

Exercise – 3F

1. Write the number of solutions of the following pair of linear equations:

$$x + 2y - 8 = 0,$$

$$2x + 4y = 16$$

Sol:

The given equations are

$$x + 2y - 8 = 0 \quad \dots\dots(i)$$

$$2x + 4y - 16 = 0 \quad \dots\dots(ii)$$

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 1, b_1 = 2, c_1 = -8, a_2 = 2, b_2 = 4$ and $c_2 = -16$

Now

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$$

Thus, the pair of linear equations are coincident and therefore has infinitely many solutions.

2. Find the value of k for which the system of linear equations has an infinite number of solutions.

$$2x + 3y - 7 = 0,$$

$$(k - 1)x + (k + 2)y = 3k$$

Sol:

The given equations are

$$2x + 3y - 7 = 0 \quad \dots\dots(i)$$

$$(k - 1)x + (k + 2)y - 3k = 0 \quad \dots\dots(ii)$$

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$$a_1 = 2, b_1 = 3, c_1 = -7, a_2 = k - 1, b_2 = k + 2 \text{ and } c_2 = -3k$$

For the given pair of linear equations to have infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$$

$$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2}, \frac{3}{k+2} = \frac{-7}{-3k} \text{ and } \frac{2}{k-1} = \frac{-7}{-3k}$$

$$\Rightarrow 2(k + 2) = 3(k - 1), 9k = 7k + 14 \text{ and } 6k = 7k - 7$$

$$\Rightarrow k = 7, k = 7 \text{ and } k = 7$$

Hence, $k = 7$.

3. Find the value of k for which the system of linear equations has an infinite number of solutions.

$$10x + 5y - (k - 5) = 0,$$

$$20x + 10y - k = 0.$$

Sol:

The given pair of linear equations are

$$10x + 5y - (k - 5) = 0 \quad \dots\dots(i)$$

$$20x + 10y - k = 0 \quad \dots\dots(ii)$$

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$$a_1 = 10, b_1 = 5, c_1 = -(k - 5), a_2 = 20, b_2 = 10 \text{ and } c_2 = -k$$

For the given pair of linear equations to have infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{10}{20} = \frac{5}{10} = \frac{-(k-5)}{-k}$$

$$\Rightarrow \frac{1}{2} = \frac{k-5}{k}$$

$$\Rightarrow 2k - 10 = k \Rightarrow k = 10$$

Hence, $k = 10$.

4. Find the value of k for which the system of linear equations has an infinite number of solutions.

$$2x + 3y = 9,$$

$$6x + (k - 2)y = (3k - 2)$$

Sol:

The given pair of linear equations are

$$2x + 3y - 9 = 0 \quad \dots\dots(i)$$

$$6x + (k - 2)y - (3k - 2) = 0 \quad \dots\dots(ii)$$

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$$a_1 = 2, b_1 = 3, c_1 = -9, a_2 = 6, b_2 = k - 2 \text{ and } c_2 = -(3k - 2)$$

For the given pair of linear equations to have infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{6} = \frac{3}{k-2} \neq \frac{-9}{-(3k-2)}$$

$$\Rightarrow \frac{2}{6} = \frac{3}{k-2}, \frac{3}{k-2} \neq \frac{-9}{-(3k-2)}$$

$$\Rightarrow k = 11, \frac{3}{k-2} \neq \frac{9}{(3k-2)}$$

$$\Rightarrow k = 11, 3(3k - 2) \neq 9(k - 2)$$

$$\Rightarrow k = 11, 1 \neq 3 \text{ (true)}$$

Hence, $k = 11$.

5. Write the number of solutions of the following pair of linear equations:

$$x + 3y - 4 = 0, 2x + 6y - 7 = 0.$$

Sol:

The given pair of linear equations are

$$x + 3y - 4 = 0 \quad \dots\dots(i)$$

$$2x + 6y - 7 = 0 \quad \dots\dots(ii)$$

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$$a_1 = 1, b_1 = 3, c_1 = -4, a_2 = 2, b_2 = 6 \text{ and } c_2 = -7$$

Now

$$\frac{a_1}{a_2} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-4}{-7} = \frac{4}{7}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the pair of the given linear equations has no solution.

6. Find the values of k for which the system of equations $3x + ky = 0$, $2x - y = 0$ has a unique solution.

Sol:

The given pair of linear equations are

$$3x + ky = 0 \quad \dots\dots(i)$$

$$2x - y = 0 \quad \dots\dots(ii)$$

Which is of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$a_1 = 3$, $b_1 = k$, $c_1 = 0$, $a_2 = 2$, $b_2 = -1$ and $c_2 = 0$

For the system to have a unique solution, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \frac{3}{2} \neq \frac{k}{-1}$$

$$\Rightarrow k \neq -\frac{3}{2}$$

Hence, $k \neq -\frac{3}{2}$.

7. The difference of two numbers is 5 and the difference between their squares is 65. Find the numbers.

Sol:

Let the numbers be x and y , where $x > y$.

Then as per the question

$$x - y = 5 \quad \dots\dots(i)$$

$$x^2 - y^2 = 65 \quad \dots\dots(ii)$$

Dividing (ii) by (i), we get

$$\frac{x^2 - y^2}{x - y} = \frac{65}{5}$$

$$\Rightarrow \frac{(x - y)(x + y)}{x - y} = 13$$

$$\Rightarrow x + y = 13 \quad \dots\dots(iii)$$

Now, adding (i) and (ii), we have

$$2x = 18 \Rightarrow x = 9$$

Substituting $x = 9$ in (iii), we have

$$9 + y = 13 \Rightarrow y = 4$$

Hence, the numbers are 9 and 4.

8. The cost of 5 pens and 8 pencils together cost Rs. 120 while 8 pens and 5 pencils together cost Rs. 153. Find the cost of a 1 pen and that of a 1 pencil.

Sol:

Let the cost of 1 pen and 1 pencil are ₹x and ₹y respectively.

Then as per the question

$$5x + 8y = 120 \quad \dots\dots(i)$$

$$8x + 5y = 153 \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$13x + 13y = 273$$

$$\Rightarrow x + y = 21 \quad \dots\dots(iii)$$

Subtracting (i) from (ii), we get

$$3x - 3y = 33$$

$$\Rightarrow x - y = 11 \quad \dots\dots(iv)$$

Now, adding (iii) and (iv), we get

$$2x = 32 \Rightarrow x = 16$$

Substituting $x = 16$ in (iii), we have

$$16 + y = 21 \Rightarrow y = 5$$

Hence, the cost of 1 pen and 1 pencil are respectively ₹16 and ₹5.

9. The sum of two numbers is 80. The larger number exceeds four times the smaller one by 5. Find the numbers.

Sol:

Let the larger number be x and the smaller number be y.

Then as per the question

$$x + y = 80 \quad \dots\dots(i)$$

$$x = 4y + 5$$

$$x - 4y = 5 \quad \dots\dots(ii)$$

Subtracting (ii) from (i), we get

$$5y = 75 \Rightarrow y = 15$$

Now, putting $y = 15$ in (i), we have

$$x + 15 = 80 \Rightarrow x = 65$$

Hence, the numbers are 65 and 15.

10. A number consists of two digits whose sum is 10. If 18 is subtracted from the number, its digits are reversed. Find the number.

Sol:

Let the ones digit and tens digit be x and y respectively.

Then as per the question

$$x + y = 10 \quad \dots\dots(i)$$

$$(10y + x) - 18 = 10x + y$$

$$x - y = -2 \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$2x = 8 \Rightarrow x = 4$$

Now, putting $x = 4$ in (i), we have

$$4 + y = 10 \Rightarrow y = 6$$

Hence, the number is 64.

11. A man purchased 47 stamps of 20p and 25p for ₹10. Find the number of each type of stamps

Sol:

Let the number of stamps of 20p and 25p be x and y respectively.

Then as per the question

$$x + y = 47 \quad \dots\dots(i)$$

$$0.20x + 0.25y = 10$$

$$4x + 5y = 200 \quad \dots\dots(ii)$$

From (i), we get

$$y = 47 - x$$

Now, substituting $y = 47 - x$ in (ii), we have

$$4x + 5(47 - x) = 200$$

$$\Rightarrow 4x - 5x + 235 = 200$$

$$\Rightarrow x = 235 - 200 = 35$$

Putting $x = 35$ in (i), we get

$$35 + y = 47$$

$$\Rightarrow y = 47 - 35 = 12$$

Hence, the number of 20p stamps and 25p stamps are 35 and 12 respectively.

12. A man has some hens and cows. If the number of heads be 48 and number of feet by 140. How many cows are there.

Sol:

Let the number of hens and cow be x and y respectively.

As per the question

$$x + y = 48 \quad \dots\dots(i)$$

$$2x + 4y = 140$$

$$x + 2y = 70 \quad \dots\dots(ii)$$

Subtracting (i) from (ii), we have

$$y = 22$$

Hence, the number of cows is 22.

13. If $\frac{2}{x} + \frac{3}{y} = -\frac{9}{xy}$ and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$, find the values of x and y.

Sol:

The given pair of equation is

$$\frac{2}{x} + \frac{3}{y} = -\frac{9}{xy} \quad \dots\dots\dots(i)$$

$$\frac{4}{x} + \frac{9}{y} = \frac{21}{xy} \quad \dots\dots\dots(ii)$$

Multiplying (i) and (ii) by xy, we have

$$3x + 2y = 9 \quad \dots\dots\dots(iii)$$

$$9x + 4y = 21 \quad \dots\dots\dots(iv)$$

Now, multiplying (iii) by 2 and subtracting from (iv), we get

$$9x - 6x = 21 - 18 \Rightarrow x = \frac{3}{3} = 1$$

Putting $x = 1$ in (iii), we have

$$3 \times 1 + 2y = 9 \Rightarrow y = \frac{9-3}{2} = 3$$

Hence, $x = 1$ and $y = 3$.

14. If $\frac{x}{4} + \frac{y}{3} = -\frac{15}{12}$ and $\frac{x}{2} + y = 1$, then find the value of $(x + y)$.

Sol:

The given pair of equations is

$$\frac{x}{4} + \frac{y}{3} = -\frac{15}{12} \quad \dots\dots\dots(i)$$

$$\frac{x}{2} + y = 1 \quad \dots\dots\dots(ii)$$

Multiplying (i) by 12 and (ii) by 4, we have

$$3x + 4y = -15 \quad \dots\dots\dots(iii)$$

$$2x + 4y = 4 \quad \dots\dots\dots(iv)$$

Now, subtracting (iv) from (iii), we get

$$x = 1$$

Putting $x = 1$ in (iv), we have

$$2 + 4y = 4$$

$$\Rightarrow 4y = 2$$

$$\Rightarrow y = \frac{1}{2}$$

$$\therefore x + y = 1 + \frac{1}{2} = \frac{3}{2}$$

Hence, the value of $x + y$ is $\frac{3}{2}$.

15. If $12x + 17y = 53$ and $17x + 12y = 63$ then find the value of $(x + y)$

Sol:

The given pair of equations is

$$12x + 17y = 53 \quad \dots\dots(i)$$

$$17x + 12y = 63 \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$29x + 29y = 116$$

$$\Rightarrow x + y = 4 \quad (\text{Dividing by } 29)$$

Hence, the value of $x + y$ is 4.

16. Find the value of k for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has infinite nonzero solutions.

Sol:

The given system is

$$3x + 5y = 0 \quad \dots\dots(i)$$

$$kx + 10y = 0 \quad \dots\dots(ii)$$

This is a homogeneous system of linear differential equation, so it always has a zero solution i.e., $x = y = 0$.

But to have a non-zero solution, it must have infinitely many solutions.

For this, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \frac{3}{k} = \frac{5}{10} = \frac{1}{2}$$

$$\Rightarrow k = 6$$

Hence, $k = 6$.

17. Find the value of k for which the system of equations $kx - y = 2$ and $6x - 2y = 3$ has a unique solution.

Sol:

The given system is

$$kx - y - 2 = 0 \quad \dots\dots(i)$$

$$6x - 2y - 3 = 0 \quad \dots\dots(ii)$$

Here, $a_1 = k$, $b_1 = -1$, $c_1 = -2$, $a_2 = 6$, $b_2 = -2$ and $c_2 = -3$

For the system, to have a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2} = \frac{1}{2}$$

$$\Rightarrow k \neq 3$$

Hence, $k \neq 3$.

18. Find the value of k for which the system of equations $2x + 3y - 5 = 0$ and $4x + ky - 10 = 0$ has infinite number of solutions.

Sol:

The given system is

$$2x + 3y - 5 = 0 \quad \dots\dots(i)$$

$$4x + ky - 10 = 0 \quad \dots\dots(ii)$$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$, $a_2 = 4$, $b_2 = k$ and $c_2 = -10$

For the system, to have an infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{4} = \frac{3}{k} = \frac{-5}{-10}$$

$$\Rightarrow \frac{1}{2} = \frac{3}{k} = \frac{1}{2}$$

$$\Rightarrow k = 6$$

Hence, $k = 6$.

19. Show that the system $2x + 3y - 1 = 0$ and $4x + 6y - 4 = 0$ has no solution.

Sol:

The given system is

$$2x + 3y - 1 = 0 \quad \dots\dots(i)$$

$$4x + 6y - 4 = 0 \quad \dots\dots(ii)$$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -1$, $a_2 = 4$, $b_2 = 6$ and $c_2 = -4$

Now,

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{-1}{-4} = \frac{1}{4}$$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ and therefore the given system has no solution.

20. Find the value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ is inconsistent.

Sol:

The given system is

$$x + 2y - 3 = 0 \quad \dots\dots(i)$$

$$5x + ky + 7 = 0 \quad \dots\dots(ii)$$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$, $a_2 = 5$, $b_2 = k$ and $c_2 = 7$.

For the system, to be consistent, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$$

$$\Rightarrow \frac{1}{5} = \frac{2}{k}$$

$$\Rightarrow k = 10$$

Hence, $k = 10$.

21. Solve for x and y : $\frac{3}{x+y} + \frac{2}{x-y} = 2$, $\frac{9}{x+y} - \frac{4}{x-y} = 1$

Sol:

The given system of equations is

$$\frac{3}{x+y} + \frac{2}{x-y} = 2 \quad \dots\dots\dots(i)$$

$$\frac{9}{x+y} - \frac{4}{x-y} = 1 \quad \dots\dots\dots(ii)$$

Substituting $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$ in (i) and (ii), the given equations are changed to

$$3u + 2v = 2 \quad \dots\dots\dots(iii)$$

$$9u - 4v = 1 \quad \dots\dots\dots(iv)$$

Multiplying (i) by 2 and adding it with (ii), we get

$$15u = 4 + 1 \Rightarrow u = \frac{1}{3}$$

Multiplying (i) by 3 and subtracting (ii) from it, we get

$$6u + 4v = 6 - 1 \Rightarrow u = \frac{5}{10} = \frac{1}{2}$$

Therefore

$$x + y = 3 \quad \dots\dots\dots(v)$$

$$x - y = 2 \quad \dots\dots\dots(vi)$$

Now, adding (v) and (vi) we have

$$2x = 5 \Rightarrow x = \frac{5}{2}$$

Substituting $x = \frac{5}{2}$ in (v), we have

$$\frac{5}{2} + y = 3 \Rightarrow y = 3 - \frac{5}{2} = \frac{1}{2}$$

Hence, $x = \frac{5}{2}$ and $y = \frac{1}{2}$.

Exercise – MCQ

1. If $2x + 3y = 12$ and $3x - 2y = 5$ then
 (a) $x = 2$, $y = 3$ (b) $x = 2$, $y = -3$ (c) $x = 3$, $y = 2$ (d) $x = 3$, $y = -2$

Answer: (c) $x = 3$, $y = 2$

Sol:

The given system of equations is

$$2x + 3y = 12 \quad \dots\dots\dots(i)$$

$$3x - 2y = 5 \quad \dots\dots\dots(ii)$$

Multiplying (i) by 2 and (ii) by 3 and then adding, we get

$$4x + 9x = 24 + 15$$

$$\Rightarrow x = \frac{39}{13} = 3$$

Now, putting $x = 3$ in (i), we have

$$2 \times 3 + 3y = 12 \Rightarrow y = \frac{12-6}{3} = 2$$

Thus, $x = 3$ and $y = 2$.

2. If $x - y = 2$ and $\frac{2}{x+y} = \frac{1}{5}$ then

(a) $x = 4, y = 2$ (b) $x = 5, y = 3$ (c) $x = 6, y = 4$ (d) $x = 7, y = 5$

Answer: (c) $x = 6, y = 4$

Sol:

The given system of equations is

$$x - y = 2 \quad \dots\dots(i)$$

$$x + y = 10 \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$2x = 12 \Rightarrow x = 6$$

Now, putting $x = 6$ in (ii), we have

$$6 + y = 10 \Rightarrow y = 10 - 6 = 4$$

Thus, $x = 6$ and $y = 4$.

3. If $\frac{2x}{3} - \frac{y}{2} + \frac{1}{6} = 0$ and $\frac{x}{2} + \frac{2y}{3} = 3$ then

(a) $x = 2, y = 3$ (b) $x = -2, y = 3$ (c) $x = 2, y = -3$ (d) $x = -2, y = -3$

Answer: (a) $x = 2, y = 3$

Sol:

The given system of equations is

$$\frac{2x}{3} - \frac{y}{2} = -\frac{1}{6} \quad \dots\dots(i)$$

$$\frac{x}{2} + \frac{2y}{3} = 3 \quad \dots\dots(ii)$$

Multiplying (i) and (ii) by 6, we get

$$4x - 3y = -1 \quad \dots\dots(iii)$$

$$3x + 4y = 18 \quad \dots\dots(iv)$$

Multiplying (iii) by 4 and (iv) by 3 and adding, we get

$$16x + 9x = -4 + 54$$

$$\Rightarrow x = \frac{50}{25} = 2$$

Now, putting $x = 2$ in (iv), we have

$$3 \times 2 + 4y = 18 \Rightarrow y = \frac{18-6}{4} = 3$$

Thus, $x = 2$ and $y = 3$.

4. If $\frac{1}{x} + \frac{2}{y} = 4$ and $\frac{3}{y} - \frac{1}{x} = 11$ then

(a) $x = 2, y = 3$ (b) $x = -2, y = 3$ (c) $x = \frac{-1}{2}, y = 3$ (d) $x = \frac{-1}{2}, y = \frac{1}{3}$

Answer: (d) $x = \frac{-1}{2}, y = \frac{1}{3}$

Sol:

The given system of equations is

$$\frac{1}{x} + \frac{2}{y} = 4 \quad \dots\dots(i)$$

$$\frac{3}{y} - \frac{1}{x} = 11 \quad \dots\dots(ii)$$

Adding (i) and (ii), we get

$$\frac{2}{y} + \frac{3}{y} = 15$$

$$\Rightarrow \frac{5}{y} = 15 \Rightarrow y = \frac{5}{15} = \frac{1}{3}$$

Now, putting $y = \frac{1}{3}$ in (i), we have

$$\frac{1}{x} + 2 \times 3 = 4 \Rightarrow \frac{1}{x} = 4 - 6 \Rightarrow x = -\frac{1}{2}$$

Thus, $x = -\frac{1}{2}$ and $y = \frac{1}{3}$.

5. If $\frac{2x+y+2}{5} = \frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$ then

(a) $x = 1, y = 1$ (b) $x = -1, y = -1$ (c) $x = 1, y = 2$ (d) $x = 2, y = 1$

Answer: (a) $x = 1, y = 1$

Sol:

Consider $\frac{2x+y+2}{5} = \frac{3x-y+1}{3}$ and $\frac{3x-y+1}{3} = \frac{3x+2y+1}{6}$. Now, simplifying these equations, we get

$$3(2x + y + 2) = 5(3x - y + 1)$$

$$\Rightarrow 6x + 3y + 6 = 15x - 5y + 5$$

$$\Rightarrow 9x - 8y = 1 \quad \dots\dots(i)$$

And

$$6(3x - y + 1) = 3(3x + 2y + 1)$$

$$\Rightarrow 18x - 6y + 6 = 9x + 6y + 3$$

$$\Rightarrow 3x - 4y = -1 \quad \dots\dots(ii)$$

Multiplying (ii) by 2 and subtracting it from (i)

$$9x - 6x = 1 + 2 \Rightarrow x = 1$$

Now, putting $x = 1$ in (ii), we have

$$3 \times 1 - 4y = -1 \Rightarrow y = \frac{3+1}{4} = 1$$

Thus, $x = 1, y = 1$.

6. If $\frac{3}{x+y} + \frac{2}{x-y} = 2$ and $\frac{9}{x+y} - \frac{4}{x-y} = 1$ then

(a) $x = \frac{1}{2}, y = \frac{3}{2}$ (b) $x = \frac{5}{2}, y = \frac{1}{2}$ (c) $x = \frac{3}{2}, y = \frac{1}{2}$ (d) $x = \frac{1}{2}, y = \frac{5}{2}$

Answer: (b) $x = \frac{5}{2}, y = \frac{1}{2}$

Sol:

The given equations are

$$\frac{3}{x+y} + \frac{2}{x-y} = 2 \quad \dots\dots(i)$$

$$\frac{9}{x+y} - \frac{4}{x-y} = 1 \quad \dots\dots(ii)$$

Substituting $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$ in (i) and (ii), the new system becomes

$$3u + 2v = 2 \quad \dots\dots(iii)$$

$$9u - 4v = 1 \quad \dots\dots(iv)$$

Now, multiplying (iii) by 2 and adding it with (iv), we get

$$6u + 9u = 4 + 1 \Rightarrow u = \frac{5}{15} = \frac{1}{3}$$

Again, multiplying (iii) by 2 and subtracting (iv) from , we get

$$6v + 4v = 6 - 1 \Rightarrow v = \frac{5}{10} = \frac{1}{2}$$

Therefore

$$x + y = 3 \quad \dots\dots(v)$$

$$x - y = 2 \quad \dots\dots(vi)$$

Adding (v) and (vi), we get

$$2x = 3 + 2 \Rightarrow x = \frac{5}{2}$$

Substituting $x = \frac{5}{2}$, in (v), we have

$$\frac{5}{2} + y = 3 \Rightarrow y = 3 - \frac{5}{2} = \frac{1}{2}$$

Thus, $x = \frac{5}{2}$ and $y = \frac{1}{2}$.

7. If $4x+6y=3xy$ and $8x +9y=5xy$ then

(a) $x = 2, y = 3$ (b) $x = 1, y = 2$ (c) $x = 3, y = 4$ (d) $x = 1, y = - 1$

Answer: (c) $x = 3, y = 4$

Sol:

The given equations are

$$4x + 6y = 3xy \quad \dots\dots(i)$$

$$8x + 9y = 5xy \quad \dots\dots(ii)$$

Dividing (i) and (ii) by xy , we get

$$\frac{6}{x} + \frac{4}{y} = 3 \quad \dots\dots(iii)$$

$$\frac{9}{x} + \frac{8}{y} = 5 \quad \dots\dots\dots(\text{iv})$$

Multiplying (iii) by 2 and subtracting (iv) from it, we get

$$\frac{12}{x} - \frac{9}{x} = 6 - 5 \Rightarrow \frac{3}{x} = 1 \Rightarrow x = 3$$

Substituting $x = 3$ in (iii), we get

$$\frac{6}{3} + \frac{4}{y} = 3 \Rightarrow \frac{4}{y} = 1 \Rightarrow y = 4$$

Thus, $x = 3$ and $y = 4$.

8. If $29x + 37y = 103$ and $37x + 29y = 95$ then
 (a) $x = 1, y = 2$ (b) $x = 2, y = 1$ (c) $x = 3, y = 2$ (d) $x = 2, y = 3$

Answer: (a) $x = 1, y = 2$

Sol:

The given system of equations is

$$29x + 37y = 103 \quad \dots\dots\dots(\text{i})$$

$$37x + 29y = 95 \quad \dots\dots\dots(\text{ii})$$

Adding (i) and (ii), we get

$$66x + 66y = 198 \\ \Rightarrow x + y = 3 \quad \dots\dots\dots(\text{iii})$$

Subtracting (i) from (ii), we get

$$8x - 8y = -8 \\ \Rightarrow x - y = -1$$

Adding (iii) and (iv), we get

$$2x = 2 \Rightarrow x = 1$$

Substituting $x = 1$ in (iii), we have

$$1 + y = 3 \Rightarrow y = 2$$

Thus, $x = 1$ and $y = 2$.

9. If $2^{x+y} = 2^{x-y} = \sqrt{8}$ then the value of y is

- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 0 (d) none of these

Answer: (c) 0

Sol:

$$\therefore 2^{x+y} = 2^{x-y} = \sqrt{8}$$

$$\therefore x + y = x - y$$

$$\Rightarrow y = 0$$

10. If $\frac{2}{x} + \frac{3}{y} = 6$ and $\frac{1}{x} + \frac{1}{2y} = 2$ then

(a) $x = 1, y = \frac{2}{3}$ (b) $x = \frac{2}{3}, y = 1$ (c) $x = 1, y = \frac{3}{2}$ (d) $x = \frac{3}{2}, y = 1$

Answer: (b) $x = \frac{2}{3}, y = 1$

Sol:

The given equations are

$$\frac{2}{x} + \frac{3}{y} = 6 \quad \dots\dots\dots(i)$$

$$\frac{1}{x} + \frac{1}{2y} = 2 \quad \dots\dots\dots(ii)$$

Multiplying (ii) by 2 and subtracting it from (i), we get

$$\frac{3}{y} - \frac{1}{y} = 6 - 4$$

$$\Rightarrow \frac{2}{y} = 2 \Rightarrow y = 1$$

Substituting $y = 1$ in (ii), we get

$$\frac{1}{x} + \frac{1}{2} = 2$$

$$\Rightarrow \frac{1}{x} = 2 - \frac{1}{2} \Rightarrow \frac{3}{2}$$

$$\Rightarrow x = \frac{2}{3}$$

11. The system of $kx - y = 2$ and $6x - 2y = 3$ has a unique solution only when

(a) $k = 0$ (b) $k \neq 0$ (c) $k = 3$ (d) $k \neq 3$

Answer: (d) $k \neq 3$

Sol:

The given equations are

$$kx - y - 2 = 0 \quad \dots\dots\dots(i)$$

$$6x - 2y - 3 = 0 \quad \dots\dots\dots(ii)$$

Here, $a_1 = k, b_1 = -1, c_1 = -2, a_2 = 6, b_2 = -2$ and $c_2 = -3$.

For the given system to have a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{6} \neq \frac{-1}{-2}$$

$$\Rightarrow k \neq 3$$

12. The system $x - 2y = 3$ and $3x + ky = 1$ have a unique solution only when ?

(a) $k = -6$ (b) $k \neq -6$ (c) $k = 0$ (d) $k \neq 0$

Answer: (b) $k \neq -6$

Sol:

The correct option is (b).

The given system of equations can be written as follows:

$$x - 2y - 3 = 0 \text{ and } 3x + ky - 1 = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -3$, $a_2 = 3$, $b_2 = k$ and $c_2 = -1$.

$$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{k} \text{ and } \frac{c_1}{c_2} = \frac{-3}{-1} = 3$$

These graph lines will intersect at a unique point when we have:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{1}{3} \neq \frac{-2}{k} \Rightarrow k \neq -6$$

Hence, k has all real values other than -6 .

13. The system $x + 2y = 3$ and $5x + ky + 7 = 0$ have no solution when?

(a) $k = 10$ (b) $k \neq 10$ (c) $k = \frac{-7}{3}$ (d) $k = -21$

Answer: (a) $k = 10$

Sol:

The correct option is (a).

The given system of equations can be written as follows:

$$x + 2y - 3 = 0 \text{ and } 5x + ky + 7 = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = -3$, $a_2 = 5$, $b_2 = k$ and $c_2 = 7$.

$$\therefore \frac{a_1}{a_2} = \frac{1}{5}, \frac{b_1}{b_2} = \frac{2}{k} \text{ and } \frac{c_1}{c_2} = \frac{-3}{7}$$

For the system of equations to have no solution, we must have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7} \Rightarrow k = 10$$

14. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(a) $\frac{-5}{4}$ (b) $\frac{2}{5}$ (c) $\frac{3}{2}$ (d) $\frac{15}{4}$

Answer: (d) $\frac{15}{4}$

Sol:

The given system of equations can be written as follows:

$$3x + 2ky - 2 = 0 \text{ and } 2x + 5y + 1 = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 3$, $b_1 = 2k$, $c_1 = -2$, $a_2 = 2$, $b_2 = 5$ and $c_2 = 1$

$$\therefore \frac{a_1}{a_2} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{2k}{5} \text{ and } \frac{c_1}{c_2} = \frac{-2}{1}$$

For parallel lines, we have:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$

$$\Rightarrow k = \frac{15}{4}$$

15. For what value of k do the equations $kx - 2y = 3$ and $3x + y = 5$ represent two lines intersecting at a unique point?
 (a) $k = 3$ (b) $k = -3$ (c) $k = 6$ (d) all real values except -6

Answer: (d) all real values except -6

Sol:

The given system of equations can be written as follows:

$$kx - 2y - 3 = 0 \text{ and } 3x + y - 5 = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = k$, $b_1 = -2$, $c_1 = -3$ and $a_2 = 3$, $b_2 = 1$ and $c_2 = -5$

$$\therefore \frac{a_1}{a_2} = \frac{k}{3}, \frac{b_1}{b_2} = \frac{-2}{1} \text{ and } \frac{c_1}{c_2} = \frac{-3}{-5} = \frac{3}{5}$$

Thus, for these graph lines to intersect at a unique point, we must have:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{k}{3} \neq \frac{-2}{1} \Rightarrow k \neq -6$$

Hence, the graph lines will intersect at all real values of k except -6 .

16. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has
 (a) a unique solution (b) exactly two solutions
 (c) infinitely many solutions (d) no solution

Answer: (d) no solution

Sol:

The given system of equations can be written as:

$$x + 2y + 5 = 0 \text{ and } -3x - 6y + 1 = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 1$, $b_1 = 2$, $c_1 = 5$, $a_2 = -3$, $b_2 = -6$ and $c_2 = 1$

$$\therefore \frac{a_1}{a_2} = \frac{1}{-3}, \frac{b_1}{b_2} = \frac{2}{-6} = \frac{1}{-3} \text{ and } \frac{c_1}{c_2} = \frac{5}{1}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given system has no solution.

17. The pair of equations $2x + 3y = 5$ and $4x + 6y = 15$ has
 (a) a unique solution (b) exactly two solutions
 (c) infinitely many solutions (d) no solution

Answer: (d) no solution

Sol:

The given system of equations can be written as:

$$2x + 3y - 5 = 0 \text{ and } 4x + 6y - 15 = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$, $a_2 = 4$, $b_2 = 6$ and $c_2 = -15$

$$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \text{ and } \frac{c_1}{c_2} = \frac{-5}{-15} = \frac{1}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the given system has no solution.

18. If a pair of linear equations is consistent, then their graph lines will be

- (a) parallel (b) always coincident
(c) always intersecting (d) intersecting or coincident

Answer: (d) intersecting or coincident

Sol:

If a pair of linear equations is consistent, then the two graph lines either intersect at a point or coincidence.

19. If a pair of linear equations is inconsistent, then their graph lines will be

- (a) parallel (b) always coincident
(c) always intersecting (d) intersecting or coincident

Answer: (a) parallel

Sol:

If a pair of linear equations in two variables is inconsistent, then no solution exists as they have no common point. And, since there is no common solution, their graph lines do not intersect. Hence, they are parallel.

20. In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$, then $\angle B = ?$

- (a) 20° (b) 40° (c) 60° (d) 80°

Answer: (b) 40°

Sol:

$$\text{Let } \angle A = x^\circ \text{ and } \angle B = y^\circ$$

$$\therefore \angle A = 3\angle B = (3y)^\circ$$

$$\text{Now, } \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x + y + 3y = 180$$

$$\Rightarrow x + 4y = 180 \quad \dots\dots(i)$$

$$\text{Also, } \angle C = 2(\angle A + \angle B)$$

$$\Rightarrow 3y = 2(x + y)$$

$$\Rightarrow 2x - y = 0 \quad \dots\dots(ii)$$

On multiplying (ii) by 4, we get:

$$8x - 4y = 0 \quad \dots\dots(iii)$$

On adding (i) and (iii) we get:

$$9x = 180 \Rightarrow x = 20$$

On substituting $x = 20$ in (i), we get:

$$20 + 4y = 180 \Rightarrow 4y = (180 - 20) = 160 \Rightarrow y = 40$$

$$\therefore x = 20 \text{ and } y = 40$$

$$\therefore \angle B = y^\circ = 40^\circ$$

21. In a cyclic quadrilateral ABCD, it is being given that $\angle A = (x + y + 10)^\circ$, $\angle B = (y + 20)^\circ$, $\angle C = (x + y - 30)^\circ$ and $\angle D = (x + y)^\circ$. Then, $\angle B = ?$
 (a) 70° (b) 80° (c) 100° (d) 110°

Answer: (b) 80°

Sol:

The correct option is (b).

In a cyclic quadrilateral ABCD:

$$\angle A = (x + y + 10)^\circ$$

$$\angle B = (y + 20)^\circ$$

$$\angle C = (x + y - 30)^\circ$$

$$\angle D = (x + y)^\circ$$

We have:

$$\angle A + \angle C = 180^\circ \text{ and } \angle B + \angle D = 180^\circ \quad [\text{Since ABCD is a cyclic quadrilateral}]$$

$$\text{Now, } \angle A + \angle C = (x + y + 10)^\circ + (x + y - 30)^\circ = 180^\circ$$

$$\Rightarrow 2x + 2y - 20 = 180$$

$$\Rightarrow x + y - 10 = 90$$

$$\Rightarrow x + y = 160 \quad \dots\dots(i)$$

$$\text{Also, } \angle B + \angle D = (y + 20)^\circ + (x + y)^\circ = 180^\circ$$

$$\Rightarrow x + 2y + 20 = 180$$

$$\Rightarrow x + 2y = 160$$

On subtracting (i) from (ii), we get:

$$y = (160 - 100) = 60$$

On substituting $y = 60$ in (i), we get:

$$x + 60 = 160 \Rightarrow x = (160 - 60) = 100$$

$$\therefore \angle B = (y + 20)^\circ = (60 + 20)^\circ = 80^\circ$$

22. 5 years hence, the age of a man shall be 3 times the age of his son while 5 years earlier the age of the man was 7 times the age of his son. The present age of the man is
 (a) 45 years (b) 50 years (c) 47 years (d) 40 years

Answer: (d) 40 years

Sol:

Let the man's present age be x years.

Let his son's present age be y years.

Five years later:

$$(x + 5) = 3(y + 5)$$

$$\Rightarrow x + 5 = 3y + 15$$

$$\Rightarrow x - 3y = 10 \quad \dots\dots\dots(i)$$

Five years ago:

$$(x - 5) = 7(y - 5)$$

$$\Rightarrow x - 5 = 7y - 35$$

$$\Rightarrow x - 7y = -30 \quad \dots\dots\dots(ii)$$

On subtracting (i) from (ii), we get:

$$-4y = -40 \Rightarrow y = 10$$

On substituting $y = 10$ in (i), we get:

$$x - 3 \times 10 = 10 \Rightarrow x - 30 = 10 \Rightarrow x = (10 + 30) = 40 \text{ years}$$

Hence, the man's present age is 40 years.

23.

Assertion (A)	Reason (R)
The system of equations $x + y - 8 = 0$ and $x - y - 2 = 0$ has a unique solutions.	The system of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.

The correct answer is: (a) / (b) / (c) / (d).

Answer: (c)

Sol:

Option (c) is the correct answer.

Clearly, Reason (R) is false.

On solving $x + y = 8$ and $x - y = 2$, we get:

$$x = 5 \text{ and } y = 3$$

Thus, the given system has a unique solution. So, assertion (A) is true.

\therefore Assertion (A) is true and Reason (R) is false.

24. The graphs of the equations $6x - 2y + 9 = 0$ and $3x - y + 12 = 0$ are two lines which are

(a) coincident

(b) parallel

(c) intersecting exactly at one point

(d) perpendicular to each other

Answer: (b) parallel

Sol:

The given equations are as follows:

$$6x - 2y + 9 = 0 \text{ and } 3x - y + 12 = 0$$

They are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 6$, $b_1 = -2$, $c_1 = 9$ and $a_2 = 3$, $b_2 = -1$ and $c_2 = 12$

$$\therefore \frac{a_1}{a_2} = \frac{6}{3} = \frac{2}{1}, \frac{b_1}{b_2} = \frac{-2}{-1} = \frac{2}{1} \text{ and } \frac{c_1}{c_2} = \frac{9}{12} = \frac{3}{4}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

The given system has no solution.

Hence, the lines are parallel.

25. The graphs of the equations $2x + 3y - 2 = 0$ and $x - 2y - 8 = 0$ are two lines which are

- (a) coincident
- (b) parallel
- (c) intersecting exactly at one point
- (d) perpendicular to each other

Answer:

Sol:

The given equations are as follows:

$$2x + 3y - 2 = 0 \text{ and } x - 2y - 8 = 0$$

They are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -2$ and $a_2 = 1$, $b_2 = -2$ and $c_2 = -8$

$$\therefore \frac{a_1}{a_2} = \frac{2}{1}, \frac{b_1}{b_2} = \frac{3}{-2} \text{ and } \frac{c_1}{c_2} = \frac{-2}{-8} = \frac{1}{4}$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The given system has a unique solution.

Hence, the lines intersect exactly at one point.

26. The graphs of the equations $5x - 15y = 8$ and $3x - 9y = \frac{24}{5}$ are two lines which are

- (a) coincident
- (b) parallel
- (c) intersecting exactly at one point
- (d) perpendicular to each other

Answer: (a) coincident

Sol:

The correct option is (a).

The given system of equations can be written as follows:

$$5x - 15y - 8 = 0 \text{ and } 3x - 9y - \frac{24}{5} = 0$$

The given equations are of the following form:

$$a_1x + b_1y + c_1 = 0 \text{ and } a_2x + b_2y + c_2 = 0$$

Here, $a_1 = 5$, $b_1 = -15$, $c_1 = -8$ and $a_2 = 3$, $b_2 = -9$ and $c_2 = -\frac{24}{5}$

$$\therefore \frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3} \text{ and } \frac{c_1}{c_2} = -8 \times \frac{5}{-24} = \frac{5}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

The given system of equations will have an infinite number of solutions.

Hence, the lines are coincident.

27. The sum of the digits of a two digit number is 15. The number obtained by interchanging the digits exceeds the given number by 9. The number is

(a) 96 (b) 69 (c) 87 (d) 78

Answer: (a) 96

Sol:

Let the tens and the units digits of the required number be x and y , respectively.

Required number = $(10x + y)$

According to the question, we have:

$$x + y = 15 \quad \dots\dots(i)$$

Number obtained on reversing its digits = $(10y + x)$

$$\therefore (10y + x) = (10x + y) + 9$$

$$\Rightarrow 10y + x - 10x - y = 9$$

$$\Rightarrow 9y - 9x = 9$$

$$\Rightarrow y - x = 1 \quad \dots\dots(ii)$$

On adding (i) and (ii), we get:

$$2y = 16 \Rightarrow y = 8$$

On substituting $y = 8$ in (i), we get:

$$x + 8 = 15 \Rightarrow x = (15 - 8) = 7$$

$$\text{Number} = (10x + y) = 10 \times 7 + 8 = 70 + 8 = 78$$

Hence, the required number is 78.