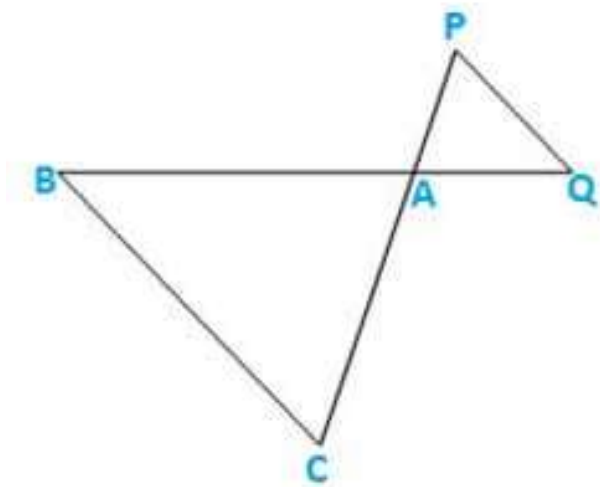


RD SHARMA
Solutions
Class 10 Maths
Chapter 4
Ex4.5

Q1: In fig. given below $\triangle ACB \sim \triangle APQ$. If $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm, and $AP = 2.8$ cm find CA and AQ .



Sol: Given,

$$\triangle ACB \sim \triangle APQ$$

$BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm, and $AP = 2.8$ cm

We need to find CA and AQ

$$\text{Since, } \triangle ACB \sim \triangle APQ$$

$$\frac{BA}{AQ} = \frac{CA}{AP} = \frac{BC}{PQ}$$

$$\text{Therefore, } 6.5AQ = 8 \times \frac{6.5}{4} = \frac{8}{4}$$

$$AQ = 6.5 \times \frac{8}{8}$$

$$AQ = 3.25 \text{ cm}$$

$$\text{Similarly, } \frac{CA}{AP} = \frac{BC}{PQ}$$

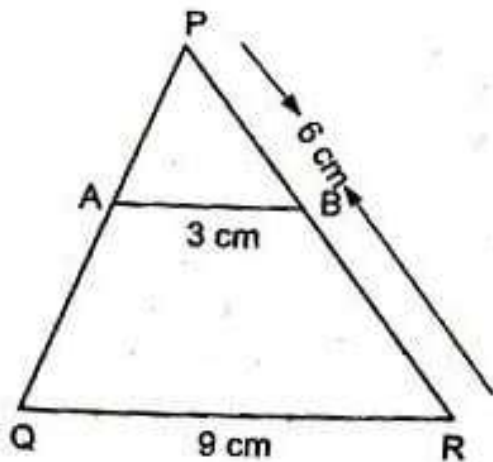
$$CA \times 2.8 = 8 \times \frac{CA}{2.8} = \frac{8}{4}$$

$$CA = 2.8 \times 2$$

$$CA = 5.6 \text{ cm}$$

Therefore, **$CA = 5.6$ cm and $AQ = 3.25$ cm.**

Q2: In fig. given, $AB \parallel QR$, find the length of PB.



Sol: Given,

$AB \parallel QR$

$AB = 3 \text{ cm}$, $QR = 9 \text{ cm}$ and $PR = 6 \text{ cm}$

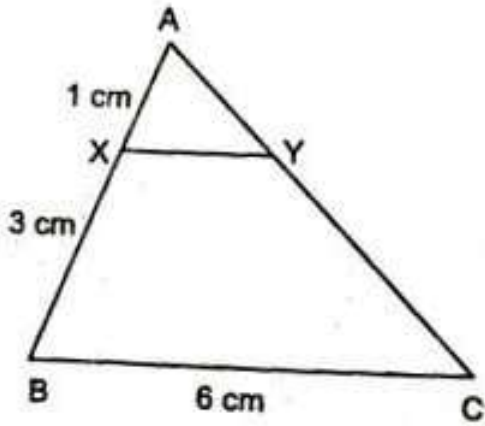
We need to find out PB,

Since, $AB \parallel QR$ $\Rightarrow \frac{AB}{QR} = \frac{PB}{PR}$

$$\text{i.e., } 3 = 9 \frac{PB}{6}$$

PB = 2 cm

Q3.) In fig. given, $XY \parallel BC$. Find the length of XY.



Sol: Given,

$XY \parallel BC$

$AX = 1 \text{ cm}$, $XB = 3 \text{ cm}$, and $BC = 6 \text{ cm}$

We need to find XY ,

Since, $\triangle AXY \sim \triangle ABC$

$$\frac{XY}{BC} = \frac{AX}{AB} \quad (AB = AX + XB = 4)$$

$$XY \cdot 6 = 14 \frac{XY}{6} = \frac{1}{4} \quad XY \cdot 1 = 64 \frac{XY}{1} = \frac{6}{4}$$

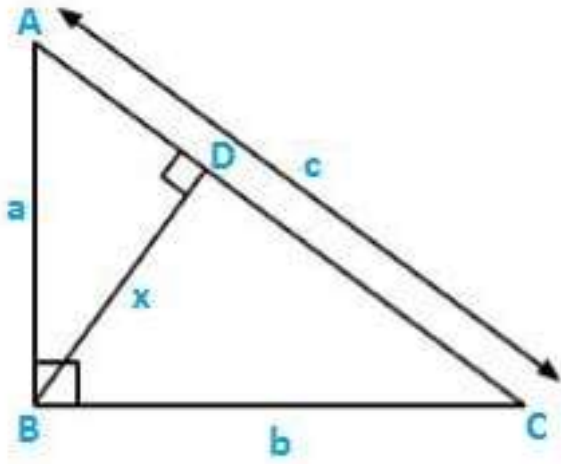
$XY = 1.5 \text{ cm}$

Q4: In a right-angled triangle with sides a and b and hypotenuse c , the altitude drawn on the hypotenuse is x . Prove that $ab = cx$.

Sol:

Let the $\triangle ABC$ be a right angle triangle having sides a and b and hypotenuse c . BD is the altitude drawn on the hypotenuse AC

We need to prove $ab = cx$



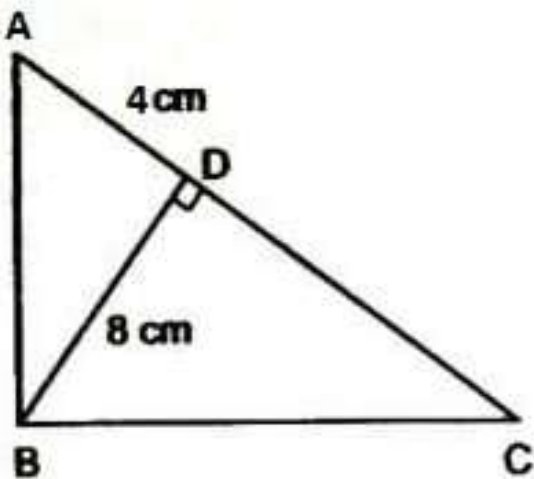
Since, the altitude is perpendicular on the hypotenuse, both the triangles are similar

$$\triangle ABD \sim \triangle BCD \Rightarrow \frac{AB}{BD} = \frac{BC}{CD} \Rightarrow ax = cb \Rightarrow \frac{a}{c} = \frac{b}{x}$$

$$xc = ab$$

$$\therefore ab = cx$$

Q5) In fig. given, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm, and $AD = 4$ cm, find CD .



Sol:

Given,

$\angle ABC = 90^\circ$ and $BD \perp AC$

When, $BD = 8$ cm, $AD = 4$ cm, we need to find CD .

Since, $\triangle ABC$ is a right angled triangle and $BD \perp AC$.

So, $\triangle DBA \sim \triangle DCB$ (A-A similarity)

$$\frac{BD}{CD} = \frac{AD}{BD}$$

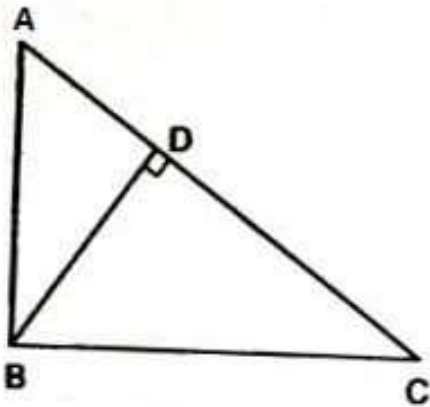
$$BD^2 = AD \times DC$$

$$(8)^2 = 4 \times DC$$

$$DC = 64/4 = 16 \text{ cm}$$

$$\therefore CD = 16 \text{ cm}$$

Q6) In fig. given, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $AC = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, Find BC .



Sol:

Given: $BD \perp AC$. $AC = 5.7$ cm, $BD = 3.8$ cm and $CD = 5.4$ cm, and $\angle ABC = 90^\circ$.

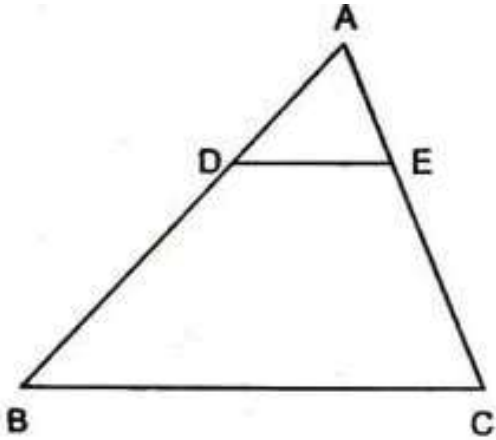
We need to find BC ,

Since, $\triangle ABC \sim \triangle BDC$

$$\frac{AB}{BD} = \frac{BC}{CD} \quad 5.7/3.8 = BC/5.4 \quad \frac{5.7}{3.8} = \frac{BC}{5.4} \quad BC = 5.7 \times 5.4 / 3.8 = \frac{5.7 \times 5.4}{3.8}$$

$$BC = 8.1 \text{ cm}$$

Q7) In the fig. given, $DE \parallel BC$ such that $AE = \frac{1}{4}AC$. If $AB = 6$ cm, find AD .



Sol:

Given, $DE \parallel BC$ and $AE = \frac{1}{4}AC$ and $AB = 6$ cm.

We need to find AD .

Since, $\triangle ADE \sim \triangle ABC$

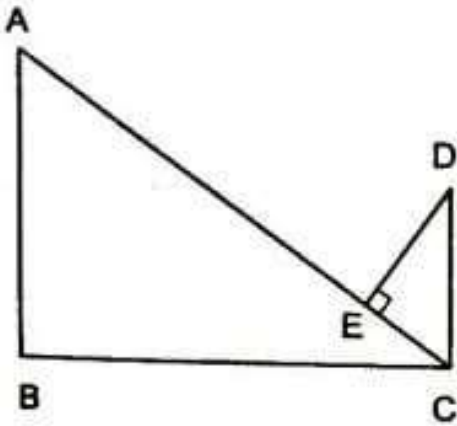
$$\frac{AD}{AB} = \frac{AE}{AC} \quad AD \cdot 6 = 14 \cdot \frac{AD}{6} = \frac{1}{4}$$

$$4 \times AD = 6$$

$$AD = \frac{6}{4}$$

$$AD = 1.5 \text{ cm}$$

Q.8) In the fig. given, if $AB \perp BC$, $DC \perp BC$, and $DE \perp AC$, prove that $\triangle CED \sim \triangle ABC$



Sol:

Given, $AB \perp BC$, $DC \perp BC$, and $DE \perp AC$

We need to prove that $\triangle CED \sim \triangle ABC$

Now,

In $\triangle ABC$ and $\triangle CED$

$$\angle B = \angle E = 90^\circ \text{ (given)}$$

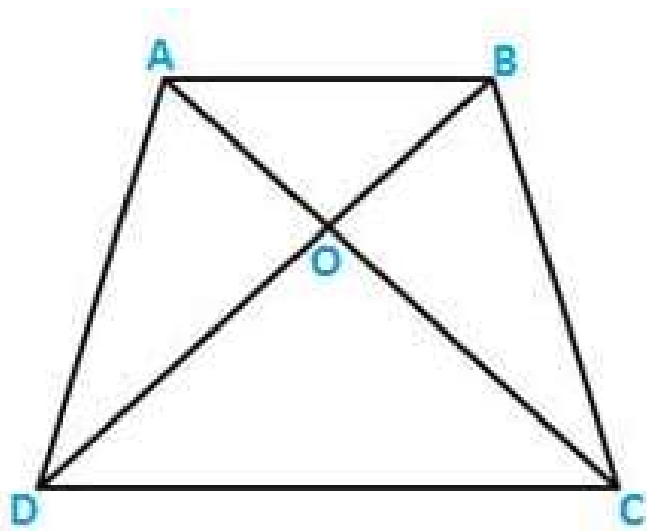
$$\angle A = \angle ECD \text{ (alternate angles)}$$

So, $\triangle CED \sim \triangle ABC$ (A-A similarity)

Q.9) Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. Using similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$

Sol: Given trapezium ABCD with $AB \parallel DC$. O is the point of intersection of AC and BD.

We need to prove $\frac{OA}{OC} = \frac{OB}{OD}$



Now, in ΔAOB and ΔCOD

$$\angle AOB = \angle COD \quad (\text{VOA})$$

$$\angle OAB = \angle OCD \quad (\text{alternate angles})$$

Therefore, $\Delta AOB \sim \Delta COD$

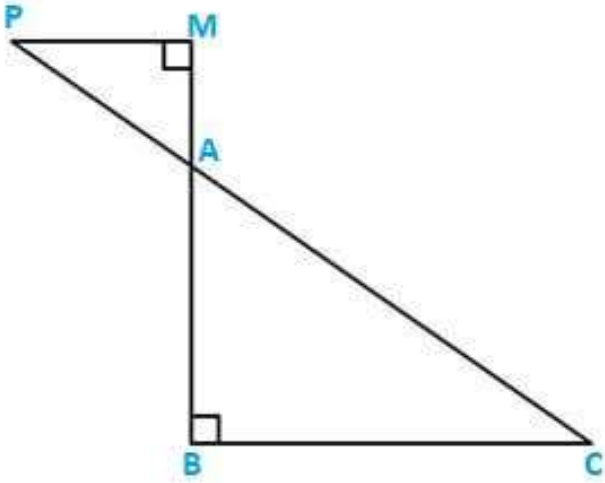
$$\text{Therefore, } \frac{OA}{OC} = \frac{OB}{OD} \quad (\text{corresponding sides are proportional})$$

Q.10) If ΔABC and ΔAMP are two right angled triangles, at angle B and M, respectively. Such that $\angle MAP = \angle BAC$. Prove that :

(i) $\Delta ABC \sim \Delta AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Sol:



(i) Given ΔABC and ΔAMP are the two right angled triangle.

$$\angle MAP = \angle BAC \quad (\text{given})$$

$$\angle AMP = \angle B = 90$$

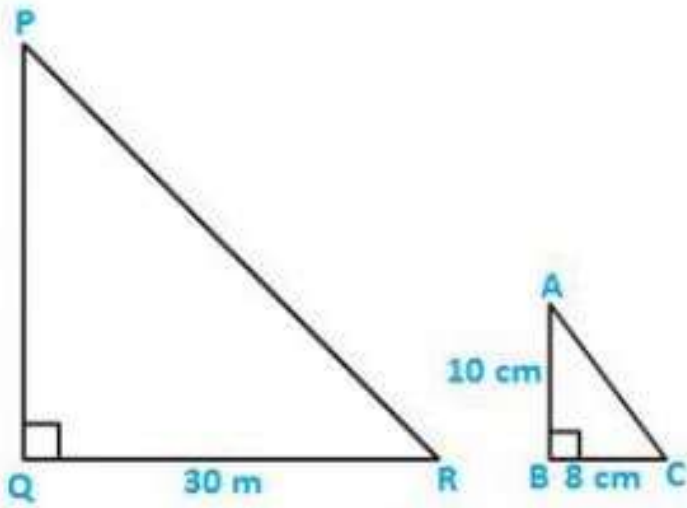
$$\Delta ABC \sim \Delta AMP \quad (\text{A-A similarity})$$

(ii) $\Delta ABC \sim \Delta AMP$

$$\text{So, } \frac{CA}{PA} = \frac{BC}{MP} \quad (\text{corresponding sides are proportional})$$

Q.11) A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.

Soln.: We need to find the height of PQ.



Now, $\Delta ABC \sim \Delta PQR$ (A-A similarity)

$$\frac{AB}{BC} = \frac{PQ}{QR} \quad 108 = PQ3000 \frac{10}{8} = \frac{PQ}{3000}$$

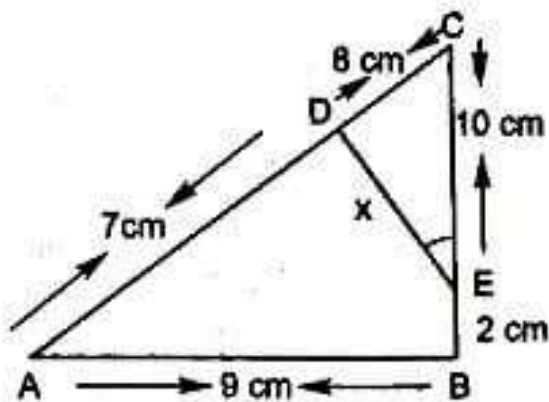
$$PQ = 3000 \times 108 \frac{3000 \times 10}{8}$$

$$PQ = 300008 \frac{30000}{8}$$

$$PQ = 3750100 \frac{3750}{100}$$

$$PQ = 37.5 \text{ m}$$

Q.12) in fig. given, $\angle A = \angle CED$, prove that $\Delta CAB \sim \Delta CED$. Also find the value of x.



Sol:

Comparing $\triangle CAB$ and $\triangle CED$

$\frac{CA}{CE} = \frac{AB}{ED}$ (similar triangles have corresponding sides in the same proportions)

$$15 = 9 \times \frac{15}{10} = \frac{9}{x} \times 10 \Rightarrow 15x = 9 \times 10 \Rightarrow x = \frac{9 \times 10}{15}$$

$$x = 6 \text{ cm}$$

Q13) The perimeters of two similar triangles are 25 cm and 15 cm, respectively. If one side of the first triangle is 9 cm, what is the corresponding side of the other triangle?

Sol:

Given perimeters of two similar triangles are 25 cm, 15 cm and one side 9 cm

We need to find the other side.

Let the corresponding side of other triangle be x cm

Since ratio of perimeter = ratio of corresponding side

$$\frac{25}{15} = \frac{9}{x}$$

$$25x = 9 \times 15$$

$$x = \frac{135}{25}$$

$$x = 5.4 \text{ cm}$$

Q14) In $\triangle ABC$ and $\triangle DEF$, it is being given that $AB = 5$ cm, $BC = 4$ cm, $CA = 4.2$ cm, $DE = 10$ cm, $EF = 8$ cm, and $FD = 8.4$ cm. If $AL \perp BC$, $DM \perp EF$, find $AL : DM$.

Sol:

Given $AB = 5$ cm, $BC = 4$ cm, $CA = 4.2$ cm, $DE = 10$ cm, $EF = 8$ cm, and $FD = 8.4$ cm

We need to find $AL : DM$

Since, both triangles are similar,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$$

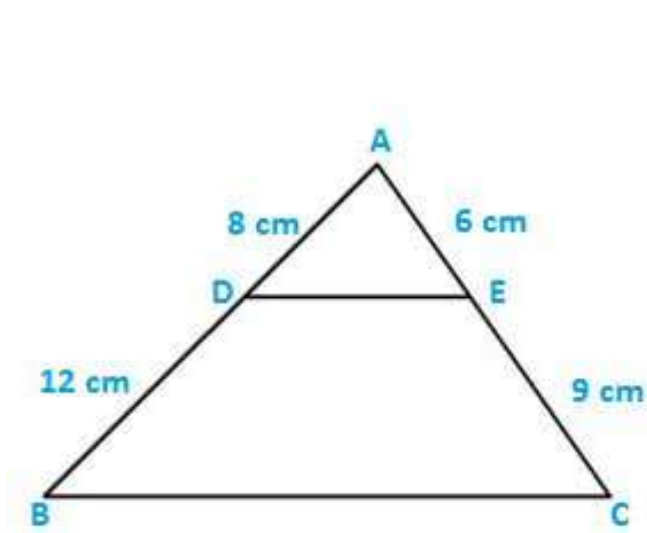
Here, we use the result that in similar triangle the ratio of corresponding altitude is same as the ratio of the corresponding sides.

Therefore, $AL : DM = 1 : 2$

Q.15) D and E are the points on the sides AB and AC respectively, of a $\triangle ABC$ such that AD = 8 cm, DB = 12 cm, AE = 6 cm, and CE = 9 cm. Prove that $BC = 5/2 DE$.

Sol: Given AD = 8 cm, AE = 6 cm, and CE = 9 cm

We need to prove that,



$$\text{Since, } \frac{AD}{AB} = \frac{AE}{AC} = \frac{8}{20} = \frac{2}{5}$$

Also, $\triangle ADE \sim \triangle ABC$ (SAS similarity)

$$\frac{BC}{DE} = \frac{AB}{AD}$$

$$\frac{BC}{DE} = \frac{1}{\left(\frac{AD}{AB}\right)} \quad \left(\frac{AD}{AB} = \frac{2}{5}\right)$$

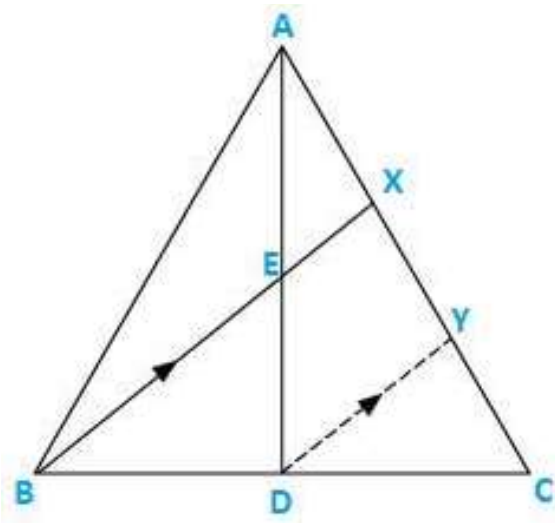
$$\frac{BC}{DE} = \frac{5}{2}$$

BC = 5/2 DE

Q.16) D is the midpoint of side BC of a $\triangle ABC$. AD is bisected at the point E and BE produced cuts AC at the point X. Prove that BE: EX = 3 : 1

Soln.: ABC is a triangle in which D is the midpoint of BC, E is the midpoint of AD. BE produced meets AC at X.

We need to prove BE: EX = 3 : 1



In ΔBCX and ΔDCY

$\angle CBX = \angle CDY$ (corresponding angles)

$\angle CXB = \angle CYD$ (corresponding angles)

$\Delta BCX \sim \Delta DCY$ (angle-angle similarity)

We know that corresponding sides of similar triangles are proportional

$$\text{Thus, } \frac{BC}{DC} = \frac{BX}{DY} = \frac{CX}{CY}$$

$$\frac{BX}{DY} = \frac{BC}{DC}$$

$$\frac{BX}{DY} = \frac{2DC}{DC} \quad (\text{As D is the midpoint of BC})$$

$$\frac{BX}{DY} = \frac{2}{1} \dots (i)$$

In ΔAEX and ΔADY ,

$\angle AEX = \angle ADY$ (corresponding angles)

$\angle AXE = \angle AYD$ (corresponding angles)

$\Delta AEX \sim \Delta ADY$ (angle-angle similarity)

We know that corresponding sides of similar sides of similar triangles are proportional

$$\text{Thus, } \frac{AE}{AD} = \frac{EX}{DY} = \frac{AX}{AY}$$

$$\frac{EX}{DY} = \frac{AE}{AD}$$

$$\frac{EX}{DY} = \frac{AE}{2AE} \quad (\text{As D is the midpoint of BC})$$

$$\frac{EX}{DY} = \frac{1}{2} \dots (\text{ii})$$

Dividing eqn. (i) by eqn. (ii)

$$\frac{BX}{EX} = \frac{4}{1}$$

$$BX = 4EX$$

$$BE + EX = 4EX$$

$$BE = 3EX$$

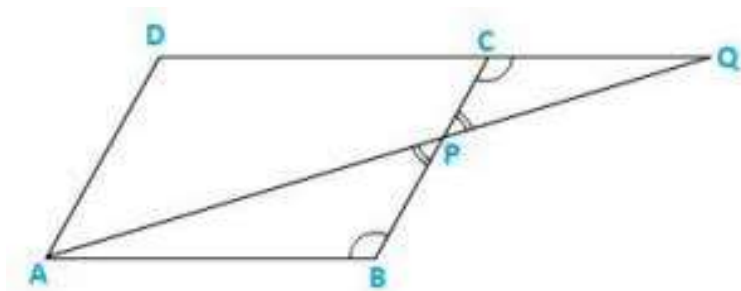
$$\mathbf{BE : EX = 3:1}$$

Q.17) ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q. Prove that the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC.

Sol:

ABCD is a parallelogram and APQ is a straight line meeting BC at P and DC produced at Q.

We need to prove, the rectangle obtained by BP and DQ is equal to the rectangle contained by AB and BC. We need to prove that $BP \times DQ = AB \times BC$



In ΔABP and ΔQCP ,

$$\angle ABP = \angle QCP \quad (\text{alternate angles as } AB \parallel DC)$$

$$\angle BPA = \angle QPC \quad (\text{VOA})$$

$$\triangle ABP \sim \triangle QCP \quad \triangle ABP \sim \triangle QCP \quad \triangle \triangle \quad (\text{AA similarity})$$

We know that corresponding sides of similar triangles are proportional

$$\text{Thus, } \frac{AE}{AD} = \frac{EX}{DY} = \frac{AX}{AY}$$

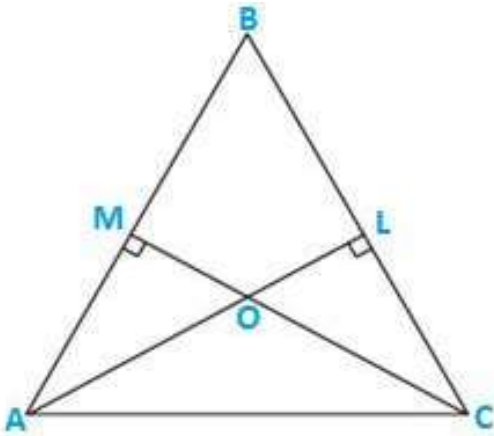
$$\frac{EX}{DY} = \frac{AE}{AD}$$

Q.18) In $\triangle ABC$, AL and CM are the perpendiculars from the vertices A and C to BC and AB respectively. If AL and CM intersect at O , prove that:

(i) $\triangle OMA \sim \triangle OLC$

(ii) $\frac{OA}{OC} = \frac{OM}{OL}$

Sol:



(i) in $\triangle OMA$ and $\triangle OLC$,

$$\angle AOM = \angle COL \quad (\text{VOA})$$

$$\angle OMA = \angle OLC \quad (90^\circ \text{ each})$$

$$\triangle OMA \sim \triangle OLC \quad (\text{A-A similarity})$$

(ii) Since, $\triangle OMA \sim \triangle OLC$ by A-A similarity, then

$$OM \cdot OC = OA \cdot OL \implies \frac{OM}{OL} = \frac{OA}{OC} = \frac{MA}{LC} \quad (\text{corresponding sides of similar triangles are proportional})$$

$$OA \cdot OC = OM \cdot OL \implies \frac{OA}{OL} = \frac{OM}{OC}$$

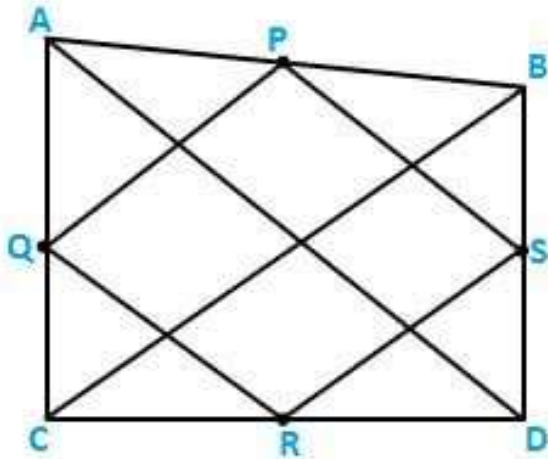
Q.19) ABCD is a quadrilateral in which AD = BC. If P, Q, R, S be the midpoints of AB, AC, CD and BD respect. Show that PQRS is a rhombus.

Soln.:

Given, ABCD is a quadrilateral in which AD = BC and P, Q, R, S are the mid points of AB, AC, CD, BD, respectively.

To prove,

PQRS is a rhombus



Proof,

In $\triangle ABC$, P and Q are the mid points of the sides AB and AC respectively

By the midpoint theorem, we get,

$$PQ \parallel BC \text{ and } PQ = \frac{1}{2} BC.$$

In $\triangle ADC$, Q and R are the mid points of the sides AC and DC respectively

By the midpoint theorem, we get,

$$QR \parallel AD \text{ and } QR = \frac{1}{2} AD = \frac{1}{2} BC \quad (AD = BC)$$

In $\triangle BCD$,

By the mid point theorem, we get,

$$RS \parallel BC \text{ and } RS = \frac{1}{2} AD = \frac{1}{2} BC \quad (AD = BC)$$

From above eqns.

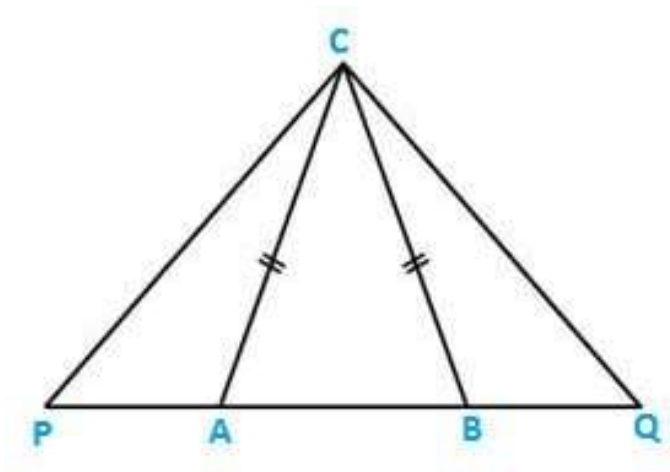
$$PQ = QR = RS$$

Thus, PQRS is a rhombus.

Q.20) In an isosceles $\triangle ABC$, the base AB is produced both ways to P and Q such that $AP \times BQ = AC^2$. Prove that $\triangle APC \sim \triangle BCQ$.

Sol: Given $\triangle ABC$ is isosceles and $AP \times BQ = AC^2$

We need to prove that $\triangle APC \sim \triangle BCQ$.



Given $\triangle ABC$ is an isosceles triangle $AC = BC$.

Now, $AP \times BQ = AC^2$ (given)

$$AP \times BQ = AC \times AC$$

$$AP \times BQ = AC \times AC \implies \frac{AP}{AC} = \frac{AC}{BQ} \implies \frac{AP}{AC} = \frac{BC}{BQ}$$

Also, $\angle CAB = \angle CBA$ (equal sides have angles opposite to them)

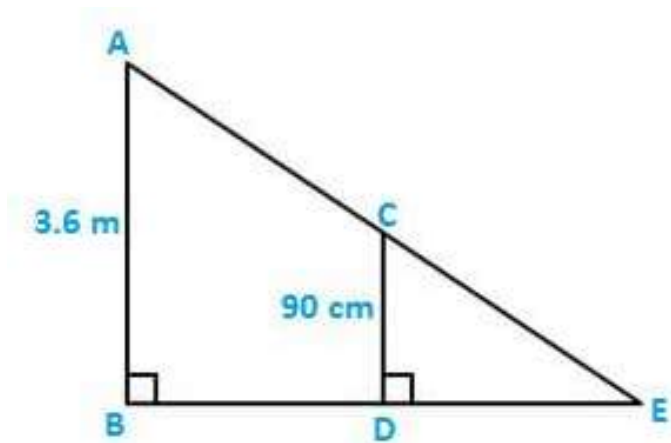
$$180 - \angle CAP = 180 - \angle CBQ$$

$$\angle CAP = \angle CBQ$$

Hence, $\triangle APC \sim \triangle BCQ$ $\triangle APC \sim \triangle BCQ$ (SAS similarity)

Q.21) A girl of height 90 cm is walking away from the base of a lamp post at a speed of 1.2 m/sec. If the lamp is 3.6m above the ground, find the length of her shadow after 4 seconds.

Soln.: Given, girl's height = 90 cm, speed = 1.2m/sec and height of lamp = 3.6 m



We need to find the length of her shadow after 4 sec.

Let, AB be the lamp post and CD be the girl

Suppose DE is the length of her shadow

Let, DE = x

and BD = 1.2 x 4

BD = 4.8 m

Now, in $\triangle ABE$ and $\triangle CDE$ we have,

$\angle B = \angle D$

$\angle E = \angle E$

So, by A-A similarity criterion,

$$\triangle ABE \sim \triangle CDE \quad \frac{BE}{DE} = \frac{AB}{CD}$$

$$4.8 + x = 3.6 \cdot \frac{4.8 + x}{x} = \frac{3.6}{0.9} = 4$$

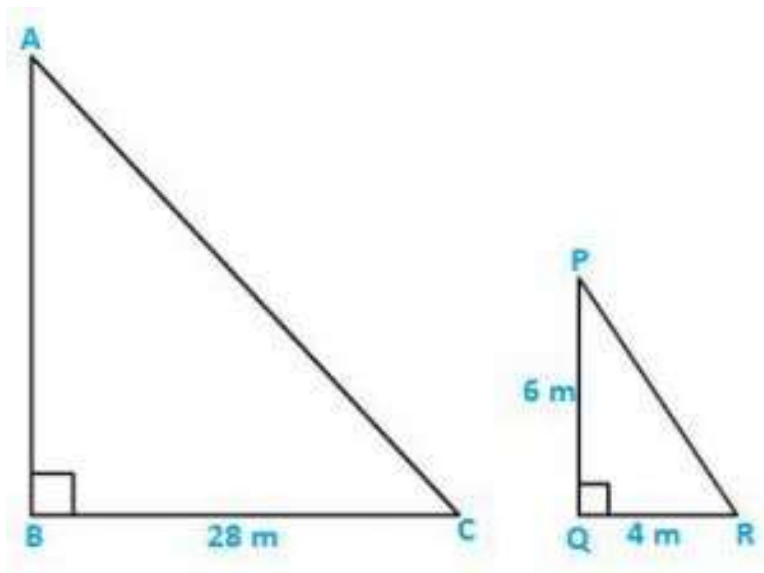
$$3x = 4.8$$

$$x = 1.6$$

hence, the length of her shadow after 4 sec. Is 1.6 m

Q.22) A vertical stick of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28m long. Find the height of the tower.

Soln.: Given length of vertical stick = 6m



We need to find the height of the tower

Suppose AB is the height of the tower and BC is its shadow.

Now, $\triangle ABC \sim \triangle PQR$ $\triangle ABC \sim \triangle PQR$ (B = Q and A = P)

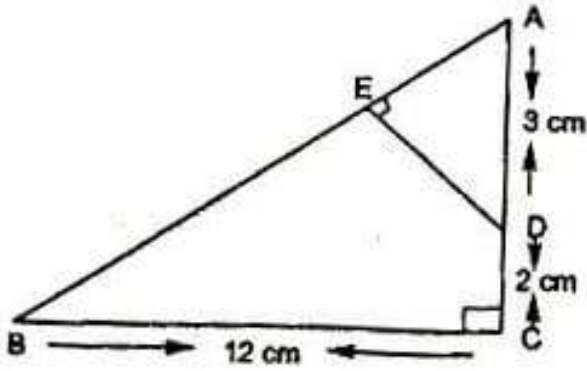
$$\frac{AB}{BC} = \frac{PQ}{QR} \quad AB \cdot 28 = 64 \cdot \frac{AB}{28} = \frac{6}{4}$$

$$AB = (28 \times 6)/4$$

$$AB = 42\text{m}$$

Hence, the height of tower is 42m.

Q.23) In the fig. given, $\triangle ABC$ is a right angled triangle at C and $DE \perp AB$. Prove that $\triangle ABC \sim \triangle ADE$.



Sol:

Given ΔACB is right angled triangle and $C = 90$

We need to prove that $\Delta ABC \sim \Delta ADE$ and find the length of AE and DE.

$\Delta ABC \sim \Delta ADE$

$\angle A = \angle A$ (common angle)

$\angle C = \angle E$ (90)

So, by A-A similarity criterion, we have

In $\Delta ABC \sim \Delta ADE$

$$\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE} \quad 13 = 12DE = 5AE \quad \frac{13}{3} = \frac{12}{DE} = \frac{5}{AE}$$

Since, $AB^2 = AC^2 + BC^2$

$$= 5^2 + 12^2$$

$$= 13^2$$

$$\therefore DE = 36/13 \text{ cm}$$

$$\text{and } AE = 15/13 \text{ cm}$$

Q.25) In fig. given, we have $AB \parallel CD \parallel EF$. If $AB = 6 \text{ cm}$, $CD = x \text{ cm}$, $EF = 10 \text{ cm}$, $BD = 4 \text{ cm}$, and $DE = y \text{ cm}$. Calculate the values of x and y .

Sol: Given AB || CD || EF.

AB = 6 cm, CD = x cm, and EF = 10 cm.

We need to calculate the values of x and y

In $\triangle ADB$ and $\triangle DEF$,

$\angle ADB = \angle EDF$ (VOA)

$\angle ABD = \angle DEF$ (alt. Interior angles)

$$\frac{EF}{AB} = \frac{OE}{OB} \quad 10 = y \cdot \frac{10}{6} = \frac{y}{4}$$

$$y = 40/6$$

$$y = 6.67 \text{ cm}$$

Similarly, in $\triangle ABE$, we have

$$\frac{OC}{AB} = \frac{OE}{OB} \quad 4.7 = x \cdot \frac{4}{6.7} = \frac{x}{6}$$

$$6.7 \times x = 6 \times 4$$

$$x = 24/6.7$$

$$x = 3.75 \text{ cm}$$

Therefore, x = 3.75 cm and y = 6.67 cm