EXERCISE-9(B)

Answer1) (i) No, because the sum of two sides of a triangle is not greater than the third side.

$5 + 4 = 9$

(ii) Yes, because the sum of two sides of a triangle is greater than the third side.

$7 + 4 > 8; 8 + 7 > 4; 8 + 4 > 7$

(iii) Yes, because the sum of two sides of a triangle is greater than the third side.

$5 + 6 > 10; 10 + 6 > 5; 5 + 10 > 6$

(iv) Yes, because the sum of two sides of a triangle is greater than the third side.

$2.5 + 5 > 7; 5 + 7 > 2.5; 2.5 + 7 > 5$

(v) No, because the sum of two sides of a triangle is not greater than the third side.

$3 + 4 < 8$

Answer2) **Given:** In ΔABC, $\angle A = 50°$ and $\angle B = 60°$

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ (Angle sum property of a triangle)

$\Rightarrow 50° + 60° + \angle C = 180°$

$\Rightarrow 110° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 110°$

$\Rightarrow \angle C = 70°$

Hence, the longest side will be opposite to the largest angle ($\angle C = 70°$) i.e. AB.

And, the shortest side will be opposite to the smallest angle ($\angle A = 50°$) i.e. BC.

Answer3) (i) **<u>Given:</u>** In ΔABC, $\angle A = 90°$

So, sum of the other two angles in triangle $\angle B + \angle C = 90°$

i.e. $\angle B, \angle C < 90°$

Since, $\angle A$ is the greatest angle.

So, the longest side is BC.

(ii) Given: $\angle A = \angle B = 45°$

Using angle sum property of triangle,

$\angle C = 90°$

Since, $\angle C$ is the greatest angle.

So, the longest side is AB.

(iii) Given: $\angle A = 100°$ and $\angle C = 50°$

Using angle sum property of triangle,

$\angle B = 30°$

Since, $\angle A$ is the greatest angle.

So, the shortest side is BC.

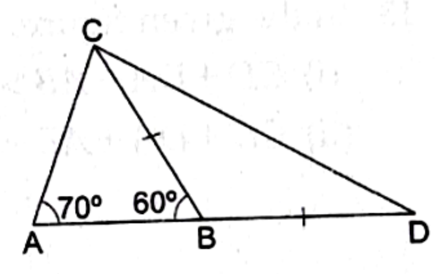
Answer4) **<u>Given:</u>** ΔABC, side AB is produced to D so that BD = BC and ∠B = 60°, ∠A = 70°

<u>To Prove:</u>

(i) AD > CD

And, (ii) AD > AC

<u>Proof:</u>



First join C and D

Now,

In ΔABC

∠A + ∠B + ∠C = 180° (Sum of all angles of triangle)

∠C = 180° – 70° – 60°= 50°

∠C = 50°
∠ACB = 50° (i)
And also in ΔBDC
∠DBC = 180° - ∠ABC (Therefore, ∠ABD is straight angle)
= 180° – 60° = 120°
BD = BC (Given)

∠BCD = ∠BDC (Therefore, angle opposite to equal sides are equal)

Now,

∠DBC + ∠BCD + ∠BDC = 180° (Sum of all sides of triangle)

120° + ∠BCD + ∠BCD = 180°

2∠BCD = 180° – 120°

2∠BCD = 60°

∠BCD = 30°

Therefore, ∠BCD = ∠BDC = 30° (ii)

Now, consider ΔBDC,

$\angle BAC = \angle DAC = 70^o$ (Given)

$\angle BDC = \angle ADC = 30^o$ [From (ii)]

$\angle ACD = \angle ACB + \angle BCD$

$= 50^o + 30^o$ [From (i) and (ii)]

$= 80^o$

Now,

$\angle ADC < \angle DAC < \angle ACD$

$AC < DC < AD$ (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

$AD > CD$

And,

$AD > AC$

Hence Proved.

We have,

$\angle ACD > \angle DAC$

And,

$\angle ACD > \angle ADC$

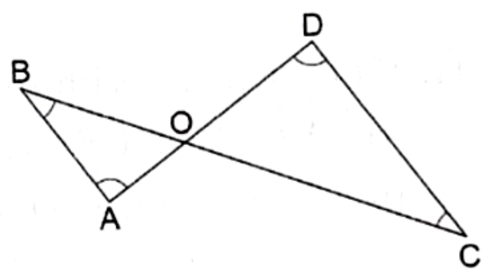
$AD > DC$

$AD > AC$ (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

Answer5)<u>**GIVEN:**</u>

$\angle B < \angle A$

$\angle C < \angle D$



TO PROVE:

AD < BC

PROOF:

∠B <∠A

SO,

OA < OB ...(1) (SIDE OPPOSITE TO SMALLER ANGLE IS SMALL)

NOW,

∠C<∠D

SO,

OD < OC ...(2) (SIDE OPPOSITE TO SMALLER ANGLE IS SMALL)

NOW,

ADDING 1 AND 2

OA + OD < OB + OC

ADDING WE GET,

AD < BC

HENCE PROVED.

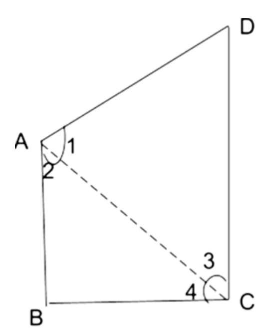
Answer6) **Given:**

In quadrilateral ABCD, AB smallest & CD is longest sides.

To Prove:∠A>∠C

&∠B>∠D

Construction: Join AC.



Mark the angles as shown in the figure..

Proof:

In $\triangle ABC$, AB is the shortest side.

$BC > AB$

$\angle 2 > \angle 4$ ...(i)

[Angle opposite to longer side is greater]

In $\triangle ADC$, CD is the longest side

$CD > AD$

$\angle 1 > \angle 3$ ...(ii)

[Angle opposite to longer side is greater]

Adding (i) and (ii), we have

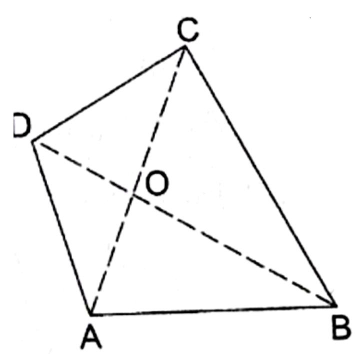
$\angle 2 + \angle 1 > \angle 4 + \angle 3$

$\Rightarrow \angle A > \angle C$

Similarly, by joining BD, we can prove that

$\angle B > \angle D$

Answer 7) **To Prove:** $(AB + BC + CD + DA) > (AC + BD)$



Proof:

ABCD is a quad.Its diagonals are AC and BD.

In triangle ACB, $AB + BC > AC$ ...(1)

In triangle BDC, $BC + CD > BD$ ...(2)

In triangle ACD, $AD + DC > AC$ ...(3)

In triangle BAD, $AB + AD > BD$ ...(4)

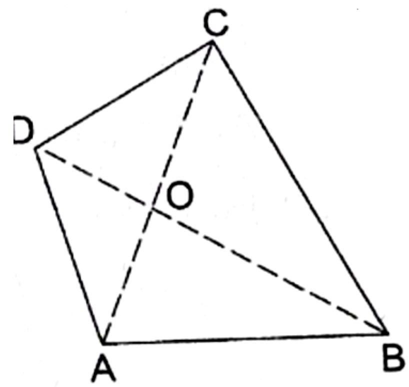
Adding 1,2,3 and 4,

$AB + BC + BC + CD + AD + DC + AB + AD > AC + BD + AC + BD$

$2AB + 2BC + 2CD + 2AD > 2AC + 2BD$

$AB + BC + CD + AD > AC + BD$.   HENCE PROVED.

Answer8) Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side



Therefore, In Δ AOB, $AB < OA + OB$ ....(i)

In Δ BOC, $BC < OB + OC$ ....(ii)

In Δ COD, $CD < OC + OD$ ....(iii)

In Δ AOD, $DA < OD + OA$ ....(iv)

$\Rightarrow AB + BC + CD + DA < 2OA + 2OB + 2OC + 2OD$

$\Rightarrow AB + BC + CD + DA < 2[(AO + OC) + (DO + OB)$

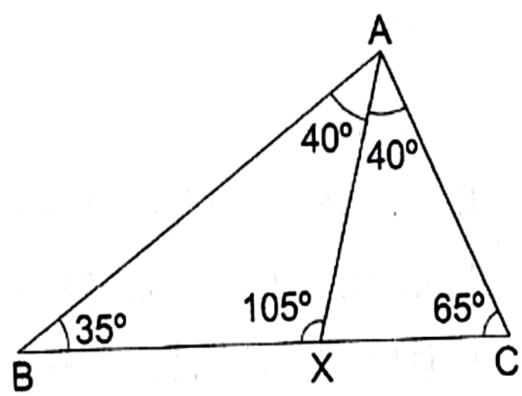
$\Rightarrow AB + BC + CD + DA < 2(AC + BD)$

Hence Proved.

Answer9) **Given:** In ΔABC, $\angle B = 35°, \angle C = 65°$ and $\angle BAX = \angle XAC$

To find: Relation between AX, BX and CX in descending order.

In ΔABC, by the angle sum property, we have



$\angle A + \angle B + \angle C = 180°$

$\angle A + 35° + 65° = 180°$

$\angle A + 100° = 180°$

$\therefore \angle A = 80°$

But $\angle BAX = \angle A = 40°$

Now in ΔABX,

$\angle B = 35°$

$\angle BAX = 40°$

And $\angle BXA = 180° - 35° - 40°$

$= 105°$

So, in ΔABX,

$\angle B$ is smallest, so the side opposite is smallest, i.e. AX is smallest side.

$\therefore AX < BX$ ...(1)

Now consider ΔAXC,

$\angle CAX = \angle A = 40°$

$\angle AXC = 180° - 40° - 65°$

$= 180° - 105° = 75°$

Hence, in ΔAXC we have,

$\angle CAX = 40°, \angle C = 65°, \angle AXC = 75°$

$\therefore \angle CAX$ is smallest in ΔAXC

So the side opposite to $\angle CAX$ is shortest

i.e. CX is shortest

$\therefore CX < AX$ .... (2)

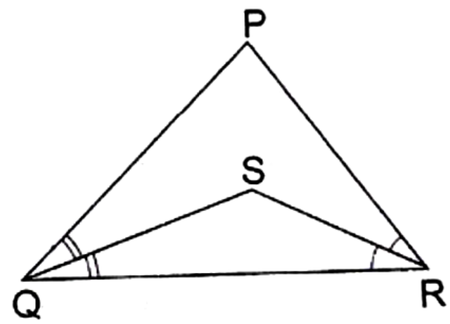
From 1 and 2 ,

$BX > AX > CX$

Answer10) **Given:** PQ>PR

QS and RS are bisector of ∠Q and ∠R Respectively

To Prove: SQ>SR



Proof:

∠ R >∠ Q (angle opposite to greater side is greater)

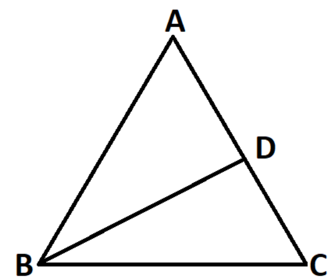
1/2*∠ R>1/2*∠Q

∠ SRQ>∠SQR

SQ>SR (Side opposite to greater angle is greater)

Answer11) **Given:** AB = AC

To prove: BD > CD



Proof :

Since AB = AC

∠ABC =
∠ACB (By
Isosceles Triangle property) ----(i)

Here clearly,

∠ABC >∠CBD

∠ACB >∠CBD ---from (i)

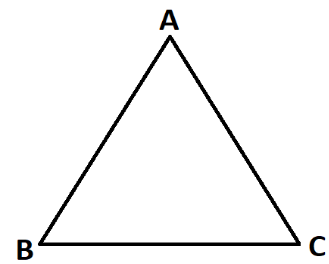
∠DCB >∠CBD

BD > CD (Angle opposite to greater side is greater in a triangle)

Hence Proved.

Answer12) Let ΔABC be a triangle in which AC is the longest side.

To prove: Angle opposite the longest side is greater than 2/3 of right angle.



Proof : $\angle B > \angle A$ ....(i)

And $\angle B > \angle C$ ....(ii)

Adding (i) and (ii), we get

→$\angle B + \angle B > \angle A + \angle C$

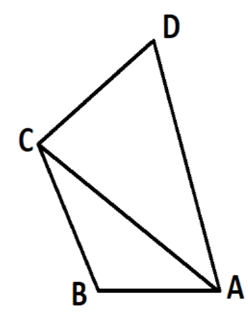
→$2\angle B > \angle A + \angle C$

→$2\angle B + \angle B > \angle A + \angle B + \angle C$

→$3\angle B > 180°$ = $\angle B > 60°$

→$\angle B > 2/3$ x right amgle. [$60° = 2/3 \times 90°$]

Answer13)



(i)**To Prove :** CD + DA + AB > BC

Proof:

ΔABC, we have

CD + DA > AC

Add AB on both sides, we get

CD + DA +AB > AC + AB > BC

CD + DA + AB > BC

Hence proved.

(ii) **To Prove:** CD + DA + AB + BC > 2AC

Proof:

In ΔABC, we have

AB + BC > AC ...(1)

In ΔADC, we have

CD + DA > AC ...(2)

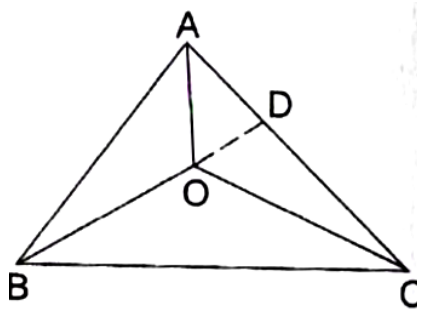
Adding (1) and (2), we get

AB + BC + CD + DA > AC + AC

CD + DA + AB + BC > 2 AC

Hence Proved.

Answer14)



<u>Given:</u>
In triangle ABC, O is any interior point.
We know that any segment from a point O inside a triangle to any vertex of the triangle cannot be longer than the two sides adjacent to the vertex.
Thus, OA cannot be longer than both AB and CA (if this is possible, then O is outside the triangle).

<u>To Prove:</u>

(i) AB + AC > OB+ OC
(ii) AB + BC + CA > OA + OB + OC
(iii) OA + OB + OC > ½ (AB + BC + CA)

<u>Proof:</u>

(i) OA cannot be longer than both AB and CA
AB>OB ...(1)

AC>OC ...(2)

Thus,

AB+AC>OB+OC ...[Adding (1) and(2)]

AB>OB ...(1)

AC>OC ...(2)

Thus,

AB+AC>OB+OC ...[Adding (1) and(2)]

(ii) AB>OA......(3)

BC>OB.....(4)

CA>OC.....(5)
Adding the above three equations, we get:

Thus, AB+BC+CA>OA+OB+OC ...(6)

OA cannot be longer than both AB and CA.
AB>OB.....(5)

AC>OC.....(6)

AB+AC>OB+OC..........[On adding (5) and (6)]

Thus, the first equation to be proved is shown correct.

(iii) Now, consider the triangles OAC, OBA and OBC.
We have:
OA+OC>AC

OA+OB>AB

OB+OC>BC

Adding the above three equations, we get:

OA+OC+OA+OB+OB+OC>AB+AC+BC

⇒2(OA+OB+OC)>AB+AC+BC

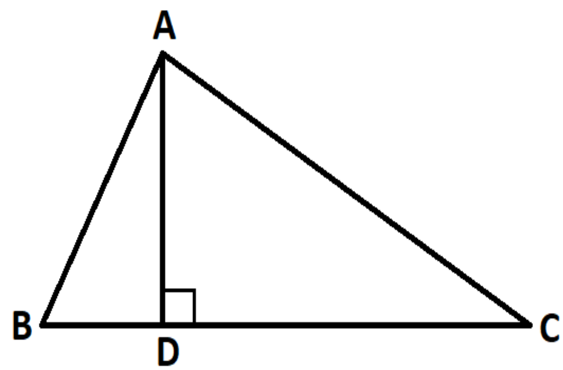
Thus, OA+OB+OC>1/2(AB+BC+CA).

Answer15) <u>Given</u> : (i) $AD \perp BC$

(ii) $CD > BD$

<u>To Prove:</u> $AC > AB$

<u>Proof</u>:



In Δ ABD ; $\angle ABD + \angle BAD + \angle BDA = 180°$

$\angle ABD + \angle BAD + 90° = 180°$

$\angle ABD + \angle BAD = 90°$

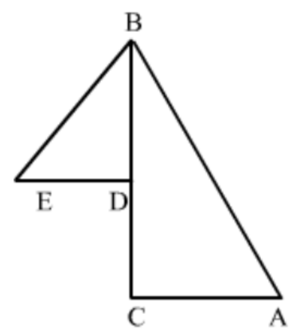
Similarly; In ΔADC ; $\angle ACB + \angle CAD = 90°$

Since; $BD < CD$; $\angle BAD < \angle CAD$

$\angle ABD > \angle ACB$

$AC > AB$

(sides opposite to greater angles are greater)



Answer16) <u>Given:</u> $CD = DE$

<u>To prove:</u> $AB + AC > BE$

<u>Proof:</u>

In ΔABC,

AB+AC>BC ...(1)

AB+AC>BC ...1

In ΔBED,

BD+CD>BE⇒BC>BE ...(2)

$BD + CD > BE \Rightarrow BC > BE$ ...2

From (1) and (2), we get

$AB + AC > BE$.

Hence Proved.