RD SHARMA
Solutions

Class 10 Maths

Chapter 4

Ex4.6

1. Triangles ABC and DEF are similar.

(i) If area of (ΔABC) = 16 cm^2 , area (ΔDEF) = 25 cm^2 and BC = 2.3 cm, find EF.

(ii) If area (ΔABC) = 9 cm^2 , area (ΔDEF) = 64 cm^2 and DE = 5.1 cm, find AB.

(iii) If AC = 19 cm and DF = 8 cm, find the ratio of the area of two triangles.

(iv) If area of (ΔABC) = 36 cm^2 , area (ΔDEF) = 64 cm^2 and DE = 6.2 cm, find AB.

(v) If AB = 1.2 cm and DE = 1.4 cm, find the ratio of the area of two triangles.

Answer:

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{BC}{EF}\right)^2 \quad \frac{16}{25} = \left(\frac{2.3}{EF}\right)^2 \quad \frac{4}{5} = \frac{2.3}{EF}$

EF = 2.875 cm

(ii) $\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2$

$\frac{9}{64} = \left(\frac{AB}{DE}\right)^2 \quad \frac{3}{8} = \frac{AB}{5.1}$

AB = 1.9125 cm

(iii) $\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AC}{DF}\right)^2$

$\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{19}{8}\right)^2 \quad \frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{361}{64}\right)$

(iv) $\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2$

$\frac{36}{64} = \left(\frac{AB}{DE}\right)^2 \quad \frac{6}{8} = \frac{AB}{6.2}$

AB = 4.65 cm

(v) $\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{AB}{DE}\right)^2$

$\frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{1.2}{1.4}\right)^2 \quad \frac{ar\Delta ABC}{ar\Delta DEF} = \left(\frac{36}{49}\right)$

2. In the fig 4.178, $\triangle ACB$ is similar to $\triangle APQ$. If BC = 10 cm, PQ = 5 cm, BA = 6.5 cm, AP = 2.8 cm, find CA and AQ. Also, find the Area of $\triangle ACB$: Area of $\triangle APQ$.

Answer:

Given: $\triangle ACB$ is similar to $\triangle APQ$

BC = 10 cm

PQ = 5 cm

BA = 6.5 cm

AP = 2.8 cm

Find:

(1) CA and AQ

(2) Area of $\triangle ACB$: Area of $\triangle APQ$

(1) It is given that $\triangle ACB \sim \triangle APQ$

We know that for any two similar triangles the sides are proportional. Hence
$\frac{AB}{AQ} = \frac{BC}{PQ} = \frac{AC}{AP}$

$\frac{AB}{AQ} = \frac{BC}{PQ}$ $\frac{6.5}{AQ} = \frac{10}{5}$

AQ = 3.25 cm

Similarly,

$\frac{BC}{PQ} = \frac{CA}{AP}$ $\frac{CA}{2.8} = \frac{10}{5}$

CA = 5.6 cm

(2) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$\frac{ar\triangle ACQ}{ar\triangle APQ} = \left(\frac{BC}{PQ}\right)^2$

$= \left(\frac{10}{5}\right)^2$

$$= (21)^2\left(\tfrac{2}{1}\right)^2$$

$$= 41\tfrac{4}{1}$$

3. The areas of two similar triangles are 81 cm² and 49 cm² respectively. Find the ration of their corresponding heights. What is the ratio of their corresponding medians?

Answer:

Given: The area of two similar triangles is $81cm^2$ and $49cm^2$ respectively.

To find:

(1) The ratio of their corresponding heights.

(2) The ratio of their corresponding medians.

(1) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$$ar(triangle1)ar(triangle2) = \left(altitude1altitude2\right)^2 \frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = \left(\frac{altitude\ 1}{altitude\ 2}\right)^2 \quad 8149 = \left(altitude1altitude2\right)^2$$

$$\frac{81}{49} = \left(\frac{altitude\ 1}{altitude\ 2}\right)^2$$

Taking square root on both sides, we get

$$97 = altitude1altitude2 \quad \frac{9}{7} = \frac{altitude\ 1}{altitude\ 2}$$

Altitude 1: altitude 2 = 9: 7

(2) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their medians.

$$ar(triangle1)ar(triangle2) = \left(median1median2\right)^2 \frac{ar\ (triangle\ 1)}{ar\ (triangle\ 2)} = \left(\frac{median\ 1}{median\ 2}\right)^2 \quad 8149 = \left(median1median2\right)^2$$

$$\frac{81}{49} = \left(\frac{median\ 1}{median\ 2}\right)^2$$

Taking square root on both sides, we get

$$97 = median1median2 \quad \frac{9}{7} = \frac{median\ 1}{median\ 2}$$

Median 1: median 2 = 9: 7

4. The areas of two similar triangles are 169 cm^2 and 121 cm^2 respectively. If the longest side of the larger triangle is 26 cm, find the longest side of the smaller triangle.

Answer:

Given:

The area of two similar triangles is 169cm2 and 121cm2 respectively. The longest side of the larger triangle is 26cm.

To find:

Longest side of the smaller triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

$$\frac{ar\ (larger\ triangle)}{ar\ (smaller\ triangle)} = \left(\frac{side\ of\ the\ larger\ triangle}{side\ of\ the\ smaller\ triangle}\right)^2$$

$$\frac{169}{121} = \left(\frac{side\ of\ the\ larger\ triangle}{side\ of\ the\ smaller\ triangle}\right)^2$$

Taking square root on both sides, we get

$$\frac{13}{11} = \frac{side\ of\ the\ larger\ triangle}{side\ of\ the\ smaller\ triangle}$$

$$\frac{13}{11} = \frac{26}{side\ of\ the\ smaller\ triangle}$$

Side of the smaller triangle = $\frac{11 \times 26}{13}$ = 22 cm

Hence, the longest side of the smaller triangle is 22 cm.

5. The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively. Find the ratio of their areas.

Answer:

Given:

The area of two similar triangles is 25cm^2 and 36cm^2 respectively. If the altitude of first triangle 2.4cm.

To find:

The altitude of the other triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\text{ar(triangle1)ar(triangle2)} = \left(\text{altitude1altitude2}\right)^2 \dfrac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \left(\dfrac{\text{altitude 1}}{\text{altitude 2}}\right)^2 \quad 2536 = \left(2.4\text{altitude2}\right)^2$

$\dfrac{25}{36} = \left(\dfrac{2.4}{\text{altitude 2}}\right)^2$

Taking square root on both sides, we get

$56 = 2.4\text{altitude2} \quad \dfrac{5}{6} = \dfrac{2.4}{\text{altitude 2}}$

Altitude 2 = 2.88 cm

Hence, the corresponding altitude of the other is 2.88 cm.

6. ABC is a triangle in which ∠ A = 90°, AN ⊥ BC, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANCΔANC **and ΔABC**ΔABC **.**

Answer:

Given:

The corresponding altitudes of two similar triangles are 6 cm and 9 cm respectively.

To find:

Ratio of areas of triangle

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$\text{ar(triangle1)ar(triangle2)} = \left(\text{altitude1altitude2}\right)^2 \dfrac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \left(\dfrac{\text{altitude 1}}{\text{altitude 2}}\right)^2 \quad \text{ar(triangle1)ar(triangle2)} = \left(69\right)^2$

$\dfrac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \left(\dfrac{6}{9}\right)^2 \quad \text{ar(triangle1)ar(triangle2)} = 3681 \dfrac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \dfrac{36}{81} \quad \text{ar(triangle1)ar(triangle2)} = 49$

$\dfrac{\text{ar (triangle 1)}}{\text{ar (triangle 2)}} = \dfrac{4}{9}$

ar (triangle 1): ar (triangle 2) = 4: 9

Hence, the ratio of the areas of two triangles is 4: 9.

7. ABC is a triangle in which ∠A°,AN⊥BC$\angle A°,\ AN \perp BC$ **, BC = 12 cm and AC = 5 cm. Find the ratio of the areas of ΔANCandΔABC**$\Delta \text{ANC and } \Delta \text{ABC}$ **.**

Answer:

Given:

In $\triangle ABC$, $\angle A = 90°$, AN $\perp$ BC, BC= 12 cm and AC = 5 cm.

To find:
Ratio of the triangles $\triangle ANC$ and $\triangle ABC$.

In $\triangle ANC$ and $\triangle ABC$,

$\angle ACN = \angle ACB$ (Common)

$\angle A = \angle ANC$ (90° 90°)

Therefore, $\triangle ANC - \triangle ABC$ (AA similarity)

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Therefore,

$\frac{Ar(\triangle ANC)}{Ar(\triangle ABC)} = \left(\frac{AC}{BC}\right)^2$ $\frac{Ar(\triangle ANC)}{Ar(\triangle ABC)} = \left(\frac{5cm}{12cm}\right)^2$

$\frac{Ar(\triangle ANC)}{Ar(\triangle ABC)} = \left(\frac{5cm}{12cm}\right)^2$ $\frac{Ar(\triangle ANC)}{Ar(\triangle ABC)} = \frac{25}{144}$

8. In Fig, DE || BC

(i) If DE = 4m, BC = 6 cm and Area $(\triangle ADE) = 16\,cm^2$, find the area of $\triangle ABC$.

(ii) If DE = 4cm, BC = 8 cm and Area $(\triangle ADE) = 25\,cm^2$, find the area of $\triangle ABC$.

(iii) If DE : BC = 3 : 5. Calculate the ratio of the areas of $\triangle ADE$ and the trapezium BCED.

Answer:

In the given figure, we have DE || BC.

In $\triangle ADE$ and $\triangle ABC$

$\angle ADE = \angle B$ (Corresponding angles)

$\angle DAE = \angle BAC \angle DAE = \angle BAC$ (Common)

So, $\triangle ADE$ and $\triangle ABC \triangle ADE$ and $\triangle ABC$ (AA Similarity)

(i) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Hence,

$\text{Ar}(\triangle ADE)\text{Ar}(\triangle ABC) = DE^2 BC^2 \dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle ABC)} = \dfrac{DE^2}{BC^2}$ $16\text{Ar}(\triangle ABC) = 4^2 6^2 \dfrac{16}{\text{Ar}(\triangle ABC)} = \dfrac{4^2}{6^2}$ $\text{Ar}(\triangle ABC) = 6^2 \times 16 4^2$

$\text{Ar}(\triangle ABC) = \dfrac{6^2 \times 16}{4^2}$

$\text{Ar}(\triangle ABC)\text{Ar}(\triangle ABC)$ = 36 cm^2

(ii) We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Hence,

$\text{Ar}(\triangle ADE)\text{Ar}(\triangle ABC) = DE^2 BC^2 \dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle ABC)} = \dfrac{DE^2}{BC^2}$ $25\text{Ar}(\triangle ABC) = 4^2 8^2 \dfrac{25}{\text{Ar}(\triangle ABC)} = \dfrac{4^2}{8^2}$ $\text{Ar}(\triangle ABC) = 8^2 \times 25 4^2$

$\text{Ar}(\triangle ABC) = \dfrac{8^2 \times 25}{4^2}$

$\text{Ar}(\triangle ABC)\text{Ar}(\triangle ABC)$ = 100 cm^2

(iii) We know that

$\text{Ar}(\triangle ADE)\text{Ar}(\triangle ABC) = DE^2 BC^2 \dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle ABC)} = \dfrac{DE^2}{BC^2}$ $\text{Ar}(\triangle ADE)\text{Ar}(\triangle ABC) = 3^2 5^2$

$\dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle ABC)} = \dfrac{3^2}{5^2}$ $\text{Ar}(\triangle ADE)\text{Ar}(\triangle ABC) = 925 \dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\triangle ABC)} = \dfrac{9}{25}$

Let the area of $\triangle ADE \triangle ADE$ = 9x sq units

area of $\triangle ABC \triangle ABC$ = 25x sq units

Now, $\text{Ar}(\triangle ADE)\text{Ar}(\text{trap} BCED) = 9x16x \dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\text{trap} BCED)} = \dfrac{9x}{16x}$

$\text{Ar}(\triangle ADE)\text{Ar}(\text{trap} BCED) = 916 \dfrac{\text{Ar}(\triangle ADE)}{\text{Ar}(\text{trap} BCED)} = \dfrac{9}{16}$

9. In $\triangle ABC \triangle ABC$, D and E are the mid- points of AB and AC respectively. Find the ratio of the areas $\triangle ADE \triangle ADE$ and $\triangle ABC \triangle ABC$.

Answer:

Given:

In $\triangle ABC$, D and E are the midpoints of AB and AC respectively.

To find:

Ratio of the areas of $\triangle ADE$ and $\triangle ABC$

It is given that D and E are the midpoints of AB and AC respectively.

Therefore, DE II BC (Converse of mid-point theorem)

Also, DE = $\frac{1}{2}$BC

In $\triangle ADE$ and $\triangle ABC$

$\angle ADE = \angle B$ (Corresponding angles)

$\angle DAE = \angle BAC$ (common)

So, $\triangle ADE - \triangle ABC$ (AA Similarity)

We know that the ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

$$\frac{Ar(\triangle ADE)}{Ar(\triangle ABC)} = \frac{AD^2}{AB^2}$$

$$\frac{Ar(\triangle ADE)}{Ar(\triangle ABC)} = \frac{1^2}{2^2}$$

$$\frac{Ar(\triangle ADE)}{Ar(\triangle ABC)} = \frac{1}{4}$$

10. The areas of two similar triangles are 100 cm2 and 49 cm2 respectively. If the altitude of the bigger triangles is 5 cm, find the corresponding altitude of the other.

Answer:

Given: the area of the two similar triangles is $100cm^2$ and $49cm^2$ respectively. If the altitude of the bigger triangle is 5cm

To find: their corresponding altitude of the other triangle

We know that the ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$$\frac{ar(\text{bigger triangle1})}{Ar(\text{triangle2})} = \left(\frac{\text{altitude of the bigger triangle1}}{\text{altitude2}}\right)^2$$

$$\left(10049\right)\left(\frac{100}{49}\right)=\left(5\text{altitude2}\right)^2\left(\frac{5}{\text{altitude2}}\right)^2$$

Taking squares on both the sides

$$\left(107\left(\frac{10}{7}\right.=\left(5\text{altitude2}\right)\left(\frac{5}{\text{altitude2}}\right)\right.$$

Altitude 2=3.5cm

11. The areas of two similar triangles are 121 cm2 and 64 cm2 respectively. If the median of the first triangle is 12.1 cm, find the corresponding median of the other.

Answer:

Given : the area of the two triangles is 121cm^2 121cm^2 and 64cm^2 64cm^2 respectively. If the merdian of the first triangle is 12.1cm

To find the corresponding medians of the other triangle

We know that ratio of the areas of the two similar triangles are equal to the ratio of the squares of their merdians

$$\left(\text{ar(triangle1)ar(triangle2)}\left(\frac{\text{ar(triangle1)}}{\text{ar(triangle2)}}\right.\right.=\left(\text{median1median2}\right)^2\left(\frac{\text{median1}}{\text{median2}}\right)^2$$

$$\left(12164\left(\frac{121}{64}\right.\right.=\left(\text{12.1median2}\right)^2\left(\frac{12.1}{\text{median2}}\right)^2$$

Taking the squareroot on the both sides

$$\left(118\left(\frac{11}{8}\right.\right.=\left(\text{12.1median2}\right)\left(\frac{12.1}{\text{median2}}\right)$$

Median2=8.8cm

12. If ΔABC~ΔDEF $\Delta ABC \sim \Delta DEF$ such that AB = 5cm, area (ΔABCΔABC) = 20 cm2 and area (ΔDEFΔDEF) = 45 cm2, determine DE.

Answer:

Given : the area of the two similar ΔABCΔABC =20cm^{2}20cm^{2}and ΔDEFΔDEF =45cm^2 45cm^2 and AB=5cm

To measure of DE

We know that the ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\text{Ar}\Delta ABC\text{Ar}\Delta DEF = \left(\text{ABDE}\right)^2 \frac{\text{Ar}\Delta ABC}{\text{Ar}\Delta DEF} = \left(\frac{AB}{DE}\right)^2$$

$2045\,\frac{20}{45} = 5DE^2\,\frac{5}{DE}^2$

$2045\,\frac{20}{45} = 5DE\,\frac{5}{DE}$

$DE^2 DE^2 = 25\times4520\,\frac{25\times45}{20}$

$DE^2 DE^2 = 2254\,\frac{225}{4}$

DE=7.5cm

13. In $\Delta ABC \triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q such that PQ || BC and PQ divides $\Delta ABC \triangle ABC$ into two parts equal in area. Find BPAB $\frac{BP}{AB}$.

Answer:

Given: in $\Delta ABC \triangle ABC$, PQ is a line segment intersecting AB at P and AC at such that $PQ\|BC$ $PQ \parallel BC$ and PQ divides $\Delta ABC \triangle ABC$ in two parts equal in area

To find : BPAB $\frac{BP}{AB}$

We have $PQ\|BC PQ \parallel BC$ and

Ar($\Delta APQ \triangle APQ$) = Ar(quad BPQC)

Ar($\Delta APQ \triangle APQ$) +Ar($\Delta APQ \triangle APQ$) =Ar(quad BPQC) +Ar($\Delta APQ \triangle APQ$)

2(Ar($\Delta APQ \triangle APQ$) =Ar($\Delta ABC \triangle ABC$)

Now $PQ\|BC PQ \parallel BC$ and BA is a transversal

In $\Delta ABC \triangle ABC$ and $\Delta APQ \triangle APQ$)

$\angle APQ=\angle B \angle APQ = \angle B$ (corresponding angles)

$\angle PAQ=\angle BAC \angle PAQ = \angle BAC$ (common)

In $\Delta ABC \sim \Delta APQ \triangle ABC \sim \triangle APQ$) (AA similarity)

We know that the ratio of the areas of the two similar triangles is used and is equal to the ratio of their squares of the corresponding sides.

Hence

$$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{AP}{AB}\right)^2 \quad \frac{Ar\triangle APQ}{2Ar\triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{AP}{AB}\right)^2 \quad \frac{1}{2} =$$

$$\left(\frac{AP}{AB}\right)^2 \quad \frac{1}{2} = \left(\frac{AP}{AB}\right)^2 \quad \sqrt{\frac{1}{2}} = \left(\frac{AP}{AB}\right)$$

$$AB = \sqrt{2}AP \quad \sqrt{2}AP$$

$$AB = \sqrt{2}(AB - BP) \quad \sqrt{2(AB - BP)}$$

$$\sqrt{2}BP \quad \sqrt{2}BP = \sqrt{2}AB - AB \quad \sqrt{2}AB - AB$$

$$\frac{BP}{AB} = \sqrt{2} - 1\sqrt{2} \quad \frac{\sqrt{2}-1}{\sqrt{2}}$$

14. The areas of two similar triangles ABC and PQR are in the ratio 9 : 16. If BC = 4.5 cm, find the length of QR.

Answer:

Given: the areas of the two similar triangles ABC and PQR are in the ratio 9:16. BC=4.5cm

To find: Length of QR

We know that the ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{Ar\triangle ABC}{Ar\triangle PQR} = \left(\frac{BC}{QR}\right)^2 = \left(\frac{BC}{QR}\right)^2$$

$$\frac{9}{16} = \left(\frac{4.5}{QR}\right)^2 \left(\frac{4.5}{QR}\right)^2$$

$$\frac{3}{4} = \frac{4.5}{QR} \quad \frac{4.5}{QR}$$

$$QR = \frac{18}{3} = 6cm$$

15. ABC is a triangle and PQ is a straight line meeting AB and P and AC in Q. If AP = 1 cm, PB = 3 cm, AQ = 1.5 cm, QC = 4.5 m, prove that area of $\triangle APQ$ is one – sixteenth of the area of $\triangle ABC$.

Answer:

Given : in $\triangle ABC$, PQ is a line segment intersecting AB at P and AC at Q. AP = 1cm , PB = 3cm, AQ= 1.5 cm and QC= 4.5cm

To find Ar($\triangle APQ$)= $\frac{1}{16} \times \triangle ABC$)

In $\triangle ABC$

$\frac{AP}{PB} = \frac{AQ}{QC}$

$\frac{1}{3} = \frac{1}{3}$

According to converse of basic proportional theorem if a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Hence,

$PQ \| BC$

Hence in $\triangle ABC$ and $\triangle APQ$

$\angle APQ$ =$\angle B$ (corresponding angles)

$\angle PAQ$ =$\angle BAC$ (common)

$\triangle ABC \sim \triangle APQ$ $\frac{Ar\triangle APQ}{Ar\triangle ABC} = (\frac{AP}{AB})^2$

$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \left(\frac{AP}{(AB+BP)}\right)^2 = \left(\frac{AP}{(AB+BP)}\right)^2$

$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \left(\frac{1}{4}\right)^2 \frac{Ar\triangle APQ}{Ar\triangle ABC} = (\frac{1}{4})^2$ (given)

$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \left(\frac{1}{16}\right) \frac{Ar\triangle APQ}{Ar\triangle ABC} = (\frac{1}{16})$

16. If D is a point on the side AB of $\triangle ABC$ such that AD : DB = 3 : 2 and E is a point on BC such that DE || AC. Find the ratio of areas of $\triangle ABC$ and $\triangle BDE$.

Answer:

Given In $\triangle ABC$, D is appoint on the side AB such that AD:DB=3:2. E is a point on side BC such that $DE \| AC$

To find

$\triangle ABC \triangle BDE \dfrac{\triangle ABC}{\triangle BDE}$

In $\triangle ABC \triangle ABC$, $\triangle BDE \triangle BDE$,

$\angle BDE \angle BDE$ $= \angle A \angle A$ (corresponding angles)

$\angle DBE \angle DBE$ $= \angle ABC \angle ABC$

$\triangle ABC \sim \triangle BDE \triangle ABC \sim \triangle BDE$

We know that the ratio of the two similar triangles is equal to the ratio of the squares of their corresponding sides

Let AD=2x and BD =3x

Hence

$\dfrac{Ar\triangle ABC}{Ar\triangle BDE} = \left(\dfrac{AB}{BD}\right)^2 \dfrac{Ar\triangle ABC}{Ar\triangle BDE} = \left(\dfrac{AB}{BD}\right)^2 \left(\dfrac{AB+DA}{BD}\right)^2 \left(\dfrac{AB+DA}{BD}\right)^2 \left(\dfrac{3x+2x}{2x}\right)^2$
$\left(\dfrac{3x+2x}{2x}\right)^2$ $\dfrac{Ar\triangle ABC}{Ar\triangle BDE} = \left(\dfrac{25}{4}\right) \dfrac{Ar\triangle ABC}{Ar\triangle BDE} = \left(\dfrac{25}{4}\right)$

17. If $\triangle ABC \triangle ABC$ and $\triangle BDE \triangle BDE$ are equilateral triangles, where D is the midpoint of BC, find the ratio of areas of $\triangle ABC \triangle ABC$ and $\triangle BDE \triangle BDE$.

Answer:

Given In $\triangle ABC \triangle ABC$, $\triangle BDE \triangle BDE$ are equilateral triangles. D is the point of BC.

To find $\dfrac{Ar\triangle ABC}{Ar\triangle BDE} \dfrac{Ar\triangle ABC}{Ar\triangle BDE}$

In $\triangle ABC \triangle ABC$, $\triangle BDE \triangle BDE$

$\triangle ABC \sim \triangle BDE \triangle ABC \sim \triangle BDE$ (AAA criteria of similarity all angles of the equilateral triangles are equal)

Since D is the mid point of BC, BD : DC=1

We know that the ratio of the areas of the two similar triangles is equal to the ratio of squares of their corresponding sides.

Let DC=x, and BD= x

Hence

$\dfrac{Ar\triangle ABC}{Ar\triangle BDE} = \left(\dfrac{BC}{BD}\right)^2 \dfrac{Ar\triangle ABC}{Ar\triangle BDE} = \left(\dfrac{BC}{BD}\right)^2$

$= \left(\dfrac{BD+DA}{DC}\right)^2 \left(\dfrac{BD+DA}{DC}\right)^2$

$$= \left(1x + 1x1x\right)^2 \left(\frac{1x+1x}{1x}\right)^2$$

$$\text{Ar}\triangle\text{ABCAr}\triangle\text{BDE} = 4{:}1 \quad \frac{\text{Ar}\triangle\text{ABC}}{\text{Ar}\triangle\text{BDE}} = 4 : 1$$

18.Two isosceles triangles have equal vertical angles and their areas are in the ratio 36 : 25. Find the ratio of their corresponding heights.

Answer:

Given:

Two isosceles triangles have equal vertical angles and their areas are in the ratio of 36: 25.

To find:

Ratio of their corresponding heights

Suppose $\triangle ABC\triangle ABC$ and $\triangle PQR\triangle PQR$ are two isosceles triangles with $\angle A = \angle P \angle A = \angle P$.

Therefore,

$$\text{ABAC} = \text{PQPR} \quad \frac{AB}{AC} = \frac{PQ}{PR}$$

In $\triangle ABC\triangle ABC$ and $\triangle PQR\triangle PQR$,

$$\angle A = \angle P\angle A = \angle P \quad \text{ABAC} = \text{PQPR} \quad \frac{AB}{AC} = \frac{PQ}{PR}$$

$\therefore \triangle ABC\triangle ABC - \triangle PQR\triangle PQR$ (SAS similarity)

Let AD and PS be the altitudes of $\triangle ABC\triangle ABC$ and $\triangle PQR\triangle PQR$, respectively.

We know that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

$$\text{ar}\triangle\text{ABCar}\triangle\text{PQR} = \left(\text{ADPS}\right)^2 \quad \frac{\text{ar }\triangle\text{ABC}}{\text{ar }\triangle\text{PQR}} = \left(\frac{AD}{PS}\right)^2 \quad 3625 = \left(\text{ADPS}\right)^2 \quad \frac{36}{25} = \left(\frac{AD}{PS}\right)^2 \quad \text{ADPS} = 65 \quad \frac{AD}{PS} = \frac{6}{5}$$

Hence, the ratio of their corresponding heights is 6: 5.

19. In the given figure. $\triangle ABC\triangle ABC$ and $\triangle DBC\triangle DBC$ are on the same base BC. If AD and BC intersect at O, Prove that

$$\textbf{Areaof}(\triangle\textbf{ABC})\textbf{Areaof}(\triangle\textbf{DBC}) = \textbf{AODO} \quad \frac{\text{Area of }(\triangle ABC)}{\text{Area of }(\triangle DBC)} = \frac{AO}{DO}$$

Answer:

Given $\triangle ABC$ and $\triangle DBC$ are on the same BC. AD and BC intersect at O.

Prove that : $\dfrac{Ar\triangle ABC}{Ar\triangle DBC} = \dfrac{AO}{DO}$

$AL \perp BC \text{ and } DM \perp BC$

Now, in $\triangle ALO$ and $\triangle ALO$ we have

$\angle ALO = \angle DMO = 90°$

$\angle AOL = \angle DOM$ (vertically opposite angles)

Therefore $\triangle ALO \sim \triangle DMO$

$\therefore \dfrac{AL}{DM} = \dfrac{AO}{DO}$

$\dfrac{Ar\triangle ABC}{Ar\triangle BCD} = \dfrac{\frac{1}{2}\times BC \times AL}{\frac{1}{2}\times BC \times DM}$

$= \dfrac{AL}{DM}$

$= \dfrac{AO}{DO}$

20. ABCD is a trapezium in which AB || CD. The Diagonal AC and BC intersect at O. Prove that :

(i) $\triangle AOB \triangle COD$

(ii) If OA = 6 cm, OC = 8 cm,

Find:

(a) $\dfrac{\text{Area of }(\triangle AOB)}{\text{Area of }(\triangle COD)}$

(b) $\dfrac{\text{Area of }(\triangle AOD)}{\text{Area of }(\triangle COD)}$

Answer: Given ABCD is the trapezium which $AB \| CD$

The diagonals AC and BD intersect at o.

To prove:

(i) $\triangle AOB \sim \triangle COD$

(ii) If OA = 6 cm, OC = 8 cm

To find :

(a) $\text{Ar}\Delta\text{AOB}\text{Ar}\Delta\text{COD}$ $\dfrac{\text{Ar}\Delta\text{AOB}}{\text{Ar}\Delta\text{COD}}$

(b) $\text{Ar}\Delta\text{AOD}\text{Ar}\Delta\text{COD}$ $\dfrac{\text{Ar}\Delta\text{AOD}}{\text{Ar}\Delta\text{COD}}$

Construction : Draw a line MN passing through O and parallel to AB and CD

Now in $\Delta\text{AOB} \text{and} \Delta\text{COD} \Delta\text{AOB} \text{and} \Delta\text{COD}$

(i) Now in $\angle\text{OAB}\angle\text{OAB}$ $=\angle\text{OCD}\angle\text{OCD}$ (Alternate angles)

(ii) $\angle\text{OBA}\angle\text{OBA}$ $=\angle\text{ODC}\angle\text{ODC}$ (Alternate angles)

$\angle\text{AOB}\angle\text{AOB}$ $=\angle\text{COD}\angle\text{COD}$ (vertically opposite angle)

$\Delta\text{AOB}\sim\Delta\text{COD} \Delta\text{AOB} \sim \Delta\text{COD}$ (A.A.Acrieteria)

a) We know that the ratio of areas of two triangles is equal to the ratio of squares of their corresponding sides.

$\text{Ar}\Delta\text{AOB}\text{Ar}\Delta\text{COD} = \left(\text{AOCO}\right)^2 \dfrac{\text{Ar}\Delta\text{AOB}}{\text{Ar}\Delta\text{COD}} = \left(\dfrac{\text{AO}}{\text{CO}}\right)^2$

$= \left(68\right)^2 \left(\dfrac{6}{8}\right)^2$

$\text{Ar}\Delta\text{AOB}\text{Ar}\Delta\text{COD} \dfrac{\text{Ar}\Delta\text{AOB}}{\text{Ar}\Delta\text{COD}} = \left(68\right)^2 \left(\dfrac{6}{8}\right)^2$

b) We know that the ratio of two similar triangles is equal to the artio of their corresponding

sides.

$\text{Ar}\Delta\text{AOB}\text{Ar}\Delta\text{COD} = \left(\text{AOCO}\right)^2 \dfrac{\text{Ar}\Delta\text{AOB}}{\text{Ar}\Delta\text{COD}} = \left(\dfrac{\text{AO}}{\text{CO}}\right)^2$

$= \left(6\text{cm}8\text{cm}\right)^2 \left(\dfrac{6\text{cm}}{8\text{cm}}\right)^2 = \left(68\right)^2 \left(\dfrac{6}{8}\right)^2$

21. In $\Delta\text{ABC}\Delta\text{ABC}$, P divides the side AB such that AP : PB = 1 : 2. Q is a point in AC such that PQ || BC. Find the ratio of the areas of $\Delta\text{APQ}\Delta\text{APQ}$ and trapezium BPQC.

Answer: Given : In $\triangle ABC$, P divides the side AB such that AP: PB =1:2, Q is a point on AC on such that PQ ||| BC

To find : The ratio of the areas of $\triangle APQ$ and the trapezium BPQC.

In $\triangle APQ$ and $\triangle ABC$

$\angle APQ = \angle B$ (corresponding angles)

$\angle PAQ = \angle BAC$ (common)

So, $\triangle APQ \sim \triangle ABC$ (AA Similarity)

We know that the ratio of areas of the twosimilar triangles is equal to the ratio of the squares of their corresponding sides.

$$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{AP}{AB}\right)^2$$

$$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \frac{1x^2}{(1x+2x)^2}$$

$$\frac{Ar\triangle APQ}{Ar\triangle ABC} = \frac{1}{9}$$

Let Area of $\triangle APQ$ =1 sq. units and Area of $\triangle ABC$ =9x sq.units

Ar[trapBCED]=Ar($\triangle ABC$) – Ar($\triangle APQ$)

=9x-1x

=8x sq units

Now,

$$\frac{Ar\triangle APQ}{Ar(trapBCED)} = \frac{xsqunits}{8xsqunits} = \frac{1}{8}$$