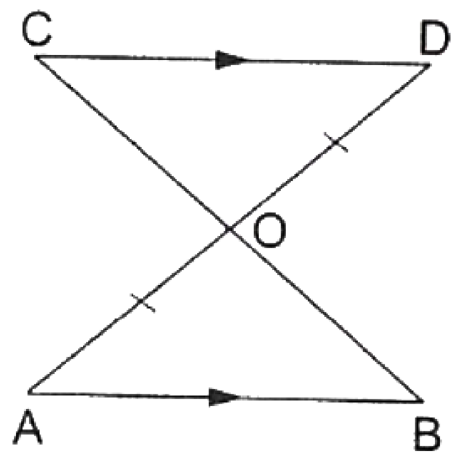
Congruence Of Triangles And Inequalities in a Triangle
CHAPTER 9

Exercise – 9 (A)

Answer1)



Given: AB || CD

To prove: i) O is the midpoint of AD

ii) ΔAOB ≅ ΔDOC

Proof:

In ΔAOB and ΔDOC

OA = OD (Given)

∠AOB = ∠ COD (vertically opposite angles)

∠OAB = ∠ODC (alternate angles)

Therefore;

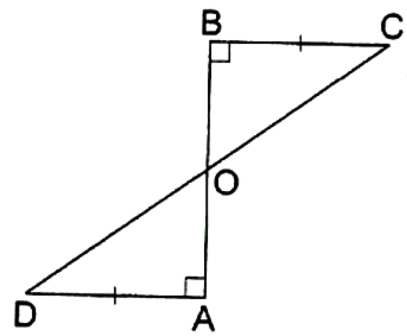
ΔAOB ≅ ΔDOC (A.A.S. criteria)

Hence;

OB = OC (c.p.c.t.)

Hence proved.

Answer2)



Given: AD=BD

To prove: CD bisects AB i.e. OA=OB

Proof:

In ΔBOC and ΔAOD

AD=BC (Given)

∠OAD=∠OBC =90 (Given)

∠AOD=∠BOC (vertically opp. Angles)

Therefore ΔBOC≅ΔAOD (A..A.S criteria)

Hence OA=OB i.e CD bisects AB

Hence proved.

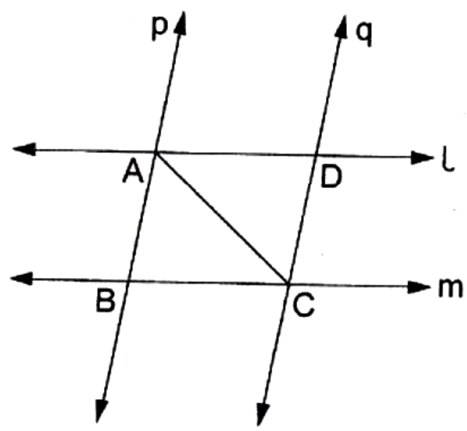
Answer3)

<u>Given</u>: (i) l || m

(ii) p || q

<u>To prove</u>: ΔABC≅ ΔCDA

<u>Proof</u>:



Taking l parallel to m AC is the trAnswerversal

∠ACB=∠CAD (Alternate angles)

Taking p parallel to q

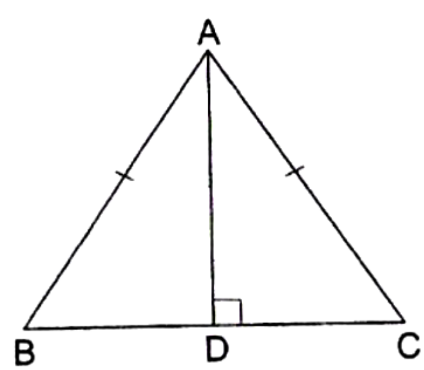
∠BAC=∠DCA (Alternate angles)

AC=AC (common)

Therefore;

ΔABC≅ ΔCDA (ASA ctiteria)

Hence Proved.



Answer4)<u>Given</u>: AB=AC

<u>To prove</u>: (i)AD bisects BC (i.e. BD=DC)

(ii) AD bisects ∠A

Prove:

In right angled ΔADB and ADC we have,

Hypotenuse AB=hypotenuse AC (Given)

AD=AD (common)

Therefore ΔADB≅ΔADC (RHS criteria)

Hence BD=DC

∠BAD =∠CAD

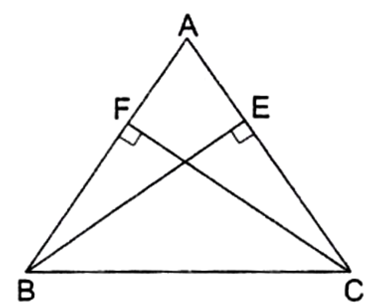
Hence AD bisects ∠A

Answer5)

Given: BE=CF

To Prove: i) ΔABE≅ΔACF

ii)AB=AC



Proof:

In ΔABE and ΔACF

∠AEB=∠AFC=90 (Given)

∠BAE=∠CAF (common)

BE=CF (Given)

Therefore ΔABE≅ΔACF (A.A.S Criteria)

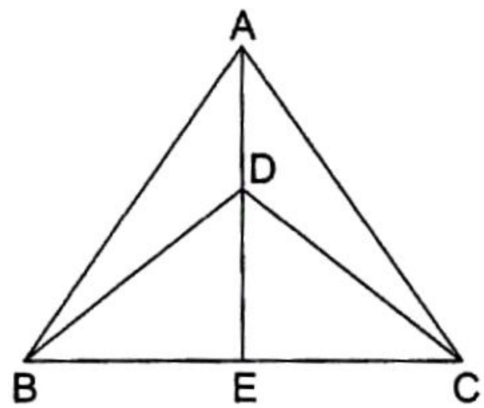
AB=AC (c.p.c.t)

Answer6)

<u>Given</u>:

(i) ΔABC and ΔDBC are two isosceles triangles in which AB=AC & BD=DC.

<u>To Prove</u>:



(i) ΔABD ≅ ΔACD

(ii) ΔABE ≅ ΔACP

(iii) AE bisects ∠A as well as ∠D.

(iv) AE is the perpendicular bisector of BC.

<u>Proof</u>:

(i) In ΔABD and ΔACD,

AD = AD (Common)

AB = AC (Given) .

BD = CD (Given)

Therefore, ΔABD ≅ ΔACD (SSS criteria)

∠BAD = ∠CAD(C.P.C.T)

∠BAE = ∠CAE

(ii) In ΔABE & ΔACE

AE= AE (Common)

∠BAE = ∠CAE

(Proved above)

AB = AC (Given)

Therefore,

$\Delta ABE \cong \Delta ACE$ (SAS criteria).

(iii) $\angle BAD = \angle CAD$ (proved in part i)

Hence, AE bisects $\angle A$.

also,

In ΔBED and ΔCED

$ED = ED$ (Common)

$BD = CD$ (Given)

$BE = CE$

($\Delta ABE \cong \Delta ACE$ so by c.p.c.t)

Therefore, $\Delta BED \cong \Delta CED$ (SSS criteria)

Thus,

$\angle BDE = \angle CDE$ (c.p.c.t)

Hence, we can say that AE bisects $\angle A$ as well as $\angle D$.

(iv) $\angle BED = \angle CED$

(by CPCT as $\Delta BED \cong \Delta CED$)

Therefore;

$BE = CE$ (c.p.c.t)

$\angle BED + \angle CED = 180°$ (BC is a straight line)

$\Rightarrow 2\angle BPD = 180°$

$\Rightarrow \angle BED = 90°$

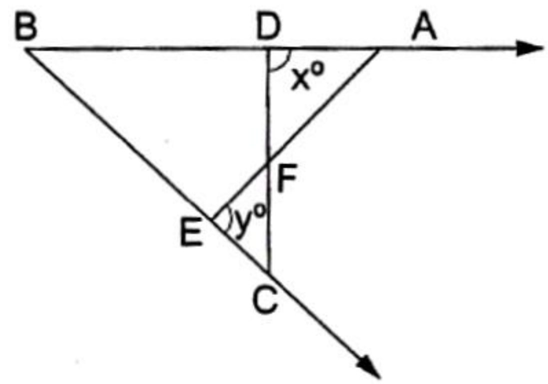
Hence, AE is the perpendicular bisector of BC.

Answer7)

<u>Given</u>: (i)x=y

(ii) AB=CB

<u>To prove</u>: AE = CD



<u>Proof</u>:

Consider the triangles AEB and CDB.

∠EBA=∠DBC∠EBA=∠DBC (Common angle) ...(i)

Further, we have:

∠BEA=180-y

∠BDC=180-x

Since x = y,

we have:

180-x =180-y

⇒∠BEA=∠BDC ...(ii)

AB = CB (Given) ...(iii)
From (i), (ii) and (iii),

we have:
△BDC≅△BEA (AAS criterion)
∴ AE = CD (CPCT)
Hence, proved.

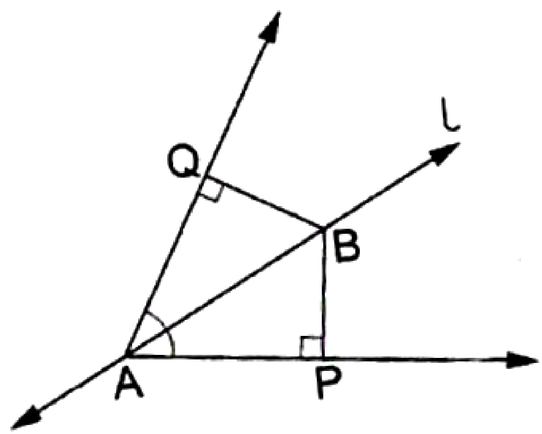
Answer8)

Given: (i) l is the bisector of an $\angle A$

(ii) BP and BQ are perpendiculars

To Prove: $\Delta APB \cong \Delta AQB$

Proof:



In ΔAPB and ΔAQB

$\angle P = \angle Q$ (Right angles)

$\angle BAP = \angle BAQ$ (l is the bisector)

$AB = AB$ (Common)

$\Delta APB \cong \Delta AQB$ (A.A.S criteria)

Hence, Proved.

(ii) $BP = BQ$ (By c.p.c.t)

Therefore,

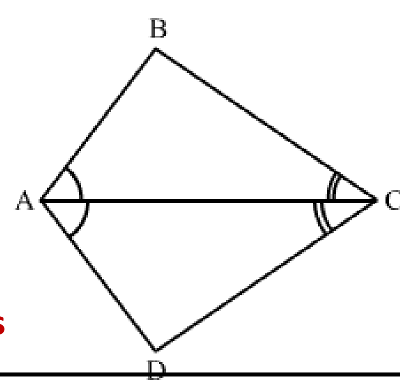
B is equidistant from the arms of $\angle A$

Answer9)

Given: AC bisects angles A and C.

To prove: (i) $AB = AD$

(ii) $CB = CD$



Proof:

Δ ABC and ΔADC,

we have:

$\angle$ CAB = $\angle$ CAD

$\angle$ BCA = $\angle$ DCA

AC = AC (common)

ΔABC≅ ΔADC

Therefore,

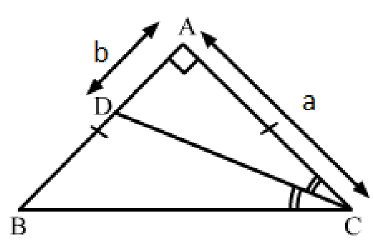
AD = AB (c.p.c.t)

CD = BC (c.p.c.t)

Answer10)Given: AB=AC

To prove: AC+AD=BC

Proof:



Let AB = AC = a and AD = b

In a right angled triangle ABC , $BC^2 = AB^2 + AC^2$

$BC^2 = a^2 + a^2$

$BC = a\sqrt{2}$

Given AD = b, we get

DB = AB – AD or DB = a – b

We have to prove that AC + AD = BC or $(a + b) = a\sqrt{2}$.

By the angle bisector theorem, we get

AD/ DB = AC / BC

$b/(a - b) = a/ a\sqrt{2}$

$b/(a - b) = 1/\sqrt{2}$

$b = (a - b)/ \sqrt{2}$

$b\sqrt{2} = a - b$

$b(1 + \sqrt{2}) = a$

$b = a/ (1 + \sqrt{2})$

Rationalizing the denominator with $(1 - \sqrt{2})$

$b = a(1 - \sqrt{2}) / (1 + \sqrt{2}) \times (1 - \sqrt{2})$

$b = a(1 - \sqrt{2})/ (-1)$

$b = a(\sqrt{2} - 1)$

$b = a\sqrt{2} - a$

$b + a = a\sqrt{2}$

or AD + AC = BC [we know that AC = a, AD = b and BC = $a\sqrt{2}$]

Hence it is proved.

Answer11)

Given: (i) OA=OB

(ii)OP=OQ

To Prove: (i) PX=QX

(ii)AX=BX

Proof:

In Δ PBO and ΔAOQ

OB=OA (Given)

OP=OQ (Given)

∠O=∠O (common)

Therefore;

Δ PBO ≅ΔQAO (S.A.S criteria)

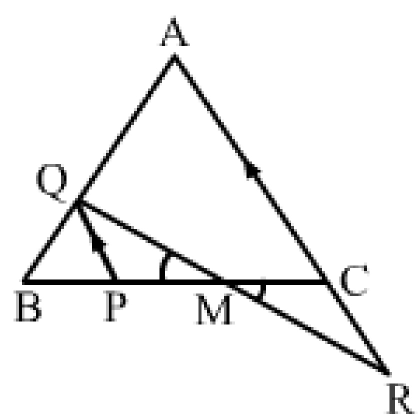
∠B=∠A (C.P.C.T)

In Δ BXQ and Δ AXP

∠B=∠A (proved above)

PX=QX (C.P.C.T.)

Hence proved



Answer12)

Given: (i) ABC is an equilateral triangle,

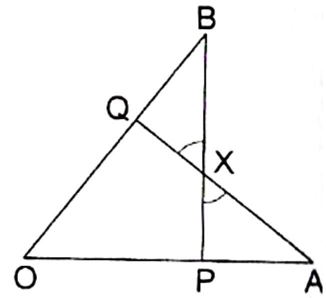
(ii) PQ ||AC

(iii) CR=BP

To prove: QR bisects PC or PM = MC

Proof:

Since, ΔABC is equilateral triangle,



$\angle A = \angle ACB = 60°$

Since, $PQ \parallel AC$ and corresponding angles are equal,

$\angle BPQ = \angle ACB = 60°$

In ΔBPQ,

$\angle B = \angle ACB = 60°$

$\angle BPQ = 60°$

Hence, ΔBPQ is an equilateral triangle.

$\therefore PQ = BP = BQ$

Since we have $BP = CR$,

We say that $PQ = CR$ ...(1)

Consider the ΔPMQ and ΔCMR,

$\angle PQM = \angle CRM$ (alternate angles)

$\angle PMQ = \angle CMR$ (vertically opposite angles)

$PQ = CR$ ... from 1

$\Delta PMQ \cong \Delta CMR$ (AAS criteria)

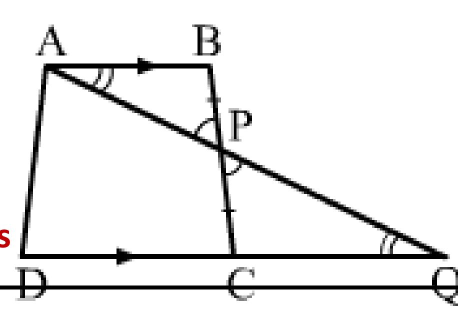
$\therefore PM = MC$ (c.p.c.t)

Hence proved.

Answer13)

Given: (i)AB ll DC

(ii) P is the midpoint of BC.



To Prove : (i) AB = CQ

(ii) DQ = DC + AB

so, AB ll DQ

so, $\angle BAQ = \angle DQA$ (alternate angles)

or $\angle BAP = \angle CQP$ ------(1)

Now, in triangle ABP and triangle QCP,

$\angle BAP = \angle CQP$ (from (1))

$\angle BPA = \angle CPQ$ (vertically opposite angles)

BP = CP (since P is the midpoint of BC)

so, triangle ABP congruent triangle QCP (by AAS congruency)

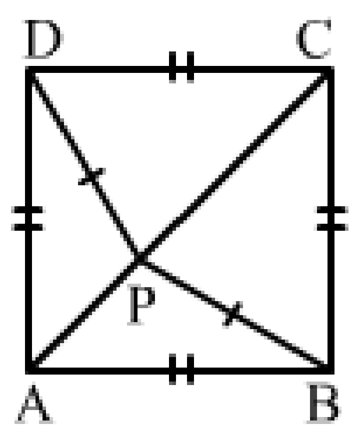
or AB = CQ (by CPCT) [proved] -----(2)

again, DQ = DC + CQ = DC + AB (from (2)) [proved]

Answer14)

Given: ABCD is a square and PB=PD

To prove: CPA is a straight line



Proof:

ΔAPD and ΔAPB,

DA = AB … (as ABCD is square)

AP = AP … (common side)

PB = PD … (Given)

$\Delta APD \cong \Delta APB$ (SSS criteria)

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle APD = \angle APB$ …(1)

Now consider ΔCPD and ΔCPB,

$CD = CB$ … ABCD is square

$CP = CP$ … common side

$PB = PD$ … Given

Thus by SSS property of congruence,

$\Delta CPD \cong \Delta CPB$

$\angle CPD = \angle CPB$ … (C.P.C.T.)………..(2)

Now,

Adding both sides of 1 and 2,

$\angle CPD + \angle APD = \angle APB + \angle CPB$ …(3)

Angels around the point P add upto $360°$

$\therefore \angle CPD + \angle APD + \angle APB + \angle CPB = 360°$ ……..(4)

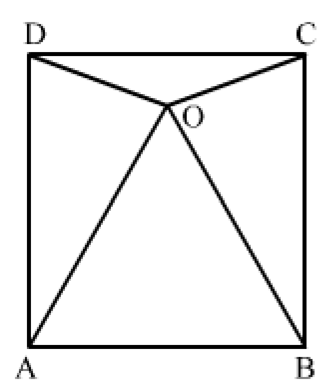
From 4,

$2(\angle CPD + \angle APD) = 360°$
$\angle CPD + \angle APD = 180°$

This proves that CPA is a straight line.

Answer15) **<u>Given:</u>** In square ABCD, ΔOAB is an equilateral triangle.

<u>To prove:</u> ΔOCD is an isosceles triangle.



<u>Proof:</u>

$\because \angle DAB = \angle CBA = 90°$ (Angles of square ABCD)

And, $\angle OAB = OBA = 60°$ (Angles of equilateral ΔOAB)

∴$\angle DAB - \angle OAB = \angle CBA - \angle OBA = 90° - 60°$

⇒$\angle OAD = \angle OBC = 30°$ .....(i)

∵$\angle DAB = \angle CBA = 90°$ Angles of square ABCD

And, $\angle OAB = OBA = 60°$

Angles of equilateral ΔOAB

∴$\angle DAB - \angle OAB = \angle CBA - \angle OBA = 90° - 60°$

⇒$\angle OAD = \angle OBC = 30°$ .....(i)

Now, in ΔDAO and ΔCBO,

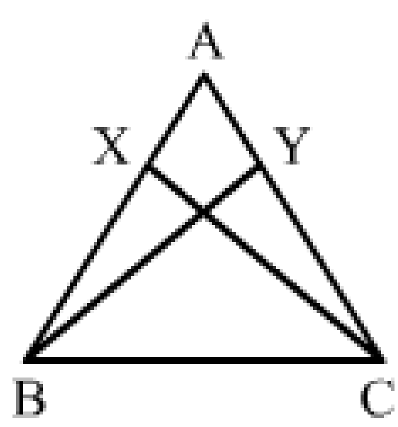
$AD = BC$ (Sides of square ABCD)
$\angle DAO = \angle CBO$ [From (i)]
$AO = BO$ (Sides of equilateral ΔOAB)

∴ By SAS congruence criteria,
$\Delta DAO \cong \Delta CBO$

So, $OD = OC$ (CPCT)
Hence, ΔOCD is an isosceles triangle.

Answer16)

Given: $AX = AY$.



To prove: $CX = BY$

Proof:

In ΔCXA and ΔBYA,

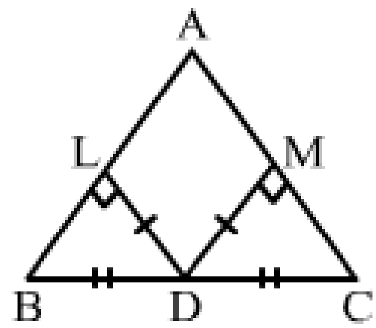
$AX = AY$ .....Given

$\angle XAC = \angle YAB$ ... common angle

AC = AB … Given,

ΔCXA ≅ ΔBYA   (S.A.S. criteria)

CX = BY   (C.P.C.T.)



Answer17)

<u>Given:</u> BD = DC and DL ⊥ AB and DM ⊥ AC such that DL=DM

<u>To prove:</u> AB = AC

<u>Proof:</u>

In right angled triangles ΔBLD and ΔCMD,

∠BLD = ∠CMD = 90°

BD = CD … Given

DL = DM … Given

Thus by right angled hypotenuse side property of congruence,

ΔBLD ≅ ΔCMD

Hence, we know that, corresponding parts of the congruent triangles are equal

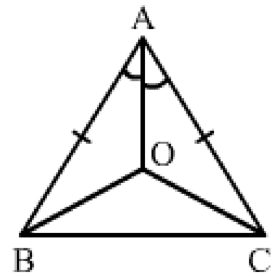
∠ABD = ∠ACD

In ΔABC, we have,

∠ABD = ∠ACD

∴ AB= AC …. Sides opposite to equal angles are equal

Answer18)



Given: In ΔABC, AB=AC and the bisectors of ∠B and ∠C meet at a point O.

To prove: BO=CO and ∠BAO = ∠CAO

Proof:

In , ΔABC we have,

∠OBC = $\frac{1}{2}$ ∠B

∠OCB = $\frac{1}{2}$ ∠C

But ∠B = ∠C ... Given

So, ∠OBC = ∠OCB

Since the base angles are equal, sides are equal

∴ OC = OB ...(1)

Since OB and OC are bisectors of angles ∠B and ∠C respectively, we have

∠ABO = $\frac{1}{2}$ ∠B

∠ACO = $\frac{1}{2}$ ∠C

∴∠ABO = ∠ACO ...(2)

Now in ΔABO and ΔACO

AB = AC ... Given

∠ABO = ∠ACO ... from (2)

BO = OC ... from (1)

Thus by SAS property of congruence,

ΔABO ≅ ΔACO

Hence, we know that, corresponding parts of the congruent triangles are equal

∠BAO = ∠CAO

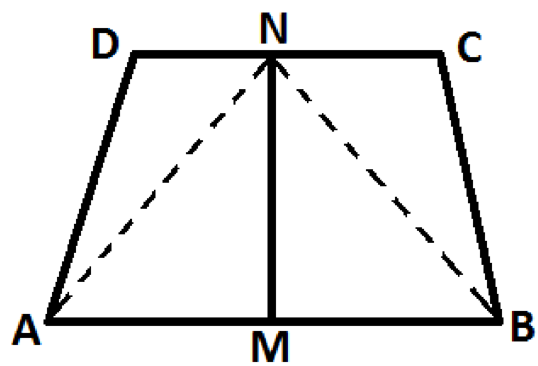
ie. AO bisects ∠A ; Hence proved.

Answer19)

<u>Given:</u> (i) ABCD is a trapezium

(ii) M is the mid point of AB

(iii) N is the mid point of CD



<u>To Prove:</u>AD = BC.

<u>Construction :</u> (i) Join B to N

(ii) Join A to N

<u>Proof :</u>

Consider ΔAMN and ΔBMN

∠AMN=∠BMN=90

AM=BM (M is the midpoint of AB)

MN=MN(common)

ΔAMN congruent to ΔBMN(SAS congruence rule)

Consider ΔADN and ΔBCN

DN=CN(N is the midpoint of CD)

AN=BN(CPCT)

∠MNA=∠BNM(CPCT) ....(1)

∠MNC=∠MND= 90 ....(2)

Subtracting Eq(2) from Eq(1)

∠MND-∠MNA=∠MNC-∠BNM

∠AND=∠BNC

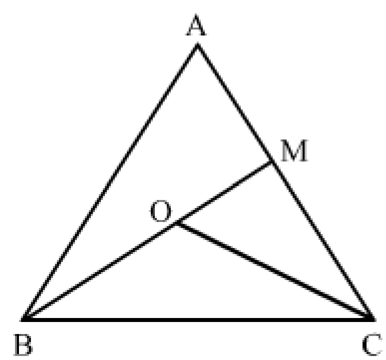
ΔAND congruent to ΔBNC

AD=BC(CPCT)

Hence proved

Answer20)

Given: Bisectors of the angles B and C of an isosceles triangle with AB = AC intersect each other at O.



To prove: ∠MOC = ∠ABC

Proof:

In ΔABC,

AB = AC (Given)

⇒ ∠ACB = ∠ABC (opposite angles to equal sides are equal)

1/2 ∠ACB = 1/2 ∠ABC (divide both sides by 2)

⇒ ∠OCB = ∠OBC ...(1) (As OB and OC are bisector of ∠B and ∠C)

Now, ∠MOC = ∠OBC + ∠OCB (as exterior angle is equal to sum of two opposite interior angle)

⇒∠MOC = ∠OBC + ∠OBC (from (1))

$\Rightarrow \angle MOC = 2\angle OBC$

$\Rightarrow \angle MOC = \angle ABC$ (because OB is bisector of $\angle B$)

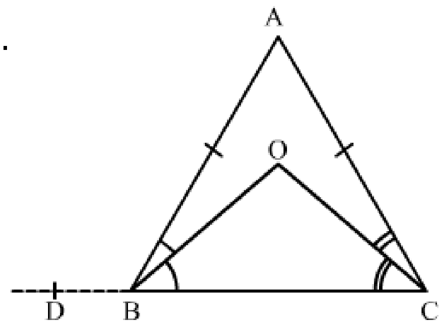
Hence proved.

Answer21)

Given: (i) In an isosceles ΔABC,

(ii) $AB = AC$,

(iii) BO and CO are the bisectors of $\angle ABC$ and $\angle ACB$.



To prove: $\angle ABD = \angle BOC$

Construction: Produce CB to point D.

Proof:

In ΔABC,

$\because AB = AC$ (Given)

$\therefore \angle ACB = \angle ABC$ (Angle opposite to equal sides are equal)

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \angle OCB = \angle OBC$ .....(i)

(Given, BO and CO are angle bisector of $\angle ABC$ and $\angle ACB$, respectively)

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABC$

$\Rightarrow \angle OCB = \angle OBC$ .....(i)

(Given, BO and CO are angle bisector of $\angle ABC$ and $\angle ACB$, respectively)

In ΔBOC,

$\angle OBC + \angle OCB + \angle BOC = 180°$ (By angle sum property of triangle)

$\Rightarrow \angle OBC + \angle OBC + \angle BOC = 180°$ [From (i)]

$\Rightarrow 2\angle OBC+\angle BOC=180°$

$\Rightarrow \angle ABC+\angle BOC=180°$ (BO is the angle bisector of $\angle ABC$) .....(ii)

$\angle OBC+\angle OCB+\angle BOC=180°$ By angle sum property of triangle

$\Rightarrow \angle OBC+\angle OBC+\angle BOC=180°$ From (i)

$\Rightarrow 2\angle OBC+\angle BOC=180°$

$\Rightarrow \angle ABC+\angle BOC=180°$ BO is the angle bisector of $\angle ABC$ .....(ii)

Also, DBC is a straight line.
So, $\angle ABC+\angle DBA=180°$ (Linear pair) .....(iii)

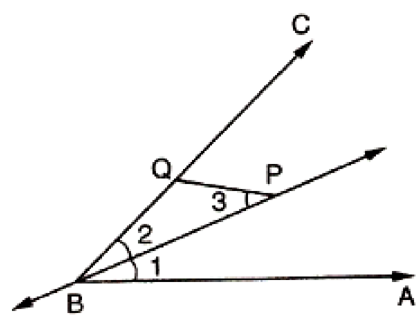
$\angle ABC+\angle DBA=180°$ (Linear pair) .....(iii)

From (ii) and (iii), we get
$\angle ABC+\angle BOC=\angle ABC+\angle DBA$

$\therefore \angle BOC=\angle DBA$

Answer22)

Given: P is the point on the bisector of an angle $\angle ABC$, and PQ || AB



To Proof: BPQ is isoscele

Since,

BP is the bisector of $\angle ABC = \angle ABP = \angle PBC$ (i)

Now,

PQ || AB

$\angle BPQ = \angle ABP$ (ii) [Alternate angles]

From (i) and (ii), we get

$\angle BPQ = \angle PBC$

Or,

$\angle BPQ = \angle PBQ$

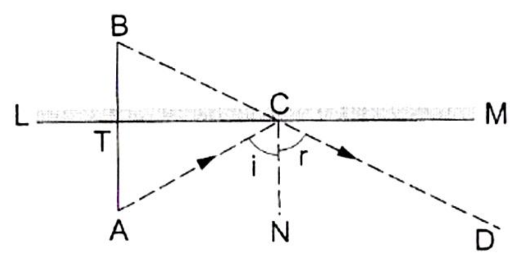
Now, in ΔBPQ

$\angle BPQ = \angle PBQ$

ΔBPQ is an isosceles triangle

Hence Proved.

Answer23) Given: A is an object in front of mirror LM,



B is the image of A and the observer is at D

AB intersects LM at T

To Prove: A and B are equidistant from LM

AT = BT

Construction: Join BD. Let it intersect LM at C

Join AC. CN be the normal at C.

Proof:

$\angle i = \angle r$ ...(1)

AB|| NC ...[Both are perpendicular to LM]

$\angle CAT = \angle CAN = \angle i$ ...(2)[Alternate angles]

$\angle CBA = \angle DCN = \angle r$ ...(3)[Corresponding angles]

From (1), (2) and (3), we get

$\angle CAT = \angle CBA$ ...(4)

In ΔCAT and ΔCBT,

$\angle CAT = \angle CBT$ ...[From (4)]

∠ATC = ∠BTC ...[Each 90°]

CT = CT ...[Common side]

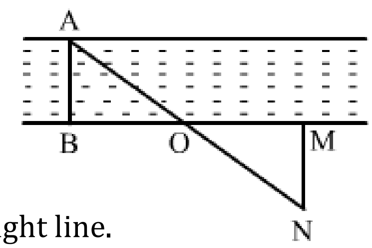
Therefore;

ΔCAT ≅ ΔCBT ...[AAA Criteria]

AT =BT ...[C.P.C.T]

Hence Proved.

Answer24)



Let AB be the breadth of the river.
M is any point situated on the bank of the river.
Let O be the mid point of BM.

Moving along perpendicular to point such that A , ,O and N are in straight line.

Then MN is the required breadth of the river.

In △OBA and △OMN,

we have:OB=OM (O is midpoint)

∠OBA=∠OMN (Each 90°)

∠AOB=∠NOM (Vertically opposite angle)

∴△OBA≅△OMN (ASA criterion)

In △OBA and △OMN,

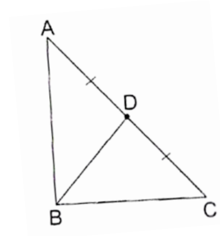
we have:OB=OM (O is midpoint)

∠OBA=∠OMN (Each 90°)

∠AOB=∠NOM (Vertically opposite angle)

∴△OBA≅△OMN (ASA criterion)
Thus, MN = AB (CPCT)
If MN is known, one can measure the width of the river without actually crossing it.

Answer25) Given: D is the midpoint of ac
BD = ½ AC



To Prove: $\angle ABC$ is 90°

In ΔADB, AD = BD

$\angle DAB = \angle DBA = \angle x$ (Opposite angles)

In ΔDCB, BD = CD

$\angle DBC = \angle DCB = \angle y$

In ΔABC we will use the angle sum property

$\angle ABC + \angle BCA + \angle CAB = 180°$

$2(\angle x + \angle y) = 180°$

$\angle x + \angle y = 90°$

$\angle ABC = 90°$

This meAnswer that ABC is the right angled triangle.

Answer26) No, because in the congruent rule, the two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle, that is, SAS criteria.

Answer 27) No,

Corresponding sides must be equal.

EXERCISE-9(B)

Answer1) (i) No, because the sum of two sides of a triangle is not greater than the third side.

$5 + 4 = 9$

(ii) Yes, because the sum of two sides of a triangle is greater than the third side.

$7 + 4 > 8; 8 + 7 > 4; 8 + 4 > 7$

(iii) Yes, because the sum of two sides of a triangle is greater than the third side.

$5 + 6 > 10; 10 + 6 > 5; 5 + 10 > 6$

(iv) Yes, because the sum of two sides of a triangle is greater than the third side.

$2.5 + 5 > 7; 5 + 7 > 2.5; 2.5 + 7 > 5$

(v) No, because the sum of two sides of a triangle is not greater than the third side.

$3 + 4 < 8$

Answer2) **Given:** In ΔABC, $\angle A = 50°$ and $\angle B = 60°$

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ (Angle sum property of a triangle)

$\Rightarrow 50° + 60° + \angle C = 180°$

$\Rightarrow 110° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 110°$

$\Rightarrow \angle C = 70°$

Hence, the longest side will be opposite to the largest angle ($\angle C = 70°$) i.e. AB.

And, the shortest side will be opposite to the smallest angle ($\angle A = 50°$) i.e. BC.

Answer3) (i) <u>**Given:**</u> In ΔABC, $\angle A = 90°$

So, sum of the other two angles in triangle $\angle B + \angle C = 90°$

i.e. $\angle B, \angle C < 90°$

Since, $\angle A$ is the greatest angle.

So, the longest side is BC.

(ii) Given: $\angle A = \angle B = 45°$

Using angle sum property of triangle,

$\angle C = 90°$

Since, $\angle C$ is the greatest angle.

So, the longest side is AB.

(iii) Given: $\angle A = 100°$ and $\angle C = 50°$

Using angle sum property of triangle,

$\angle B = 30°$

Since, $\angle A$ is the greatest angle.

So, the shortest side is BC.

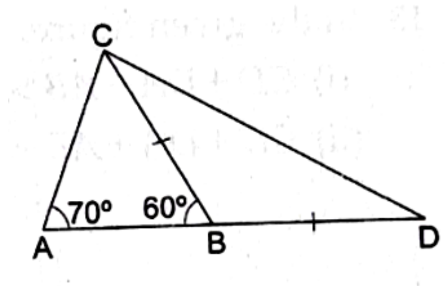
Answer4) <u>Given:</u> ΔABC, side AB is produced to D so that BD = BC and ∠B = 60°, ∠A = 70°

<u>To Prove:</u>

(i) AD > CD

And, (ii) AD > AC

<u>Proof:</u>



First join C and D

Now,

In ΔABC

∠A + ∠B + ∠C = 180° (Sum of all angles of triangle)

∠C = 180° – 70° – 60°= 50°

∠C = 50°
∠ACB = 50° (i)
And also in ΔBDC
∠DBC = 180° - ∠ABC (Therefore, ∠ABD is straight angle)
= 180° – 60° = 120°
BD = BC (Given)

∠BCD = ∠BDC (Therefore, angle opposite to equal sides are equal)

Now,

∠DBC + ∠BCD + ∠BDC = 180° (Sum of all sides of triangle)

120° + ∠BCD + ∠BCD = 180°

2∠BCD = 180° – 120°

2∠BCD = 60°

∠BCD = 30°

Therefore, ∠BCD = ∠BDC = 30° (ii)

Now, consider ΔBDC,

$\angle BAC = \angle DAC = 70^o$ (Given)

$\angle BDC = \angle ADC = 30^o$ [From (ii)]

$\angle ACD = \angle ACB + \angle BCD$

$= 50^o + 30^o$ [From (i) and (ii)]

$= 80^o$

Now,

$\angle ADC < \angle DAC < \angle ACD$

$AC < DC < AD$ (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

$AD > CD$

And,

$AD > AC$

Hence Proved.

We have,

$\angle ACD > \angle DAC$

And,

$\angle ACD > \angle ADC$

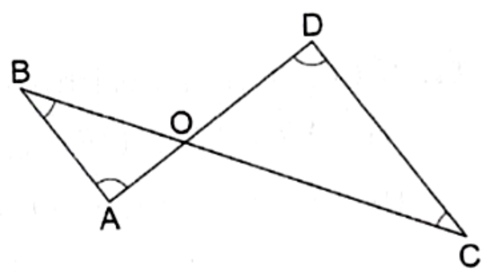
$AD > DC$

$AD > AC$ (Therefore, side opposite to greater angle is longer and smaller angle is smaller)

Answer5)<u>**GIVEN:**</u>

$\angle B < \angle A$

$\angle C < \angle D$



TO PROVE:

$AD < BC$

PROOF:

$\angle B < \angle A$

SO,

$OA < OB$ ...(1) (SIDE OPPOSITE TO SMALLER ANGLE IS SMALL)

NOW,

$\angle C < \angle D$

SO,

$OD < OC$ ...(2) (SIDE OPPOSITE TO SMALLER ANGLE IS SMALL)

NOW,

ADDING 1 AND 2

$OA + OD < OB + OC$

ADDING WE GET,

$AD < BC$

HENCE PROVED.

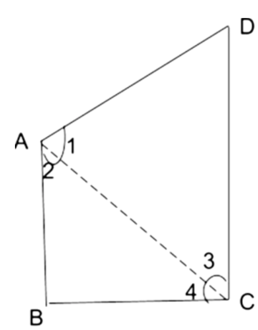
Answer6) Given:

In quadrilateral ABCD, AB smallest & CD is longest sides.

To Prove: $\angle A > \angle C$

& $\angle B > \angle D$

Construction: Join AC.



Mark the angles as shown in the figure..

Proof:

In $\triangle ABC$, AB is the shortest side.

$BC > AB$

$\angle 2 > \angle 4$ ...(i)

[Angle opposite to longer side is greater]

In $\triangle ADC$, CD is the longest side

$CD > AD$

$\angle 1 > \angle 3$ ...(ii)

[Angle opposite to longer side is greater]

Adding (i) and (ii), we have

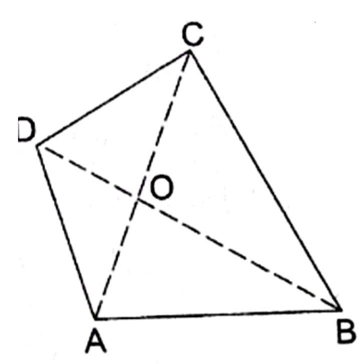
$\angle 2 + \angle 1 > \angle 4 + \angle 3$

$\Rightarrow \angle A > \angle C$

Similarly, by joining BD, we can prove that

$\angle B > \angle D$

Answer 7) **To Prove:** $(AB + BC + CD + DA) > (AC + BD)$



Proof:

ABCD is a quad.Its diagonals are AC and BD.

In triangle ACB, $AB + BC > AC$ ...(1)

In triangle BDC, $BC + CD > BD$ ...(2)

In triangle ACD, $AD + DC > AC$ ...(3)

In triangle BAD, $AB + AD > BD$ ...(4)

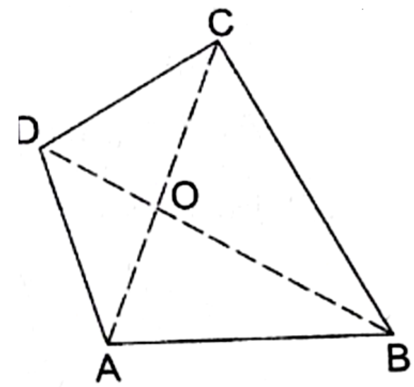
Adding 1,2,3 and 4,

AB + BC + BC + CD + AD + DC + AB + AD > AC + BD + AC + BD

2AB + 2BC + 2CD + 2AD > 2AC + 2BD

AB + BC + CD + AD > AC + BD.   HENCE PROVED.

Answer8) Since, the sum of lengths of any two sides in a triangle should be greater than the length of third side



Therefore, In Δ AOB, AB < OA + OB ……….(i)

In Δ BOC, BC < OB + OC ……….(ii)

In Δ COD, CD < OC + OD ……….(iii)

In Δ AOD, DA < OD + OA ……….(iv)

$\Rightarrow$ AB + BC + CD + DA< 2OA + 2OB + 2OC + 2OD

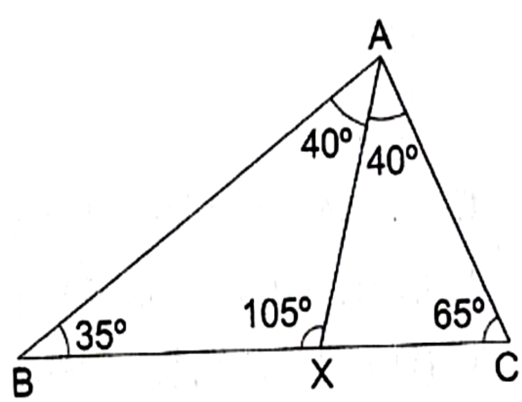
$\Rightarrow$ AB + BC + CD + DA< 2[(AO + OC) + (DO + OB)

$\Rightarrow$ AB + BC + CD + DA< 2(AC + BD)

Hence Proved.

Answer9) **Given:** In ΔABC, $\angle$B=35°,$\angle$C=65° and $\angle$BAX = $\angle$XAC

To find: Relation between AX, BX and CX in descending order.



In ΔABC, by the angle sum property, we have

$\angle A + \angle B + \angle C = 180°$

$\angle A + 35° + 65° = 180°$

$\angle A + 100° = 180°$

$\therefore \angle A = 80°$

But $\angle$BAX =$\angle$A= 40°

Now in ΔABX,

$\angle B = 35°$

$\angle BAX = 40°$

And $\angle BXA = 180° - 35° - 40°$

$= 105°$

So, in ΔABX,

$\angle B$ is smallest, so the side opposite is smallest, i.e. AX is smallest side.

$\therefore AX < BX$ ...(1)

Now consider ΔAXC,

$\angle CAX = \angle A = 40°$

$\angle AXC = 180° - 40° - 65°$

$= 180° - 105° = 75°$

Hence, in ΔAXC we have,

$\angle CAX = 40°, \angle C = 65°, \angle AXC = 75°$

$\therefore \angle CAX$ is smallest in ΔAXC

So the side opposite to $\angle CAX$ is shortest

i.e. CX is shortest

$\therefore CX < AX$ .... (2)

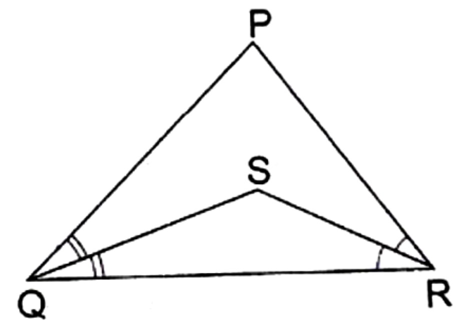
From 1 and 2 ,

$BX > AX > CX$

Answer10) **Given:** PQ>PR

QS and RS are bisector of ∠Q and ∠R Respectively

To Prove: SQ>SR



Proof:

∠ R >∠ Q (angle opposite to greater side is greater)

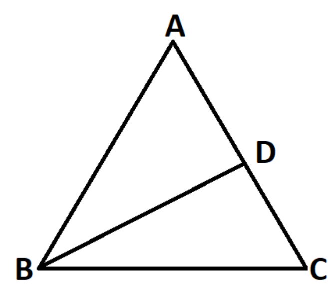
1/2*∠ R>1/2*∠Q

∠ SRQ>∠SQR

SQ>SR (Side opposite to greater angle is greater)

Answer11) **Given:** AB = AC

To prove: BD > CD



Proof :

Since AB = AC

∠ABC = ∠ACB (By Isosceles Triangle property) ----(i)

Here clearly,

∠ABC >∠CBD

∠ACB >∠CBD ---from (i)

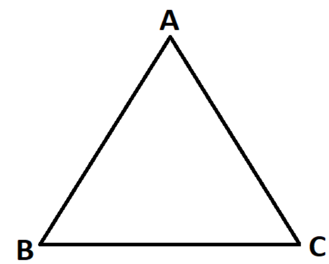
∠DCB >∠CBD

BD > CD (Angle opposite to greater side is greater in a triangle)

Hence Proved.

Answer12) Let ΔABC be a triangle in which AC is the longest side.

To prove: Angle opposite the longest side is greater than 2/3 of right angle.



Proof : $\angle B > \angle A$ ....(i)

And $\angle B > \angle C$ ....(ii)

Adding (i) and (ii), we get

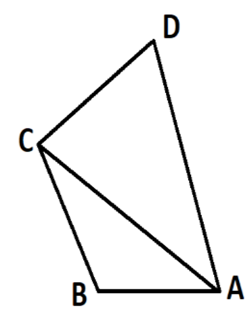
$\rightarrow \angle B + \angle B > \angle A + \angle C$

$\rightarrow 2\angle B > \angle A + \angle C$

$\rightarrow 2\angle B + \angle B > \angle A + \angle B + \angle C$

$\rightarrow 3\angle B > 180° \quad = \quad \angle B > 60°$

$\rightarrow \angle B >$ 2/3 x right amgle. [$60° = 2/3 \times 90°$]



Answer13)

(i)**To Prove :** CD + DA + AB > BC

Proof:

ΔABC, we have

CD + DA > AC

Add AB on both sides, we get

CD + DA +AB > AC + AB > BC

CD + DA + AB > BC

Hence proved.

(ii) **To Prove:** CD + DA + AB + BC > 2AC

Proof:

In ΔABC, we have

AB + BC > AC ...(1)

In ΔADC, we have

CD + DA > AC ...(2)

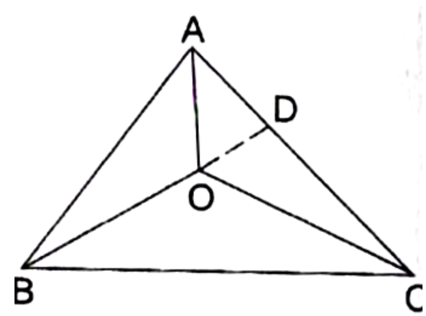
Adding (1) and (2), we get

AB + BC + CD + DA > AC + AC

CD + DA + AB + BC > 2 AC

Hence Proved.

Answer14)



Given:
In triangle ABC, O is any interior point.
We know that any segment from a point O inside a triangle to any vertex of the triangle cannot be longer than the two sides adjacent to the vertex.
Thus, OA cannot be longer than both AB and CA (if this is possible, then O is outside the triangle).

To Prove:

(i) AB + AC > OB+ OC
(ii) AB + BC + CA > OA + OB + OC
(iii) OA + OB + OC > ½ (AB + BC + CA)

Proof:

(i) OA cannot be longer than both AB and CA
AB>OB ...(1)

AC>OC ...(2)

Thus,

AB+AC>OB+OC ...[Adding (1) and(2)]

AB>OB ...(1)

$AC > OC$ ...(2)

Thus,

$AB+AC > OB+OC$ ...[Adding (1) and(2)]

(ii) $AB > OA$......(3)

$BC > OB$.....(4)

$CA > OC$.....(5)
Adding the above three equations, we get:

Thus, $AB+BC+CA > OA+OB+OC$ ...(6)

OA cannot be longer than both AB and CA.
$AB > OB$.....(5)

$AC > OC$.....(6)

$AB+AC > OB+OC$..........[On adding (5) and (6)]

Thus, the first equation to be proved is shown correct.

(iii) Now, consider the triangles OAC, OBA and OBC.
We have:
$OA+OC > AC$

$OA+OB > AB$

$OB+OC > BC$

Adding the above three equations, we get:

$OA+OC+OA+OB+OB+OC > AB+AC+BC$

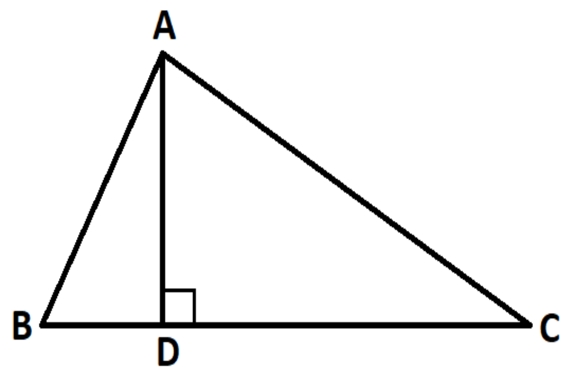
$\Rightarrow 2(OA+OB+OC) > AB+AC+BC$

Thus, $OA+OB+OC > 1/2(AB+BC+CA)$.

Answer15) <u>**Given**</u> : (i) AD⊥BC

(ii) CD > BD

<u>**To Prove:**</u> AC > AB



<u>**Proof**</u>:

In Δ ABD ;∠ABD + ∠BAD + ∠BDA = 180°

∠ABD + ∠BAD + 90°= 180°

∠ABD + ∠BAD = 90°

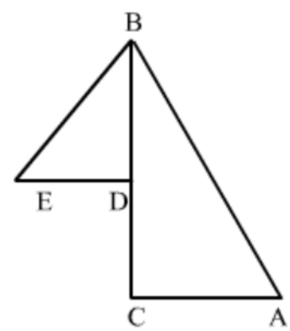
Similarly; In ΔADC ;∠ACB + ∠CAD = 90°

Since; BD <CD ;∠BAD <∠CAD

∠ABD >∠ACB

AC > AB

(sides opposite to greater angles are greater)



Answer16) <u>**Given:**</u> CD = DE

<u>**To prove:**</u> AB + AC > BE

<u>**Proof:**</u>

In ΔABC,

AB+AC>BC ...(1)

AB+AC>BC ...1

In ΔBED,

BD+CD>BE⇒BC>BE ...(2)

$BD+CD>BE \Rightarrow BC>BE$ ...2

From (1) and (2), we get

$AB + AC > BE$.

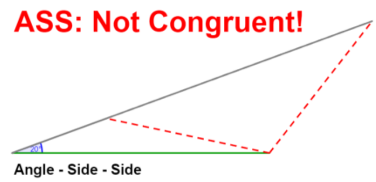
Hence Proved.

MULTIPLE CHOICE QUESTIONS

Answer1) (a)

SSA is not a criteria for congruency of triangles. SSA would mean for example, that in triangles ABC and DEF, angle A = angle D, AB = DE, and BC = EF.

With these assumptions it is *not* true that triangle ABC is congruent to triangle DEF. In general there are two sets of congruent triangles with the same SSA data.



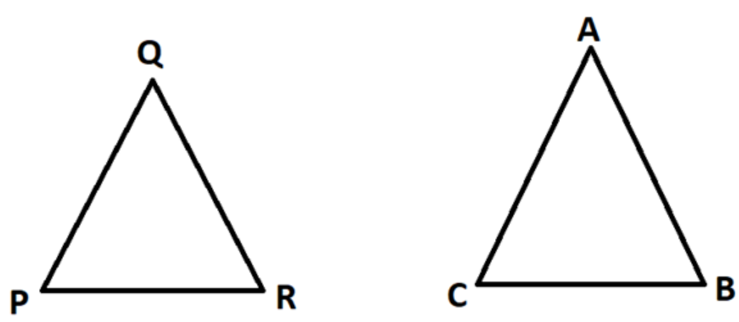
Answer2) (c)

Given: (i) AB = QR

(ii) BC = RP

(iii) CA = PQ

From the above given information following figures can be drawn.

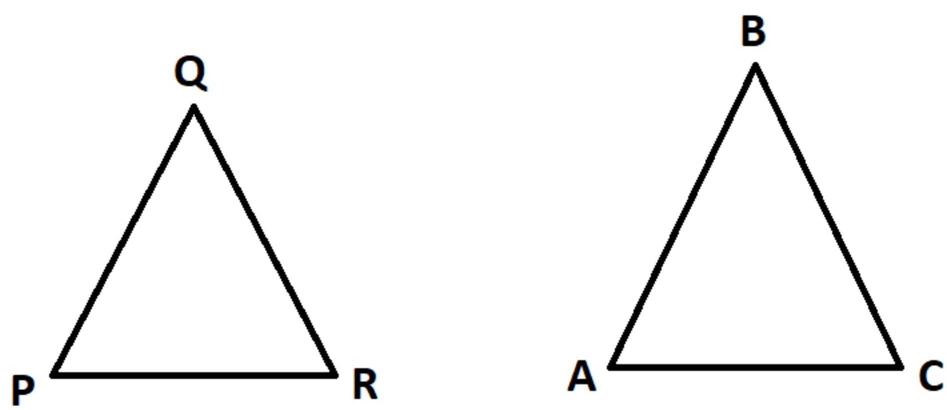


Hence, ΔPQR ≅ ΔCAB.

Answer3 (a)

Given : ΔABC ≅ ΔPQR

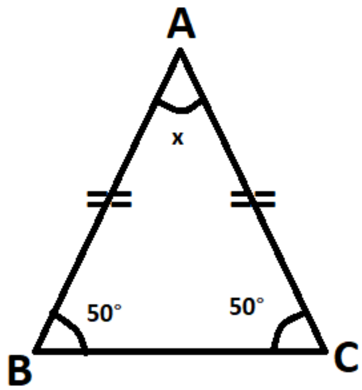
From the above given information following figures can be drawn.



If ΔABC ≅ ΔPQR, then BC=QR.
BC = RQ is not correct.

Answer4 (c)
Given : (i) AB = AC
(ii) ∠B=50°

From the above given information following figure can be drawn.



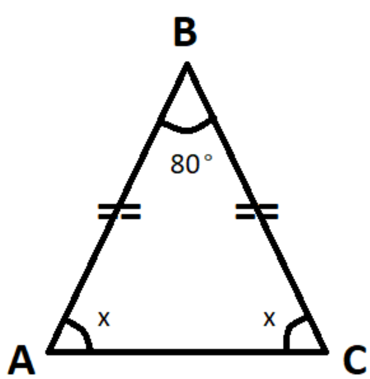
Since; Δ ABC is an isosceles triangle
Hence; ∠B=∠C=50°

∠A + ∠B + ∠C = 180°
∠x + 50° + 50° = 180°
∠x+100°=180°
∠x =180° - 100°
∠x = ∠A = 80°

Answer5) (a)

Given: (i) BC = AB
(ii) ∠B =80°

From the above given information following figure can be drawn.



Since; Δ ABC is an isosceles triangle
∠A = ∠C = ∠x

Hence; ∠A+∠B+∠C=180°

$\angle x + \angle x + 80° = 180°$

$2\angle x = 180° - 80°$

$\angle x = \frac{100°}{2}$

$\angle x = 50°$

$\angle x = \angle A = 50°$

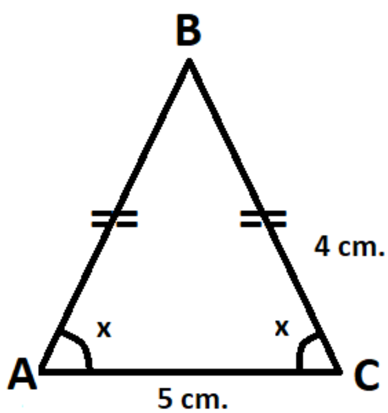
Answer6) **(a)**

Given: (i) $\angle A = \angle C$

(ii) $BC = 4$ cm

(iii) $AC = 5$ cm

From the above given information following figure can be drawn.



Since; $\angle A = \angle C$ are equal.

Hence; Δ ABC is an isosceles triangle.

So; $AB = BC = 4$ cm.

Answer7) **(b)**

Given: (i) side 1 = 4 cm.

(ii) side 2 = 2.5 cm.

Since; the sum of two sides in a triangle must be greater than the third side.

So; the third side should be less than the sum of the other two sides.

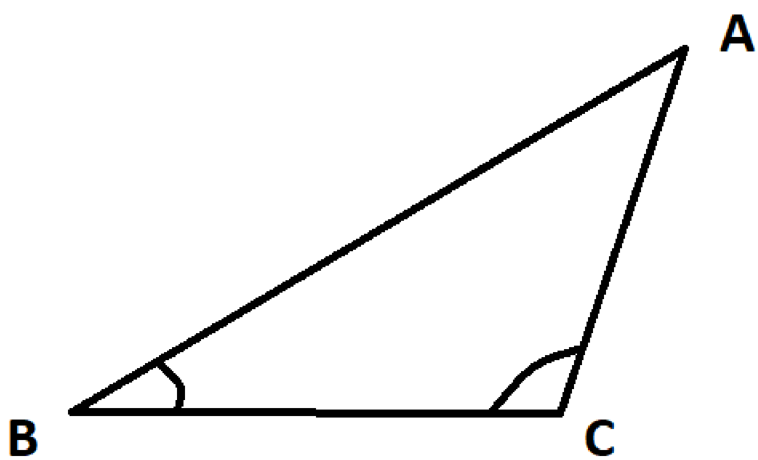
the third side should be < 4 cm $+ 2.5$ cm , i.e. 6.5 cm.

Hence; the third side cannot be 6.5 cm.

Answer 8) **(b)**

Given :$\angle C > \angle B$

From the above given information following figure can be drawn.



From the above figure it can be determined that

$AB > AC$

Answer9) **(b)**

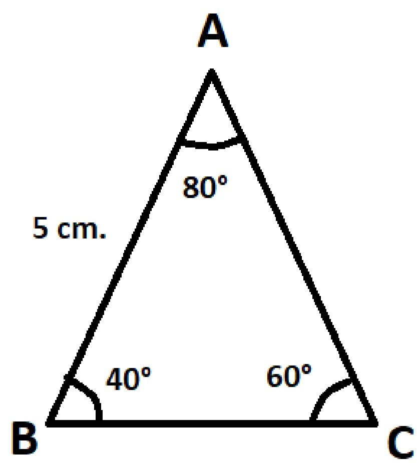
Given: (i) Δ ABC $\cong \Delta$ FDE

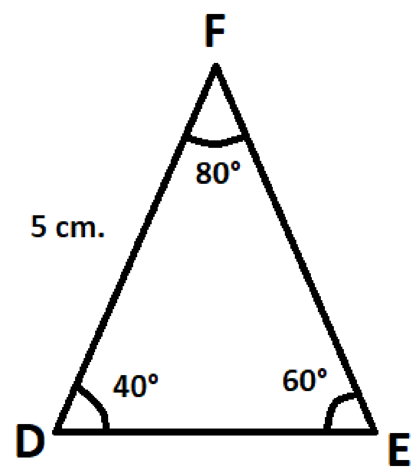
(ii) $AB = 5$ cm.

(iii) $\angle A = 80°$

(iv) $\angle B = 40°$

From the above given information following figures can be drawn .





$\angle A + \angle B + \angle C = 180°$

$80° + 40° + \angle C = 180°$

$\angle C = 180° - 120°$

$\angle C = 60°$

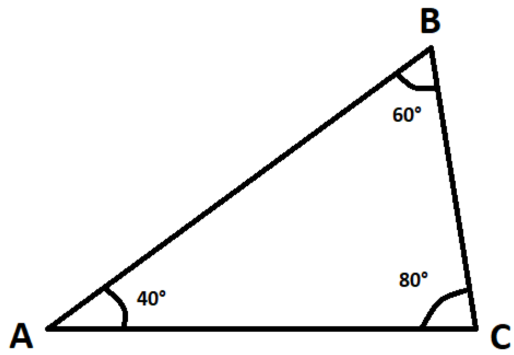
Corresponding angle the Δ FDE is $\angle E = 60°$.

Answer10) (c)

Given : (i) $\angle A = 40°$

(ii) $\angle B = 60°$

From the above given information following figure can be drawn.



$\angle A + \angle B + \angle C = 180°$

$40° + 60° + \angle C = 180°$

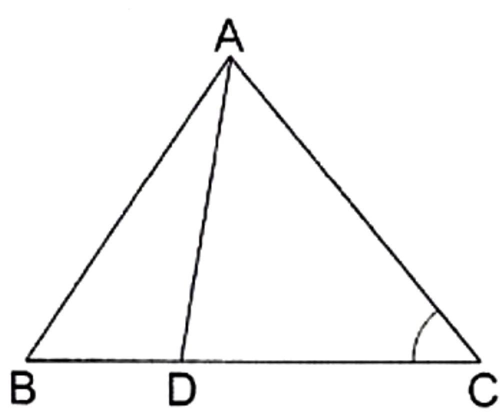
$\angle C = 180° - 100°$

$\angle C = 80°$

Since; side opposite of greater angle is greater. So, the side AB is greatest.

Answer11 (c)

Given: $AB > AC$



We know that the angle opposite to the larger side is larger.

So, $AB > AC = \angle ACB > \angle ABC$

$= \angle ACD > \angle ABD$ . .....(i)

Again side CD of Δ ACD has been produced to B.

So, ext. $\angle ADB > \angle ACD$.

From (i) and (ii), we get

$\angle ADB > \angle ACD > \angle ABD$

$\angle ADB > \angle ABD$

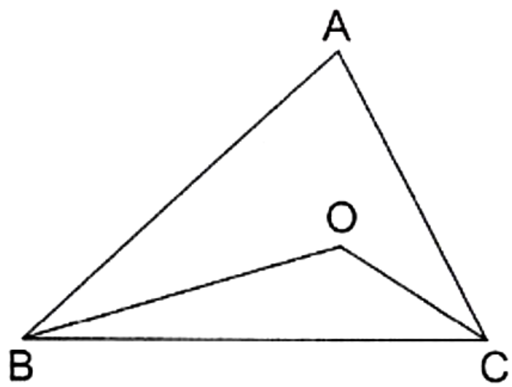
$AB > AD$ (side opposite of greater angle is greater).

Hence, $AB > AD$.

Answer12) (b)

Given: (i) $AB > AC$.

(ii) BO and CO are the bisectors of $\angle B$ and $\angle C$.



$\angle ACB > \angle ABC$

Angles opposite to greater sides are greater.

$\frac{\angle ACB}{2} > \frac{\angle ABC}{2}$

$\angle OCB > \angle OBC$

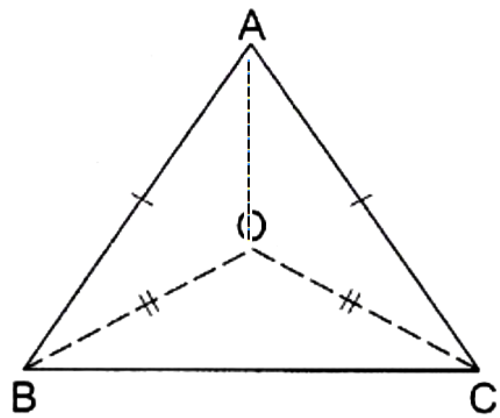
Hence, $OB > OC$.

Answer13) (a)

Given: (i) $AB = AC$

(ii) OB = OC.

It is given in the question that,



In ΔOAB and ΔOAC, we have

$AB = AC$

$OB = OC$

$OA = OA$ (Common)

$\therefore$ By SSS congruence criterion

$\Delta OAB \cong \Delta OAC$

$\therefore \angle ABO = \angle ACO$

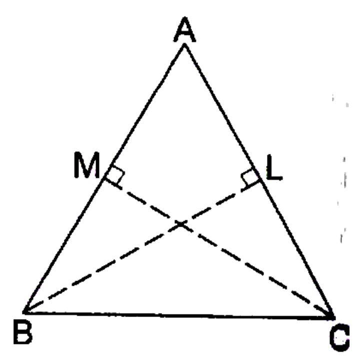
So, $\angle ABO : \angle ACO = 1:1$

Answer14) (b)

Given: (i) $BL \perp AC$

(ii) $CM \perp AB$

(iii) $BL = CM$.



In ΔABL and ΔACM, we have

$BL = CM$ (given)

$\angle BAL = \angle CAM$ (common)

$\angle ALB = \angle AMC$ (each $90°$)

$\Delta ABL \cong \Delta ACM$ and hence $AB = AC$.

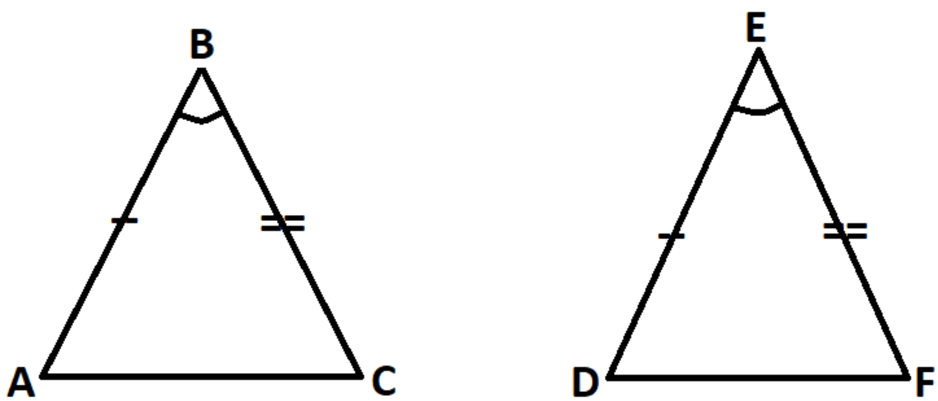
ΔABC is isosceles.

Answer15) **<u>(b)</u>**

<u>Given :</u> (i) AB = DE

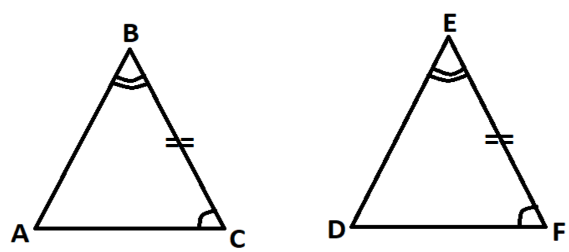
(ii) BC = EF

From the above given information following figures can be drawn.



For $\Delta ABC \cong \Delta DEF$, $\angle B$ should be equal to $\angle E$.
Hence if $\angle B = \angle E$, then $\Delta ABC \cong \Delta DEF$ by S.A.S. criterion.

Answer16) **<u>(c)</u>**
<u>Given:</u> (i) $\angle B = \angle E$
(ii) $\angle C = \angle F$



From the above given information following figures can be drawn.

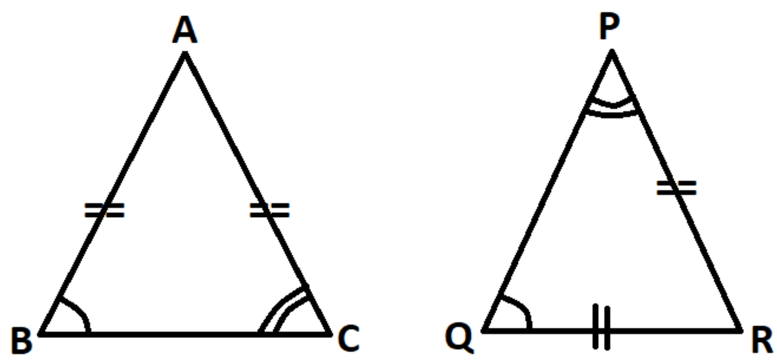
For $\Delta ABC \cong \Delta DEF$, BC should be equal to EF.
Hence if BC = EF ,then $\Delta ABC \cong \Delta DEF$ by A.S.A. criterion.

Answer17) **(a)**

Given: (i) AB = AC

(ii) ∠C = ∠P

(iii) ∠B = ∠Q



AB = AC

∠C = ∠B

∠P = ∠Q (since ∠C =∠P and ∠B = ∠Q)

QR = PQ.

Thus both the triangles are isosceles but not congruent.

Answer18) **(c)**

Two right angles would up to 180º so the third angle becomes zero. This is not possible. Therefore, the triangle cannot have two right angles.A triangle can't have two obtuse angles as obtuse angle meAnswer more than 90º. So, the sum of the two sides exceeds more than 180º which is not possible. As the sum of all three angles of a triangle is 180º.A triangle can have two acute angle as acute angle meAnswer less than 90º.And ***External angle*** of ***triangle*** is greater ***than either opposite angles***.

Answer19) a) (Sum of any sides of a triangle) **greater than(>)** (third side).

b) (Difference of any two sides of a triangle) **less than(<)** (third side).

c) (Sum of three altitudes of a triangle)**less than(<)**(sum of three sides).

d) (Sum of any two sides of a triangle) **greater than(>)** (twice the median to the third side).

e) (Perimeter of a triangle) **greater than(>)** (sum of its three median).

Answer20) a) Each angle of an equilateral triangle measures **60°**.

b) MediAnswer of an equilateral triangle are **equal**.

c) In a right angle triangle, the hypotenuse is the**longest**side.

d) Drawing a ΔABC with $AB = 3$ cm., $BC = 4$ cm. and $CA = 7$ cm. is **not possible**.