EXERCISE 2D

Answer 1
g(x)= x- 2
⇒ x = 2
Then, p(x) = $x^3 - 8 = 2^3 - 8 = 0$ (given x=2)
Yes, g(x) is factor of p(x)

Answer 2
g(x)= x – 3
⇒ x = 3
Then, p(x) = $2x^3 + 7x^2 - 24x - 45 = 2(3^3) + 7(3^2) - 24 \times 3 - 45$
⇒ 54 + 63 – 72 – 45 = 0
Yes, g(x) is factor of p(x)

Answer 3
g(x)= x -1
⇒ x = 1
Then, p(x) = $2x^4 + 9x^3 + 6x^2 - 11x - 6 = 2 \times 1^4 + 9 \times 1^3 + 6 \times 1^2 - 11 \times 1 - 6$
= 2 +9+6 -11 – 6
= 0
Yes, g(x) is factor of p(x)

Answer 4
g(x)= x +2
⇒ x = -2
Then, p(x) = $x^4 - x^2 - 12 = (-2)^4 - (-2)^2 - 12$
= 16 – 4 -12 = 0
Yes, g(x) is factor of p(x)

Answer 5
g(x) = x +3
⇒ x = -3
Then, p(x) = $69 + 11x - x^2 + x^3 = 69 + 11(-3) - (-3)^2 + (-3)^3$
⇒ 69 - 33 – 9 – 27 = 0
Yes, g(x) is factor of p(x)

Answer 6
g(x) = x+5
⇒ x = -5
Then, p(x) = $2x^3 + 9x^2 - 11x - 30 = 2(-5)^3 + 9(-5)^2 - 11(-5) - 30$
= -250 + 45 + 55 -30
= -180
Yes, g(x) is not factor of p(x)

Answer7
g(x) = 2x-3

$\Rightarrow x = \frac{3}{2}$

Then, p(x) $= 2x^4 + x^3 - 8x^2 - x + 6 = 2(\frac{3}{2})^4 + (\frac{3}{2})^3 - 8(\frac{3}{2})^2 - \frac{3}{2} + 6$

$$= \left(2\ \frac{81}{16}\right) + \frac{27}{8} - (8\ \frac{9}{4}) - \frac{3}{2} + 6$$

$$= \frac{81}{8} + \frac{27}{8} - \frac{144}{8} - \frac{12}{8} + \frac{48}{8}$$

$$= 0$$

Yes, g(x) is factor of p(x)

Answer 8

g(x)= 3x-2

$\Rightarrow$ x = $\frac{2}{3}$

Then, p(x) $= 3x^3 + x^2 - 20x + 12 = 3(\frac{2}{3})^3 + (\frac{2}{3})^2 - 20\frac{2}{3} + 12$

$$= \left(3\frac{8}{27}\right) + \frac{4}{9} - (20\frac{2}{3}) + 12$$

$$= \frac{8}{9} + \frac{4}{9} - \frac{120}{9} + \frac{108}{9}$$

$$= 0$$

Yes, g(x) is factor of p(x)

Answer 9

g (x) = $x = \sqrt{2}$

$\Rightarrow$ x = $\sqrt{2}$

Then, p(x) $= 7x^2 - 4\sqrt{2}x - 6 = 7(\sqrt{2})^2 - 4(\sqrt{2} \times \sqrt{2}) - 6$

$\Rightarrow$ (7 x 2)–(4 x 2) – 6 = 14 – 8 – 6 = 0

Answer 10

g(x) = $x + \sqrt{2}$

$\Rightarrow x = -\sqrt{2}$

Then , p(x) = $2\sqrt{2}x^2 + 5x + \sqrt{2} = 2\sqrt{2}(-\sqrt{2})^2 + 5 \times (-\sqrt{2}) + \sqrt{2}$

$$= 4\sqrt{2} - 5\sqrt{2} + \sqrt{2}$$

$$= 0$$

Answer11

Let g(p) = $(p^{10} - 1)$ and

And h(p) = $(p^{11} - 1)$

Let f(p) = (p-1)
then, ⇒ p – 1 = 0
⇒ p = 1

Now, g(1) = $[(p^{10} - 1)] = (1^{10} - 1)$
= (1-1) = 0
Hence, f(p-1) is factor of g(p)
h(p) = $(p^{11} - 1)$
h(1) = $[(p^{11} - 1)] = (1^{11} - 1) = (1 - 1) = 0$
hence, f(p-1) is also factor of h(p)

Answer12
Here f(x) = x-1
⇒ x = 1
Now, given p(x) = $2x^3 + 9x^2 + x + k$
⇒ $= 2(1)^3 + 9(1)^2 + 1 + k$
$k = -2 - 9 - 1 = -12$

Answer13
Here f(x) = x – 4
⇒ x = 4
Now, given p(x) = $2x^3 - 3x^2 - 18x + a$
⇒ $= 2(4^3) - 3(4^2) - 18(4) + a$
$= 2 \times 64 - 48 - 72 + a$
a = - 8

Answer14
Here, f(x) = x + 1
⇒ x = -1
Now, given p(x) = $ax^3 + x^2 - 2x + 4a - 9$
$= a(-1)^3 + (-1)^2 - 2(-1) + 4a - 9$
$a - 4a = 1 + 2 - 9$
$\Rightarrow 3a = 6$
⇒a = 2

Answer15
Here, f(x) = x+ 2a
⇒ x = -2a
Now, given p(x) = $x^5 - 4a^2x^3 + 2x + 2a + 3$
$= (-2a)^5 - 4a^2(-2a)^3 + 2(-2a) + 2a + 3$
$= -32a^5 + 32a^5 - 4a + 2a + 3$
$2a = 3$
$a = \frac{3}{2}$

Answer16
Here, f(x) = 2x-1

$\Rightarrow \qquad x = \frac{1}{2}$

Now, given p(x) $= 8x^4 + 4x^3 - 16x^2 + 10x + m$

$$= 8(\tfrac{1}{2})^4 + 4\left(\tfrac{1}{2}\right)^3 - 16(\tfrac{1}{2})^2 + 10\left(\tfrac{1}{2}\right) + m$$

$$-m = \left(8 \times \tfrac{1}{16}\right) + \left(4 \times \tfrac{1}{8}\right) - \left(16 \times \tfrac{1}{4}\right) + \left(10 \times \tfrac{1}{2}\right)$$

$$-m \Rightarrow \tfrac{1}{2} + \tfrac{1}{2} - 4 + 5 = \tfrac{4}{2}$$

m= -2

Answer17

Here, f(x) = x+3

$\Rightarrow$ x= -3

Now, given p(x) $= x^4 - x^3 - 11x^2 - x + a$

$$-a = (-3)^4 - (-3)^3 - 11(-3)^2 - (-3)$$

$$-a \Rightarrow 81 + 27 - 99 + 3 = 12$$

a = -12

Answer18

Here, f(x)= $x^2 + 2x - 3$

$= (x^2 + 3x - x - 3) = (x + 3)(x - 1)$

Now, p(x) will be divisible by f(x) only when it is divisible by (x-1) as well as by (x+3)

Now, (x – 1 = 0 $\Rightarrow$ x = 1) and (x+3 $\Rightarrow$ x = -3)

By the factor theorem, p(x) will be divisible by f(x) , if p(1) = 0 and p(-3) = 0

p(x)= $x^3 - 3x^2 - 13x + 15$

p(1) =$(1)^3 - 3(1)^2 - 13(1) + 15 = 1 - 3 - 13 + 15 = 0$

p(-3) = $(-3)^3 - 3(-3)^2 - 13(-3) + 15 = -27 - 27 + 39 + 15 = 0$

Answer19

p(x) = $x^3 + ax^2 + bx + 6$, g(x) = x -2 and h(x) = x -3 , then,

g(x) = 0 $\Rightarrow$ x -2 = 0 $\Rightarrow$ x = 2

h(x) = 0 $\Rightarrow$ x – 3 = 0 $\Rightarrow$ x = 3

(x-2) is factor of p(x) $\Rightarrow$ p(2) = 0

Now, p(2) = 0 $\Rightarrow [(2)^3 + a(2)^2 + b(2) + 6] = 8 + 4a + 2b + 6$

$\Rightarrow 4a + 2b = -14$……..(1)

Since, it is given that factor (x-3) leaves the remainder 3

Now, p(3) = 3 $\Rightarrow [(3)^3 + a(3)^2 + b(3) + 6] = 27 + 9a + 3b + 6$

$\Rightarrow$ 9a + 3b = - 30…….(2)

Solving both equation,

4a+2b = -14…..(divide each term by 2)

9a +3b = -30…(divide each term by 3)

We get, 2a + b = - 7…..(3)

3a + b = - 10……(4)

On solving (3) and (4) we get, a = -3 and b=-1

Answer20

Here, f(x)= x-1 and x – 2
⇒ x = 1 and x= 2
Now, p(x) will be divisible by both (x-1) and (x-2)
p(x) $= x^3 - 10x^2 + ax + b$
p(1)= $(1)^3 - 10(1)^2 + a(1) + b$
= (1) – 10 +a+b
a +b=9 ……………(1)

$$p(2) \;=\; (2)^3 - 10(2)^2 + a(2) + b$$
$$= \; 8 - 40 + 2a + b$$

2a +b = 32…………..(2)
Solving (1) and (2)

a+ b = 9
2a +b = 32

By subtracting , we get

⇒ a = 32 -9 = 23

And another equation is ⇒ b = 9-23 = -14

Answer 21
p(x)= $(x^4 + ax^3 - 7x^2 - 8x + b)$
g(x)=(x+2)
h(x)=(x+3)
since g(x) and h(x) both are exactly divisible, these are the factor of p(x)
so,
p(x)=p(-2)=$(-2^4) + a(-2^3) - 7(-2^2) - 8(-2) + b$=0
p(x)=16+a(-8)-7(4)+16+b=0
p(x)=8a-b=4…..(1)
and now,

p(x)=p(-3)=$(-3^4) + \; a(-3^3) - 7(-3^2) - 8(-3) + b = 0$
p(x)=81-27a-63+24+b=0
p(x)=27a-b=42…..(2)
on solving equation (1) and (2) we get,

8a-b = 4
27a – b =42

On subtracting equation, we get

⇒ 19a = 38
⇒ a = $\frac{38}{19}$ = 2
b = 8a – 4 = 8(2) – 4 = 12
a=2 and b=12

Answer22
Let g(x) = (x-2)
⇒ x = 2
And h(x) = $\left(x - \frac{1}{2}\right)$
⇒ x = $\frac{1}{2}$

And p(x) = $px^2 + 5x + r$
Put the value p(2)= 0 ⇒ $p(2)^2 + 5(2) + r$
⇒ 4p + r = -10..........(1)
P($\frac{1}{2}$) = 0 ⇒ $p(\frac{1}{2})^2 + 5\left(\frac{1}{2}\right) + r$
⇒ $\frac{p}{4} + \frac{5}{2} + r$
⇒ $\frac{p}{4} + r = -\frac{5}{2}$
⇒ p+4r = -10...............(2)
Solving equation 1 and 2
4p +r = -10
p + 4r = - 10
hence , 4p +r = p+4r
⇒ 3p = 3r
⇒ p = r

Answer23
Here, f(x)= $x^2 - 3x + 2$
$= (x^2 - 2x - x + 2) = (x - 2)(x - 1)$
Now, p(x) will be divisible by f(x) only when it is divisible by (x-2) as well as by (x-1)
Now, (x – 2 = 0 ⇒ x = 2) and (x-1 ⇒ x = 1)
By the factor theorem, p(x) will be divisible by f(x) , if p(2) = 0 and p(1) = 0
p(x)= $2x^4 - 5x^3 - 2x^2 - x + 2$

p(2) =$2(2)^4 - 5(2)^3 + 2(2)^2 - (2) + 2 = 32 - 40 + 8 - 2 + 2 = 0$
p(-3) = $2(1)^4 - 5(1)^3 + 2(1)^2 - (1) + 2 = 2 - 5 + 2 - 1 + 2 = 0$

Answer24
Here, f(x)= x- 2
⇒ x = 2
And p(x) = $2x^4 - 5x^3 + 2x^2 - x - 3 = 2(2)^4 - 5(2)^3 + 2(2)^2 - 2 - 3$
⇒ 32 – 40+ 8-2-3 = 5
Hence, 5 be added to exactly divisible.

Answer25
When, the given polynomial is divided by a quadratic polynomial, then the remainder is a liner expression, say (ax+b)
Let, p(x)= $(x^4 + 2x^3 - 2x^2 + 4x + 6)$ –(ax-b) and f(x)= $x^2 + 2x - 3$
Then, p(x) = $(x^4 + 2x^3 - 2x^2 + (4 - a)x + (6 + b)$
f(x)= $x^2 + 2x - 3$
then, $= (x^2 + 3x - x - 3) = (x + 3)(x - 1)$
Now, p(x) will be divisible by f(x) only when it is divisible by (x-1) as well as by (x+3)
Now, (x – 1 = 0 ⇒ x = 1) and (x+3 ⇒ x = -3)
By the factor theorem, p(x) will be divisible by f(x) , if p(1) = 0 and p(-3) = 0
p(x)= $x^4 + 2x^3 - 2x^2 + (4 - a)x + (6 + b)$

p(1) =$(1)^4 + 2(1)^3 - 2(1)^2 + (4 - a)(1) + (6 + b) = 1 + 2 - 2 + 4 + 6 - a + b$
⇒ a +b = 11.....(1)

$p(-3) = (-3)^4 + 2(-3)^3 - 2(-3)^2 + (4-a)(-3) + (6+b) = 81 - 54 - 18 - 12 + 3a + 6 + b$

⇒ 3a - b = -3.....(2)

Solve 1 and 2 equations

a = 11-b

then, 3(11-b)-b = -3

⇒ -4b = -33-3 = -36

⇒ b =9

And a ⇒ 11-b = 11-9 = 2

Hence, the required expression is (2x- 9)

Answer26

Let f(x)= (x+a)

⇒ x= - a

Then, p(x) = $x^n + a^n$, where n is positive odd integer

Now, p(-a) = 0

p(-a) ⇒ $(-a)^n + a^n = [(-1)^n a^n + a^n] = [(-1)^n + 1]a^n$

⇒ $(-1 + 1)a^n = 0$.......[∵ n being odd, $(-1)^n = -1$]

Hence proved.