EXERCISE -3E

Formulae Used: ***(i)*** $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(ii) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

Answer.1.

(i) $(3x+2)^3$

$(3x+2)^3 = (3x)^3 + (2)^3 + 3(3x)(2)(3x+2)$

$= 27x^3 + 8 + 18x(3x) + 18x(2)$

$= 27x^3 + 8 + 54x^2 + 36x$

(ii) $\left(3a + \frac{1}{4b}\right)^3$

$\left(3a + \frac{1}{4b}\right)^3 = (3a)^3 + \left(\frac{1}{4b}\right)^3 + 3(3a)\left(\frac{1}{4b}\right)\left(3a + \frac{1}{4b}\right)$

$= 27a^3 + \frac{1}{64b^3} + \frac{9a}{4b}(3a) + \frac{9a}{4b}\left(\frac{1}{4b}\right)$

$= 27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}$

(iii) $\left(1 + \frac{2}{3}a\right)^3$

$\left(1 + \frac{2}{3}a\right)^3 = (1)^3 + \left(\frac{2}{3}a\right)^3 + 3(1)\left(\frac{2}{3}a\right)\left(1 + \frac{2}{3}a\right)$

$= 1^3 + \frac{8}{27}a^3 + 2a(1) + 2a\left(\frac{2}{3}a\right)$

$= 1 + \frac{8}{27}a^3 + 2a + \frac{4}{3}a^2$

$= 1 + \frac{8}{27}a^3 + \frac{4}{3}a^2 + 2a$

Answer.2.

(i) $(5a - 3b)^3$

$(5a-3b)^3 = (5a)^3 - (3b)^3 - 3(5a)(3b)(5a-3b)$

$= 125a^3 - 27b^3 - 45ab(5a) + 45ab(3b)$

$= 125a^3 - 27b^3 - 225a^2b + 135ab^2$

(ii) $\left(3x - \frac{5}{x}\right)^3$

$\left(3x - \frac{5}{x}\right)^3 = (3x)^3 - \left(\frac{5}{x}\right)^3 - 3(3x)\left(\frac{5}{x}\right)\left(3x - \frac{5}{x}\right)$

$= 27x^3 - \frac{125}{x^3} - 45(3x) + 45\left(\frac{5}{x}\right)$

$= 27x^3 - \frac{125}{x^3} - 135x + \frac{225}{x}$

(iii) $\left(\frac{4}{5}a - 2\right)^3$

$\left(\frac{4}{5}a - 2\right)^3 = \left(\frac{4}{5}a\right)^3 - (2)^3 - 3\left(\frac{4}{5}a\right)(2)\left(\frac{4}{5}a - 2\right)$

$= \frac{64}{125}a^3 - 8 - \frac{24}{5}a\left(\frac{4}{5}a\right) + \frac{24}{5}a(2)$

$= \frac{64}{125}a^3 - 8 - \frac{96}{25}a^2 + \frac{48}{5}a$

Answer.3. $8a^3 + 27b^3 + 36a^2b + 54ab^2$

$8a^3 + 27b^3 + 36a^2b + 54ab^2 = (2a)^3 + (3b)^3 + 3(2a)(3b)(2a) + 3(2a)(3b)(3b)$

$$= (2a)^3 + (3b)^3 + 3(2a)(3b)(2a + 3b)$$
$$= (2a + 3b)^3$$
$$= (2a + 3b)(2a + 3b)(2a + 3b)$$

Answer.4. $\boldsymbol{64a^3 - 27b^3 - 144a^2b + 108ab^2}$

$$64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3(4a)(3b)(4a) + 3(4a)(3b)(3b)$$
$$= (4a)^3 - (3b)^3 - 3(4a)(3b)(4a - 3b)$$
$$= (4a - 3b)^3$$
$$= (4a - 3b)(4a - 3b)(4a - 3b)$$

Answer.5. $\boldsymbol{1 + \frac{27}{125}a^3 + \frac{9a}{5} + \frac{27a^2}{25}}$

$$1 + \frac{27}{125}a^3 + \frac{9a}{5} + \frac{27a^2}{25} = (1)^3 + \left(\frac{3}{5}a\right)^3 + 3(1)\left(\frac{3}{5}a\right)(1) + 3(1)\left(\frac{3}{5}a\right)\left(\frac{3}{5}a\right)$$
$$= (1)^3 + \left(\frac{3}{5}a\right)^3 + 3(1)\left(\frac{3}{5}a\right)\left(1 + \frac{3}{5}a\right)$$
$$= \left(1 + \frac{3}{5}a\right)^3$$
$$= \left(1 + \frac{3}{5}a\right)\left(1 + \frac{3}{5}a\right)\left(1 + \frac{3}{5}a\right)$$

Answer.6. $\boldsymbol{125x^3 - 27y^3 - 225x^2y + 135xy^2}$

$$125x^3 - 27y^3 - 225x^2y + 135xy^2 = (5x)^3 - (3y)^3 - 3(5x)(3y)(5x) + 3(5x)(3y)(3y)$$
$$= (5x)^3 - (3y)^3 - 3(5x)(3y)(5x - 3y)$$
$$= (5x - 3y)^3$$
$$= (5x - 3y)(5x - 3y)(5x - 3y)$$

Answer.7. $\boldsymbol{a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3}$

$$.a^3x^3 - 3a^2bx^2 + 3ab^2x - b^3 = a^3x^3 - b^3 - 3a^2bx^2 + 3ab^2x$$
$$= (ax)^3 - (b)^3 - 3(ax)(b)(ax) + 3(ax)(b)(b)$$
$$= (ax)^3 - (b)^3 - 3(ax)(b)(ax - b)$$
$$= (ax - b)^3$$
$$= (ax - b)(ax - b)(ax - b)$$

Answer.8. $\boldsymbol{\frac{64}{125}a^3 - \frac{96}{25}a^2 + \frac{48}{5}a - 8}$

$$.\frac{64}{125}a^3 - \frac{96}{25}a^2 + \frac{48}{5}a - 8 = \frac{64}{125}a^3 - 8 - \frac{96}{25}a^2 + \frac{48}{5}a$$
$$= \left(\frac{4}{5}a\right)^3 - (2)^3 - 3\left(\frac{4}{5}a\right)(2)\left(\frac{4}{5}a\right) + 3\left(\frac{4}{5}a\right)(2)(2)$$
$$= \left(\frac{4}{5}a\right)^3 - (2)^3 - 3\left(\frac{4}{5}a\right)(2)\left(\frac{4}{5}a - 2\right)$$
$$= \left(\frac{4}{5}a - 2\right)^3$$
$$= \left(\frac{4}{5}a - 2\right)\left(\frac{4}{5}a - 2\right)\left(\frac{4}{5}a - 2\right)$$

Answer.9. $\boldsymbol{a^3 - 12a(a - 4) - 64}$

$.a^3 - 12a(a-4) - 64 = (a)^3 - (4)^3 - 3(a)(4)(a-4)$
$= (a-4)^3$
$= (a-4)(a-4)(a-4)$

Answer.10.

(i) $(\mathbf{103})^3$

$(103)^3 = (100+3)^3$
$= (100)^3 + (3)^3 + 3 \times 100 \times 3 \times (100+3)$
$= 1000000 + 27 + (900 \times 103)$
$= 1000027 + 92700$
$= 1092727$

(ii) $(\mathbf{99})^3$

$(99)^3 = (100-1)^3$
$= (100)^3 - (1)^3 - 3 \times 100 \times 1 \times (100-1)$
$= 1000000 - 1 - (300 \times 99)$
$= 999999 - 29700$
$= 970299$