3 + 64b^3 = (3a)^3 + (4b)^3$
$= (3a + 4b)(9a^2 - 12ab + 16b^2)$

Answer.3. $125a^3 + \frac{1}{8}$

$.125a^3 + \frac{1}{8} = (5a)^3 + \left(\frac{1}{8}\right)^3$
$= \left(5a + \frac{1}{8}\right)\left(25a^2 - \frac{5a}{2} + \frac{1}{4}\right)$

Answer.4. $216x^3 + \frac{1}{125}$

$.216x^3 + \frac{1}{125} = (6x)^3 + \left(\frac{1}{5}\right)^3$
$= \left(6x + \frac{1}{5}\right)\left(36x^2 - \frac{6x}{5} + \frac{1}{25}\right)$

Answer.5. $16x^4 + 54x$

$.16x^4 + 54x = 2x(8x^3 + 27)$
$= 2x\{(2x)^3 + (3)^3\}$
$= 2x(2x + 3)(4x^2 - 6x + 6)$

Answer.6. $7a^3 + 56b^3$

$.7a^3 + 56b^3 = 7(a^3 + 8b^3)$
$= 7\{(a)^3 + (2b)^3\}$
$= 7(a + 2b)(a^2 - 2ab + 4b^2)$

Answer.7. $x^5 + x^2$

$.x^5 + x^2 = x^2(x^3 + 1)$
$= x^2\{(x)^3 + (1)^3\}$
$= x^2(x + 1)(x^2 - x + 1)$

Answer.8. $a^3 + 0.008$

$.a^3 + 0.008 = (a)^3 + (0.2)^3$
$= (a + 0.2)(a^2 - 0.2a + 0.04)$

Answer.9. $1 - 27a^3$

$.1 - 27a^3 = (1)^3 - (3a)^3$
$= (1 - 3a)(1 + 3a + 9a^2)$

Answer.10. $64a^3 - 343$

$.64a^3 - 343 = (4a)^3 - (7)^3$
$= (4a - 7)(16a^2 + 28a + 49)$

Answer.11. $x^3 - 512$

$.x^3 - 512 = (x)^3 - (8)^3$

$\qquad = (x - 8)(x^2 + 8x + 64)$

Answer.12. $a^3 - 0.064$

$.a^3 - 0.064 = (a)^3 - (0.4)^3$

$\qquad = (a - 0.4)(a^2 + 0.4a + 0.16)$

Answer.13. $8x^3 - \frac{1}{27y^3}$

$.8x^3 - \frac{1}{27y^3} = (2x)^3 - \left(\frac{1}{3y}\right)^3$

$\qquad = \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

Answer.14. $\frac{x^3}{216} - 8y^3$

$.\frac{x^3}{216} - 8y^3 = \left(\frac{x}{6}\right)^3 - (2y)^3$

$\qquad = \left(\frac{x}{6} + 2y\right)\left(\frac{x^2}{36} + \frac{2xy}{6} + 4y^2\right)$

$\qquad = \left(\frac{x}{6} + 2y\right)\left(\frac{x^2}{36} + \frac{xy}{3} + 4y^2\right)$

Answer.15. $x - 8xy^3$

$.x - 8xy^3 = x(1 - 8y^3)$

$\qquad = x\{(1)^3 - (2y)^3\}$

$\qquad = x(1 - 2y)(1 + 2y + 4y^2)$

Answer.16. $32x^4 - 500x$

$.32x^4 - 500x = 4x(8x^3 - 125)$

$\qquad = 4x\{(2x)^3 - (5)^3\}$

$\qquad = 4x(2x - 5)(4x^2 + 10x + 25)$

Answer.17. $3a^7b - 81a^4b^4$

$3a^7b - 81a^4b^4 = 3a^4b(a^3 - 27b^3)$

$\qquad = 3a^4b\{(a)^3 - (3b)^3\}$

$\qquad = 3a^4b(a - 3b)(a^2 + 3ab + 9b^2)$

Answer.18. $x^4y^4 - xy$

$.x^4y^4 - xy = xy(x^3y^3 - 1)$

$\qquad = xy\{(xy)^3 - (1)^3\}$

$\qquad = xy(xy - 1)(x^2y^2 + xy + 1)$

Answer.19. $8x^2y^3 - x^5$

$8x^2y^3 - x^5 = x^2(8y^3 - x^3)$

$\qquad = x^2\{(2y)^3 - (x)^3\}$

$\qquad = x^2(2y - x)(4y^2 + 2xy + x^2)$

Answer.20. $1029 - 3x^3$

$.1029 - 3x^3 = 3(343 - x^3)$

$\qquad = 3\{(7)^3 - (x)^3\}$

$$= 3(7 - x)(49 + 7x + x^2)$$

Answer.21. $x^6 - 729$

$$x^6 - 729 = (x^2)^3 - (9)^3$$
$$= (x^2 - 9)\{(x^2)^2 + 9x^2 + 81)\}$$
$$= (x - 3)(x + 3)(x^4 + 9x^2 + 81)$$
$$= (x - 3)(x + 3)\{(x^4 + 9x^2 + 9x^2 + 81) - (3x)^2\}$$
$$= (x - 3)(x + 3)\{(x^2 + 9)^2 - (3x)^2\}$$
$$= (x - 3)(x + 3)(x^2 + 3x + 9)(x^2 - 3x + 9)$$

Answer.22. $x^9 - y^9$

$$x^9 - y^9 = (x^3)^3 - (y^3)^3$$
$$= (x^3 - y^3)\{(x^3)^2 + x^3y^3 + (y^3)^2\}$$
$$= \{(x)^3 - (y)^3\}(x^6 + x^3y^3 + y^6)$$
$$= (x - y)(x^2 + xy + y^2)(x^6 + x^3y^3 + y^6)$$

Answer.23. $(a + b)^3 - (a - b)^3$

Let $(a + b) = x$ *and* $(a - b) = y$

$$(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$$

Now, putting value of x and y

$$RHS = (x - y)(x^2 + xy + y^2)$$
$$= \{(a + b) - (a - b)\}\{(a + b)^2 + (a + b)(a - b) + (a - b)^2\}$$
$$= (a + b - a + b)\{a^2 + b^2 + 2ab + a^2 - ab + ab - b^2 + a^2 + b^2 - 2ab\}$$
$$= (2b)(3a^2 + b^2)$$

So,

$$(a + b)^3 - (a - b)^3 = 2b(3a^2 + b^2)$$

Answer.24. $8a^3 - b^3 - 4ax + 2bx$

$$8a^3 - b^3 - 4ax + 2bx = (2a)^3 - (b)^3 - 2x(2a - b)$$
$$= (2a - b)(4a^2 + 2ab + b^2) - 2x(2a - b)$$
$$= (2a - b)(4a^2 + 2ab + b^2 - 2x)$$

Answer.25. $a^3 + 3a^2b + 3ab^2 + b^3 - 8$

$$a^3 + 3a^2b + 3ab^2 + b^3 - 8 = \{a^3 + b^3 + 3ab(a + b)\} - (2)^3$$
$$= (a + b)^3 - (2)^3$$
$$= (a + b - 2)[(a + b)^2 + 2(a + b) + 4]$$

Answer.26. $a^3 - \frac{1}{a^3} - 2a + \frac{2}{a}$

$$a^3 - \frac{1}{a^3} - 2a + \frac{2}{a} = \left(a^3 - \frac{1}{a^3}\right) - \left(2a + \frac{2}{a}\right)$$
$$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2}\right) - 2\left(a - \frac{1}{a}\right)$$
$$= \left(a - \frac{1}{a}\right)\left(a^2 + 1 + \frac{1}{a^2} - 2\right)$$
$$= \left(a - \frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right)$$

Answer.27. $2a^3 + 16b^3 - 5a - 10b$

$$2a^3 + 16b^3 - 5a - 10b = 2(a^3 + 8b^3) - 5(a + 2b)$$
$$= 2(a + 2b)(a^2 - 2ab + 4b^2) - 5(a + 2b)$$
$$= (a + 2b)\{2(a^2 - 2ab + 4b^2) - 5\}$$
$$= (a + 2b)(2a^2 - 4ab + 8b^2 - 5)$$

Answer.28. $a^6 + b^6$

$$a^6 + b^6 = (a^2)^3 + (b^2)^3$$

$$= (a^2 + b^2)\{(a^2)^2 - a^2b^2 + (b^2)^2\}$$
$$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$$

Answer.29. $a^{12} - b^{12}$

$$a^{12} - b^{12} = (a^6)^2 - (b^6)^2$$
$$= (a^6 - b^6)(a^6 + b^6)$$
$$= [(a^3)^2 - (b^3)^2][(a^2)^3 + (b^2)^3]$$
$$= (a^3 - b^3)(a^3 + b^3)[(a^2 + b^2)(a^4 - a^2b^2 + b^4)]$$
$$= (a - b)(a^2 + ab + b^2)(a + b)(a^2 - ab + b^2)(a^2 + b^2)(a^4 - a^2b^2 + b^4)$$
$$= (a - b)(a + b)(a^2 + b^2)(a^2 + ab + b^2)(a^2 - ab + b^2)(a^4 - a^2b^2 + b^4)$$

Answer.30. $x^6 - 7x^3 - 8$

Let $x^3 = a$

$$x^6 - 7x^3 - 8 = a^2 - 7a - 8$$
$$= a^2 - 8a + a - 8$$
$$= a(a - 8) + 1(a - 8)$$
$$= (a + 1)(a - 8)$$

So, $x^6 - 7x^3 - 8 = (a + 1)(a - 8)$

$$= (x^3 + 1)(x^3 - 8) \qquad\qquad [\because a = x^3]$$
$$= [(x)^3 + (1)^3][(x)^3 - (2)^3]$$
$$= (x + 1)(x^2 - x + 1)(x - 2)(x^2 + x + 4)$$
$$= (x - 2)(x + 1)(x^2 - x + 1)(x^2 + x + 4)$$

Answer.31. $x^3 - 3x^2 + 3x + 7$

$$x^3 - 3x^2 + 3x + 7 = (x^3 - 3x^2 + 3x - 1) + 8$$
$$= [x^3 - 1^3 - 3x(x - 1)] + (2)^3$$
$$= (x - 1)^3 + (2)^3$$
$$= [(x - 1) + 2][(x - 1)^2 - (x - 1) \times (2) + (2)^2]$$
$$= (x + 1)[(x^2 - 2x + 1) - 2x + 2 + 4]$$
$$= (x + 1)(x^2 - 4x + 7)$$

Answer.32. $(x + 1)^3 + (x - 1)^3$

$$(x + 1)^3 + (x - 1)^3 = [(x + 1) + (x - 1)][(x + 1)^2 - (x + 1)(x - 1) + (x - 1)^2]$$
$$= (2x)[(x^2 + 2x + 1) - x^2 + 1 + (x^2 - 2x + 1)]$$
$$= 2x[x^2 + 2x + 1 - x^2 + 1 + x^2 + 1 - 2x]$$
$$= 2x(x^2 + 3)$$

Answer.33. $(2a + 1)^3 + (a - 1)^3$

$$(2a + 1)^3 + (a - 1)^3 = [(2a + 1) + (a - 1)][(2a + 1)^2 - (2a + 1)(a - 1) + (a - 1)^2]$$
$$= (3a)[(4a^2 + 4a + 1) - 2a^2 + 2a - a + 1 + (a^2 + 1 - 2a)]$$
$$= 3a[4a^2 + 4a + 1 - 2a^2 + a + 1 + a^2 + 1 - 2a]$$
$$= 3a(3a^2 + 3a + 3)$$
$$= 9a(a^2 + a + 1)$$

Answer.34. $8(x + y)^3 - 27(x - y)^3$

$$8(x + y)^3 - 27(x - y)^3 = \{2(x + y)\}^3 - \{3(x - y)\}^3$$
$$= \{2(x + y) - 3(x - y)\}[\{2(x + y)\}^2 + \{2(x + y)\}\{3(x - y)\} + \{3(x - y)\}^2]$$
$$= [2x + 2y - 3x + 3y][\{4(x^2 + 2xy + y^2)\} + 6(x^2 - y^2) + \{9(x^2 - 2xy + y^2)\}]$$
$$= (-x + 5y)[4x^2 + 8xy + 4y^2 + 6x^2 - 6y^2 + 9x^2 - 18xy + 9y^2]$$
$$= (-x + 5y)(19x^2 - 10xy + 7y^2)$$

Answer.35. $(x + 2)^3 + (x - 2)^3$

$.(x + 2)^3 + (x - 2)^3 = [(x + 2) + (x - 2)][(x + 2)^2 - (x + 2)(x - 2) + (x - 2)^2]$
$= (2x)[(x^2 + 4x + 4) - x^2 + 4 + (x^2 - 4x + 4)]$
$= 2x[x^2 + 4x + 4 - x^2 + 4 + x^2 - 4x + 4]$
$= 2x(x^2 + 12)$

Answer.36. $(x + 2)^3 - (x - 2)^3$

$.(x + 2)^3 - (x - 2)^3 = [(x + 2) - (x - 2)][(x + 2)^2 + (x + 2)(x - 2) + (x - 2)^2]$
$= (4)[(x^2 + 4x + 4) + x^2 - 4 + (x^2 - 4x + 4)]$
$= 4[x^2 + 4x + 4 + x^2 - 4 + x^2 - 4x + 4]$
$= 4(3x^2 + 4)$

Answer.37. *Prove that :* $\dfrac{0.85 \times 0.85 \times 0.85 + 0.15 \times 0.15 \times 0.15}{0.85 \times 0.85 - 0.85 \times 0.15 + 0.15 \times 0.15} = 1$

$L.H.S. \Rightarrow \dfrac{0.85 \times 0.85 \times 0.85 + 0.15 \times 0.15 \times 0.15}{0.85 \times 0.85 - 0.85 \times 0.15 + 0.15 \times 0.15} = \dfrac{(0.85)^3 + (0.15)^3}{0.85 \times 0.85 - 0.85 \times 0.15 + 0.15 \times 0.15}$

$= \dfrac{(0.85 + 0.15)\{(0.85)^2 - 0.85 \times 0.15 + (0.15)^2\}}{(0.85)^2 - 0.85 \times 0.15 + (0.15)^2}$

$= (0.85 + 0.15)$

$= 1$

$= R.H.S$

$Hence, L.H.S = R.H.S$

Answer.38. *Prove that :* $\dfrac{59 \times 59 \times 59 - 9 \times 9 \times 9}{59 \times 59 + 59 \times 9 + 9 \times 9} = 50$

$L.H.S. \Rightarrow \dfrac{59 \times 59 \times 59 - 9 \times 9 \times 9}{59 \times 59 + 59 \times 9 + 9 \times 9} = \dfrac{(59)^3 - (9)^3}{59 \times 59 + 59 \times 9 + 9 \times 9}$

$= \dfrac{(59 - 9)\{(59)^2 + 59 \times 9 + (9)^2\}}{59 \times 59 + 59 \times 9 + 9 \times 9}$

$= (59 - 9)$

$= 50$

$= R.H.S$

$Hence, L.H.S = R.H.S$